MATH. SCAND. 2 (1954), 193–194

REMARKS TO THE PAPER: ON THE FLUCTUATIONS OF SUMS OF RANDOM VARIABLES

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1. In the review of the above mentioned paper (Math. Scand. 1 (1953), 263–285) in Mathematical Reviews 15 (1954), 444, K. L. Chung has pointed out a misprint on page 267 and a mistake in the formulation of the argument in the first lines of page 268.

On p. 267, l. 18 read

$$\bigcup_{j=0}^{i-1} \bigcup_{k=i}^{n} \quad \text{for} \quad \bigcap_{j=0}^{i-1} \bigcap_{k=i}^{n} .$$

On p. 268 the first nine lines should be replaced by the following:

$$(*) \qquad B = \bigcap_{h=1}^{n} [S_h \leqq 0]\,[S_i = 0]\,[S_{n+1} = 0]$$
$$\subseteq \bigcup_{m=0}^{i-1} [L_n = 0]\,[S_{m+1} = 0] \bigcap_{h=1}^{m} [S_h < 0]\,[S_{n+1} = 0] .$$

From (3.3) on p. 266 and (*), it follows that

$$(**) \quad \Pr\{BC\} \leqq \sum_{m=0}^{i-1} \Pr\{[L_n = 0]\,[S_{m+1} = 0] \bigcap_{h=1}^{m} [S_h < 0]\,[S_{n+1} = 0]\, C\}$$
$$= \sum_{m=0}^{i-1} \big(\Pr\{[L_n = m]\,[S_{n+1} = 0]\, C\} - \Pr\{[L_n = m+1]\,[S_{n+1} = 0]\, C\}\big)$$
$$= \Pr\{[L_n = 0]\,[S_{n+1} = 0]\, C\} - \Pr\{[L_n = i]\,[S_{n+1} = 0]\, C\} .$$

Since we have assumed that (3.1) holds, it follows from (**) that $\Pr\{BC\} = 0$. From Lemma 1 it then follows that $\Pr\{AC\} = 0$. This completes the proof of Theorem 2.

2. On p. 269 the proof of Theorem 3 is incomplete. As it stands, the proof is valid only if the event C can be defined by means of

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$X_1^{(n)}, \ldots, X_{n+1}^{(n)}$, i. e. if the event C has the property that the points $(x_1+t, \ldots, x_{n+1}+t)$ belong to C, for any t, if $(x_1, \ldots, x_{n+1})$ belong to C. We can complete the proof as follows:

Since the proof, as it stands, is correct when $C = E$, it follows that Theorem 3 is true for $C = E$ and any probability field $\Pr\{A\}$ in which $X_1, \ldots, X_{n+1}$ are symmetrically dependent. The conditional probabilities $\Pr\{A|C\}$ with an arbitrary C subject only to the condition $\Pr\{C\} > 0$, define a new probability field. The random variables $X_1, \ldots, X_{n+1}$ are symmetrically dependent also with respect to this new probability field when C is symmetric with respect to $X_1, \ldots, X_{n+1}$. Therefore Theorem 3 holds for this new probability field. Thus it holds in general.

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