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A REMARK ON RINGS WITH PRIMARY IDEALS AS MAXIMAL IDEALS

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M. Satyanarayana in [1] defines a P-ring to be a commutative ring with identity in which every primary ideal is a maximal ideal. Below is a characterisation of such rings.

THEOREM. *A commutative ring R with identity is a P-ring if and only if R is regular (in the von Neumann sense).*

PROOF. Let R be a P-ring and M any maximal ideal of R. The quotient ring R_M has a unique prime ideal M^e other than R_M ([2, Theorem 19, Chapter IV]). Let $I \neq R_M$ be any ideal of R_M. This is a primary ideal; so I^c (see [2] for notation) is a primary and hence a maximal ideal of R. Consequently $I^c = M$ whence $I = M^e$. In particular $(0) = M^e$; so that R_M is a field for every maximal ideal M of R. Clearly every invertible element of R is regular. So let r be any non-invertible element of R and $T = \{t \in R : rt = 0\}$. This is an ideal of R which is not contained in any maximal ideal M, which contains r, as R_M is a field. So $T + (r) = R$ which implies $1 = t + rx$ where $t \in T$ and $x \in R$. This gives $r = r^2 x$ and thus R is a regular ring. The other part is immediate.

REFERENCES

1. M. Satyanarayana, *Rings with primary ideals as maximal ideals*, Math. Scand. 20 (1967), 52–54.
2. O. Zariski and P. Samuel, *Commutative Algebra* I, D. van Nostrand Co., Princeton, New Jersey, 1958.

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