MATH. SCAND. 32 (1973), 22–30

LATTICES AND MODELS OF FIELDS OF GENUS 0

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0. Introduction.

Let A be a Dedekind ring of characteristic $\neq 2$, F the field of fractions of A and E a finitely generated regular extension of F of transcendence degree 1 and of genus 0. We say that an S-scheme $\alpha : M \to S$ where $S = \mathrm{Spec}(A)$, is a model of E/F if α is proper and dominant, the ring $R(M)$ of rational functions on M is E and the homomorphism $\alpha^* : R(S) \to R(M)$ induced by α is an inclusion of $F(=R(S))$ into $E(=R(M))$. We say that a model M of E/F is relatively minimal if M is regular and for every regular model M' of E/F every S-morphism $M \to M'$ is an isomorphism.

A. Białynicki-Birula has proved ([1], [2]) that if A is a discrete valuation ring and the residue field of A is perfect then for every regular extension E/F of genus 0 there is a quadratic form $q(X_0, X_1, X_2)$ in $A[X_0, X_1, X_2]$ such that the S-scheme $\mathrm{Proj}(A[X_0, X_1, X_2]/(q))$ is a relatively minimal model of E/F. As a global equivalent of this theorem we shall prove that if A is a Dedekind ring with perfect residue fields then for every regular extension E/F of genus 0 there is a regular quadratic space (V, Q) over F and a lattice L on V (over A) such that L defines a relatitively minimal model of E/F (Theorem 2). Locally the model which corresponds to L is described by a quadratic form as above. Moreover we shall prove that the elements of the set of the isomorphism classes of relatively minimal models of E/F are in one to correspondence with the classes of lattices contained in the proper genus of L (Theorem 3). Thus the paper presents global equivalents of some local results contained in [1], [2], [3]. These local results play a very essential role here. On the other hand the paper clarify and generalize somewhat unnatural results of [5] and [6].

1. Schemes associated with lattices.

Let A be a Dedekind ring of characteristic $\neq 2$, F the field of fractions of A, (V, Q) a regular quadratic space over F (see[9, § 42 B]) and L a lattice on V (see[9, § 81 A]). We shall denote by $\tilde{A}$ the structure sheaf on $\mathrm{Spec}(A)$ and by $\tilde{M}$ or $M^{\sim}$ the $\tilde{A}$-Module defined by an A-module M. If V is a linear space over F (in particular $V = F$) and M is an A-module

Received April 25, 1972.

contained in V, then we shall assume that all localizations of M with respect to all prime ideals in A are contained in V. Hence, if U is an open subset in $\mathrm{Spec}(A)$, then the sheaves $\tilde{A}$, $\tilde{L}$, $\tilde{\mathfrak{a}}$ ($\mathfrak{a}$ an arbitrary fractional ideal in F) can be defined as follows:

$$\tilde{A}(U) = \bigcap\nolimits_{x\in U} A_x, \quad \tilde{L}(U) = \bigcap\nolimits_{x\in U} L_x, \quad \tilde{\mathfrak{a}}(U) = \bigcap\nolimits_{x\in U} \mathfrak{a}_x$$

with evident restrictions for $U' \subseteq U$, U' open in $\mathrm{Spec}(A)$.

For every $x \in \mathrm{Spec}(A)$ we can choose an open neighbourhood U such that $\tilde{L}(U)$ and $(\mathfrak{s}L)^{\sim}(U)$ are free $\tilde{A}(U)$-modules, where $\mathfrak{s}L$ denotes the scale of L (see [9, § 82 E]).

Let $(\mathfrak{s}L)^{\sim}(U) = (s_U)$ and let q_U be a quadratic form which corresponds to the quadratic mapping $(1/s_U)Q$ in some base of $\tilde{L}(U)$ over $\tilde{A}(U)$ (see [9, § 41 C]). The form q_U belongs to a polynomial ring

$$\tilde{A}(U)[X_0, X_1, \ldots, X_n] = \tilde{A}(U)[X] \quad (n = \dim V - 1)$$

and the ideal (q_U) does not depend on a choice of s_U. Let

$$M_U(L) = \mathrm{Proj}\big(\tilde{A}(U)[X]/(q_U)\big) \tag{1}$$

and let $\alpha_U : M_U(L) \to \mathrm{Spec}(\tilde{A}(U))$ be defined by a natural injection $\tilde{A}(U) \to \tilde{A}(U)[X]/(q_U)$.

