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EXPANSIVE AUTOMORPHISMS IN COMPACT GROUPS

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An automorphism θ of a topological group G is expansive iff there is neighborhood V of the identity such that for any two distinct elements $x,y \in G$, there is an integer n with $\theta^n(xy^{-1}) \notin V$. For the notions of expansive homeomorphism on topological space, see [1], [2].

In [1], M. Eisenberg proved that when G is a compact connected Lie group and G admits an expansive automorphism, then G is abelian. He also shows the existence of expansive automorphism on n-dimensional torus group for all positive integer $n > 1$. In this note, we shal prove the following theorem.

THEOREM. *If F is a compact connected finite dimensional topological group and G admits an expansive automorphisms, then G is abelian.*

PROOF. Let G be a compact connected finite dimensional topological group. It is known that G is isomorphic to $(L \times H)/D$, where L is a compact simply connected semi-simple Lie group, H is compact connected abelian group, and D is a finite normal subgroup of the direct product $L \times H$. (Cf. [4, Example 107]). Since D is finite,

$$\frac{LD}{D} \approx \frac{L}{L \cap D}$$

is a compact connected semi-simple Lie group. (Accurately, we should write $(L \times \{\mu\})D$ for LD, where μ is the identity of H.) Let θ be an automorphism of $(L \times H)/D$. Then $(LD)/D$ is invariant under θ. This can be seen by the following diagram:

$$\begin{array}{ccc} L \times H & \xrightarrow{\quad \pi \quad} & H \\ \varphi \big\downarrow & & \big\downarrow \psi \\ (L \times H)/D & \xrightarrow[F]{\quad\quad} & H/\pi(D) \end{array}$$

Here, π is the projection, φ and ψ are the quotient maps, F is defined by

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$F(\varphi(l,h)) = \psi(h)$. One could show that F is well defined and F is a continuous homomorphism. Since $\theta((LD)/D)$ is also a compact connected semi-simple Lie group and $H/\pi(D)$ is abelian, it follows that $F\theta((LD)/D) = \pi(D) \in H/\pi(D)$. Since $\ker F = (LD)/D$ we get

$$\theta((LD)/D) \subseteqq (LD)/D .$$

Hence θ induces an automorphism on $(LD)/D$. Now, by a theorem in [3], the bi-continuous automorphism group $A((LD)/D)$ of $(LD)/D$ is compact when $A((LD/D))$ is topological by compact open topology. So the natural action of $A((LD)/D)$ on $(LD)/D$ is equicontinuous, a fortiori, no automorphism on $(LD)/D$ can be expansive unless $(LD)/D$ is degenerate. Thus, we can conclude if θ is an expansive automorphism on G, then $(LD)/D$ is degenerate, and G is abelian. The proof is complete.

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