The open sets U form a covering of $\mathrm{Spec}(A)$ and for every two sets U_1, U_2 there is an $\tilde{A}(U_1 \cap U_2)$-isomorphism $\varphi_{U_1U_2}$ of $\tilde{A}(U_1 \cap U_2)[X]/(q_{U_1})$ onto $\tilde{A}(U_1 \cap U_2)[X]/(q_{U_2})$ induced by an equivalence of forms q_{U_1} and q_{U_2} over $\tilde{A}(U_1 \cap U_2)$. Since $\varphi_{UU} = \mathrm{id}$ and $\varphi_{U_1U_2} \circ \varphi_{U_2U_3} = \varphi_{U_1U_3}$ we can patch together all maps

$$\alpha_U : M_U(L) \to \mathrm{Spec}(\tilde{A}(U))$$

(see [7, § 3]). The $\mathrm{Spec}(A)$-scheme obtained in this way we shall denote by

$$\alpha_L : M(L) \to \mathrm{Spec}(A)$$

Remark. $M(\mathfrak{a}L) = M(L)$ for an arbitrary fractional ideal $\mathfrak{a}$ in F and $M(L^{\alpha}) = M(L)$ (see [9, § 82 J]) for an arbitrary element $\alpha \in F^*$. Since $\mathfrak{s}(\mathfrak{a}L) = \mathfrak{a}^2\mathfrak{s}L$ (see [9, § 82 F]) and $\mathfrak{s}L^{\alpha} = \alpha\mathfrak{s}L$ (see [9, § 82 J]) we can always choose L' such that $M(L') = M(L)$ and $\mathfrak{s}L'$ is a given fractional ideal with the image in $\mathrm{Cl}(A)/\mathrm{Cl}(A)^2$ equal to the image of $\mathfrak{s}L$ ($\mathrm{Cl}(A)$ denotes the ideal class group of A).

Definition. If L, L' are lattices on quadratic spaces (V, Q), (V', Q') respectively, then a linear mapping $f: V \to V'$ is a similarity of L onto L' if $f|L$ maps L onto L' and $Q'(f(v)) = a_f Q(v)$ where $a_f \in A^*$. Two similarities

f, f' of L onto L' are equivalent if there is $a \in A^*$ such that $f' = af$. The set of equivalence classes of this relation we shall denote by $\mathrm{Bir}(L, L')$ ("Bir" means birational or biregular; see Proposition 1 and Theorem 1). If $V' = V$, $Q' = Q$ and $L' = L$ then we shall write $\mathrm{Bir}(L)$. $\mathrm{Bir}(L)$ is a group.

We shall apply this definition also in the special case $A = F$, $L = V$, $L' = V'$.

From now on we shall consider only the quadratic spaces of dimension 3 (except Propositions 2 and 3).

PROPOSITION 1. *If L, L' are lattices on regular quadratic spaces (V, Q), (V', Q') respectively, then there is a one to one correspondence between the birational S-maps of $M(L)$ into $M(L')$ and the elements of* $\mathrm{Bir}(V, V')$. *If $V' = V$, $Q' = Q$* and *$L' = L$ then this correspondence is an isomorphism of groups.*

PROOF. There is a one to one correspondence between the birational S-maps of $M(L)$ into $M(L')$ and the F-isomorphisms of the ring of rational functions $R(M(L'))$ onto the ring $R(M(L))$. This is an isomorphisms of groups if $M(L') = M(L)$.

If q, q' are quadratic forms corresponding to (V, Q), (V', Q') respectively, then

$$R(M(L)) = (F[X_0, X_1, X_2]/(q))_0^0$$

(the field of fractions of degree 0 in the field of fractions of $F[X_0, X_1, X_2]/(q)$) and analogously for $R(M(L'))$. If $f: V \to V'$ is a similarity then f induces a linear automorphism φ of $F[X_0, X_1, X_2]$ such that $\varphi(q') = a_f q$, hence an isomorphism of $R(M(L'))$ onto $R(M(L))$. It is easy to see that two similarities give the same isomorphism of fields if and only if they are equivalent. Hence we have an injection of $\mathrm{Bir}(V, V')$ in the set of F-isomorphisms of $R(M(L'))$ onto $R(M(L))$. This is also a surjection since every F-isomorphism of $(F[X_0, X_1, X_2]/(q'))_0^0$ onto $(F[X_0, X_1, X_2]/(q))_0^0$ is induced by a linear automorphism φ of $F[X_0, X_1, X_2]$ such that $\varphi(q') = aq$.

THEOREM 1. *Let L, L' be lattices on a regular quadratic space (V, Q), such that the S-schemes $M(L)$, $M(L')$ are regular. Then*

(a) *if $M(L)$, $M(L')$ are S-isomorphic then the scales $\mathfrak{s}L$, $\mathfrak{s}L'$ define the same element in* $\mathrm{Cl}(A)/\mathrm{Cl}(A)^2$,

(b) *if $\mathfrak{s}L = \mathfrak{s}L'$ then there is a one to one correspondence between the S-isomorphisms of $M(L)$ onto $M(L')$ and the elements of* $\mathrm{Bir}(L, L')$. *If $L' = L$ then this correspondence is an isomorphism of groups.*

Remark. The assumption that $M(L)$, $M(L')$ are regular puts some restrictions on L, L'. These restrictions can be obtained e.g. from Theorem 2 in [3].

Proof. We shall prove (a) and (b) at the same time.

Let $f: L \to L'$ be a similarity. Then of course the scales of L and L' are equal. The open sets U such that $\tilde{L}(U)$, $\tilde{L}'(U)$ and $(\mathfrak{s}L)^{\sim}(U) = (\mathfrak{s}L')^{\sim}(U)$ are free over $\tilde{A}(U)$ form a covering of $\operatorname{Spec}(A)$ and for every such set there is an $\tilde{A}(U)$-isomorphism $M_U(f)$ of $M_U(L)$ onto $M_U(L')$ induced by an equivalence of $a_f q_U$ and q'_U over $\tilde{A}(U)$. Since

$$M_{U_1}(f) \mid U_1 \cap U_2 = M_{U_2}(f) \mid U_1 \cap U_2$$

we can patch together all $M_U(f)$ and we get an S-isomorphism

$$M(f) : M(L) \to M(L') .$$

If f, f' are equivalent then of course $M(f) = M(f')$, hence we have an injection of $\operatorname{Bir}(L, L')$ into the set of S-isomorphisms of $M(L)$ onto $M(L')$. We shall show (a) and that this mapping is surjective.

Let $\Phi: M(L) \to M(L')$ be an S-isomorphism. Let $f: V \to V$ satisfying $Q(f(v)) = a_f Q(v)$ be a similarity such that Φ as a birational map is defined by f (f exists by Proposition 1).

Let $x \in \operatorname{Spec}(A)$, $(\mathfrak{s}L)_x = (s_x)$, $(\mathfrak{s}L')_x = (s'_x)$ and $\mathfrak{a}_x = (a_x)$ where

$$\mathfrak{a} = \mathfrak{a}_{L,L'}(f) = \{a \in F : (af|L) : L \to L'\} .$$

This is a fractional ideal of F.

We shall prove that

$$\det(a_x f) \in A_x^* \quad \text{and} \quad a_f s_x a_x^2 / s'_x \in A_x^* . \tag{2}$$

This proves (a) and the remaining part of (b). In fact, (2) is equivalent to:

$$\mathfrak{a}^3 (\det f) = A \quad \text{and} \quad (a_f) \mathfrak{s}L \mathfrak{a}^2 = \mathfrak{s}L' .$$

Hence $\mathfrak{s}L$ and $\mathfrak{s}L'$ define the same element in $\operatorname{Cl}(A)/\operatorname{Cl}(A)^2$ and if $\mathfrak{s}L = \mathfrak{s}L'$ then $\mathfrak{a}^2 (a_f) = A$ which gives $a = a_f / \det f$. Now if $\mathfrak{a} = (a_f / \det f)$ then

$$af: L \to L', \quad \det(af) \in A^*; \quad Q(af(v)) = a^2 a_f Q(v), \quad a^2 a_f \in A^* .$$

This proves that af is a similarity of L onto L'.

The statement (2) is in fact contained in the proof of Theorem 4 in [3] but for the completeness we shall give an account of the proof.

Since Φ is an S-isomorphism we have an A_x-isomorphism

$$\Phi_x : M(L) \times_{\operatorname{Spec}(A)} \operatorname{Spec}(A_x) \to M(L') \times_{\operatorname{Spec}(A)} \operatorname{Spec}(A_x)$$

that is

$$\Phi_x : \mathrm{Proj}\,(A_x[X_0, X_1, X_2]/(q_x)) \to \mathrm{Proj}\,(A_x[X_0, X_1, X_2]/(q'_x))$$

where q_x, q'_x are quadratic forms corresponding to quadratic mappings $(1/s_x)Q$ and $(1/s'_x)Q$ respectively. Let φ_x be a linear homomorphism of $A_x[X_0, X_1, X_2]$ corresponding to $a_x f$ such that $\varphi_x(q'_x) = \alpha_x q_x$. Since

$$Q(a_x f(v)) = a_f a_x^2 Q(v)$$

we get $\alpha_x = a_f a_x^2 s_x/s'_x$ and φ_x induces a homomorphism φ_x^* of rings

$$A_x[x_0', x_1', x_2'] = A_x[X_0, X_1, X_2]/(q'_x) \to A_x[X_0, X_1, X_2]/(q_x) = \\ = A_x[x_0, x_1, x_2]\,.$$

This homomorphism and Φ_x define the same birational morphism of $M(L) \times A_x$ on $M(L') \times A_x$. Hence they are equal on $U = \bigcup D(\varphi_x^*(x_i'))$. In order to simplify notations we shall omit all indexes x.

Let

$$\varphi(X_i) = \sum a_{ij} X_j\,, \quad a_{ij} \in A\,, \quad \det(a_{ij}) \neq 0\,,$$

where at least one $a_{ij} \in A^*$ and $\varphi(q') = \alpha q$. We have to show that $\det \varphi \in A^*$ and $\alpha \in A^*$.

Let π be a generator of a maximal ideal in A, let $k = A/(\pi)$ and for every $a \in A$ let $\bar{a}$ be its image in k. If $G \in A[X_0, X_1, X_2]$ then by $\bar{G}$ we shall denote the image of G in $k[X_0, X_1, X_2]$.

We shall prove that the rank of the matrix $[\bar{a}_{ij}]$ is at least 2. This rank is of course $\geqq 1$ since there is a pair (i,j) with $a_{ij} \in A^*$. Let us suppose that this rank is 1. Then the forms $\bar{\varphi}(X_0)$, $\bar{\varphi}(X_1)$, $\bar{\varphi}(X_2)$ are linearly dependent over k. Let $\bar{\varphi}(X_1) = \bar{a}_1 \bar{\varphi}(X_0), \bar{\varphi}(X_2) = \bar{a}_2 \bar{\varphi}(X_0)$. Hence

$$\varphi^{-1}((\pi)) \supseteqq (\pi, X_1 - a_1 X_0, X_2 - a_2 X_0) \tag{3}$$

We shall consider two cases:

(i) $\bar{q}$ is irreducible that is (π) is a prime ideal in $A[x_0, x_1, x_2]$, φ^* must be defined in the point (π) since $(\pi) \in U$ and by (3) this image is a closed point of $M(L') \times A$. This is impossible since Φ is an isomorphism and (π) is not closed in $M(L) \times A$.

(ii) $\bar{q}$ is reducible that is $\bar{q} = \bar{q}_1 \bar{q}_2$ and $\bar{q}_1 \neq \bar{q}_2$ since $M(L) \times A$ is regular (to check this see e.g. Theorem 2 in [3]). The ideals (π, q_1) and (π, q_2) are prime and their intersection is (π). Hence only one of them can contain $(\pi, \varphi(X_0), \varphi(X_1), \varphi(X_2))$ and φ^* is defined in at least one of these points, say (π, q_1). By (3) the image of this point is closed. This is a contradiction since (π, q_1) is not closed.

Hence the rank of $[\bar{a}_{ij}]$ is at least 2. Now if Ψ is the inverse of Φ and ψ

with a matrix $[b_{ij}]$ corresponds to Ψ in the same way as φ to Φ, then the rank of $[\bar{b}_{ij}]$ is also at least 2. But the product of $[a_{ij}]$ and $[b_{ij}]$ corresponds to the composition of Φ and Ψ that is to the identity. Hence this product is a diagonal matrix (by Proposition 1). By the well known properties of the rank we get the rank of the product $[\bar{a}_{ij}][\bar{b}_{ij}]$ is at least 1. Since this is a diagonal matrix this rank must be equal to 3. Hence $\det\varphi \in A^*$. If $\psi(q)=\alpha' q'$ then $\varphi\psi(q)=\alpha\alpha' q$. Since $\alpha\alpha' \in A$ and $(\alpha\alpha')^3 = \det(\varphi\psi)^2 \in A^*$ we get that $\alpha\alpha'$ is invertible in A.

COROLLARY 1. *Let L be a lattice on the regular quadratic space (V,Q) over F such that $M(L)$ is a regular S-scheme. Let* $\operatorname{Aut} M(L)$ *be the sheaf of S-automorphisms of $M(L)$ and let* $\operatorname{Bir}(\tilde{L})$ *be a presheaf such that for U open in* $\operatorname{Spec}(A)$, $\operatorname{Bir}(\tilde{L})(U)=\operatorname{Bir}(\tilde{L}(U))$. *Then* $\operatorname{Bir}(\tilde{L})$ *is a sheaf isomorphic with* $\operatorname{Aut} M(L)$.

2. Relatively minimal models

THEOREM 2. *Let A be a Dedekind ring of characteristic $\neq 2$ with perfect residue fields. If F is a field of fractions of A and E/F regular extension of genus 0, then there is a regular quadratic space (V,Q) over F and a lattice L on V such that*

$$\alpha_L : M(L) \to \operatorname{Spec}(A)$$

is a relatively minimal model of E/F.

PROOF. Let $E=F(x,y)$ where

$$a_0+a_1x^2+a_2y^2 = 0, \quad a_0a_1a_2 \neq 0,\ a_i \in A$$

(see [8, § 25]) and let (V,Q) be quadratic space over F such that

$$V = Fe_0+Fe_1+Fe_2 \quad \text{and} \quad Q(x_0e_0+x_1e_1+x_2e_2) = a_0x_0{}^2+a_1x_1{}^2+a_2x_2{}^2 .$$

If $L'=Ae_0+Ae_1+Ae_2$ then for every prime ideal x in A which does not contain 2 and $(a_0,a_1,a_2)_x=A_x$ there is an open neighbourhood U such that $M_U(L')$ is a relatively minimal model of E/F ([1, Theorem on p. 302]). Let $X \subseteq \operatorname{Spec}(A)$ be the set of such prime ideals x_i that L' does not define a relatively minimal model over some open neighbourhood of x_i. Let $X=\{x_1,\dots,x_r\}$ and let q_i be a quadratic form in $A_{x_i}[X_0,X_1,X_2]$ such that

$$M_i = \operatorname{Proj}(A_{x_i}[X_0, X_1, X_2]/(q_i))$$

is a relatively minimal A_{x_i}-model of E/F. The existence of q_i follows from [1] and [2]. Let (V_i,Q_i) be a quadratic space over F defined by q_i that is

$$V_i = Fe_0^{(i)} + Fe_1^{(i)} + Fe_2^{(i)}, \quad Q_i(x_0e_0^{(i)} + x_1e_1^{(i)} + x_2e_2^{(i)}) = q_i(x_0, x_1, x_2)$$

and let

$$L_i = A_{x_i}e_0^{(i)} + A_{x_i}e_1^{(i)} + A_{x_i}e_2^{(i)}.$$

Since the spaces (V,Q) and (V_i, Q_i) are similar (by Proposition 1) there is $f_i: V_i \to V$ such that $Q(f_i(v)) = a_iQ_i(v)$. Let $L_i' = f_i(L_i)$ and let L be a lattice on (V,Q) (over A) such that $L_x = L_x'$ if $x \notin X$ and $L_x = L_i'$ if $x = x_i$ (see [9, § 81 E, 81:14]). $M(L)$ is a relatively minimal model of E/F.

Lemma. *If (V,Q) is a regular quadratic space over F of odd dimension and $Q(f(v)) = a_fQ(v)$ then $a_f = b_f^2$ where $b_f = \det f/a_f^r, 2r+1 = \dim V$.*

Proposition 2. *If L, L' are lattices on a regular quadratic space (V,Q) over F, $\dim V$ is odd and $f: L \to L'$ is a similarity then there is $b_f \in A^*$ such that $(1/b_f)f$ is a rotation.*

Proof. Since $Q(f(v)) = a_fQ(v)$ where $a_f \in A^*$ and A is integrally closed we get that b_f defined in the Lemma belongs to A^*. Hence one of the mappings $(1/b_f)f$ or $-(1/b_f)f$ is a rotation.

Proposition 3. *If L is a lattice on a regular quadratic space (V,Q) over F and $\dim V$ is odd, then $\mathrm{Bir}(L) \approx O^+(L)$ where $O^+(L)$ denotes the group of rotations of L.*

Proof. Of course there is a natural injection $O^+(L) \to \mathrm{Bir}(L)$. If f is a similarity of L, then f is an automorphism of a free A_x-lattice L_x. Hence $\det f$ is invertible in A, since it is invertible in A_x for every x. If $Q(f(v)) = a_fQ(v)$, $a_f \in A^*$, then $b_f = \det f/a_f^r \in A^*$ (notations as in Lemma). Hence $(1/b_f)f$ or $-(1/b_f)f$ is an element of $O^+(L)$ and the image of this element is the class of f.

Remark. The assumption that A is integrally closed is not used in this proof.

Theorem 3. *Let A be a Dedekind ring of characteristic $\neq 2$ with perfect residue fields, F the field of fractions of A, E a regular extension of transcendence degree 1 and of genus 0 of F and L a lattice on a regular quadratic space (V,Q) over F such that $M(L)$ is a relatively minimal model of E/F.*

Then there is a one to one correspondence between the classes of S-isomorphic relatively minimal models of E/F and the classes of lattices in the proper genus of L.

PROOF. The classes of S-isomorphic relatively minimal models of E/F are in one to one correspondence with the elements of $\check{H}^1(\mathrm{Spec}(A), \mathrm{Aut}\,M(L))$. This follows from Theorem 1 in [2] (or Corollary in [4]). Corollary 1 gives $\mathrm{Aut}\,M(L) \approx \mathrm{Bir}(\tilde{L})$ and from Proposition 3, $\mathrm{Bir}(\tilde{L}) \approx O^+(\tilde{L})$, where $O^+(\tilde{L})(U) = O^+(\tilde{L}(U))$. Hence

$$\check{H}_1(\mathrm{Spec}(A), \mathrm{Aut}\,M(L)) \approx \check{H}^1(\mathrm{Spec}(A), O^+(\tilde{L})) .$$

The elements of the last set are in one to one correspondence with the classes of lattices in the proper genus of L, since according to the definition of proper genus ([9, § 102 A]) a lattice L' is in the proper genus of L if for every $x \in \mathrm{Spec}(A)$ there is an open neighbourhood U and a rotation f_U such that $f_U(L) = L'$.

COROLLARY. *Let A be a Dedekind ring of characteristic $\neq 2$ with perfect residue fields such that for every lattice on a regular quadratic space over the field of fractions of A the number of classes in the proper genus is finite e.g. A is a Dedekind ring such that its field of fractions F is a global field* ([9, § 103, *Theorem* 103:4]). *Then for every regular extension E/F of genus* 0 *the number of classes of* Spec(A)*-isomorphic relatively minimal models of E/F is finite and in every such class there is a model of the form $M(L)$ where L is a lattice.*

PROOF. If $M(L)$ is a relatively minimal model of E/F and L_1, L_2 are in proper genus of L, then the scales of these lattices are equal. Hence if $M(L_1) \approx M(L_2)$ then by Theorem 1 and Proposition 2, there is a rotation f such that $f(L_1) = L_2$. Hence L_1 and L_2 are in the same (proper) class.

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