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EDWARDS' SEPARATION THEOREM FOR COMPLEX LINDENSTRAUSS SPACES WITH APPLICATION TO SELECTION AND EMBEDDING THEOREMS

GUNNAR HANS OLSEN

1. Introduction.

The aim of this note is to extend the Edwards' separation theorem to the class of complex Banach spaces whose duals are $L_1(\mu)$ spaces for some measure μ, also termed Lindenstrauss spaces. In the real case this was done by Lindenstrauss and Lazar [6]. They used this theorem to prove a selection theorem, which generalized a result of Lazar [7] for simplex spaces, which in turn partially generalized a selection theorem of Michael [10]. This theorem has important consequences. Among these is the result that every separable Banach space whose dual is a non-separable $L_1(\mu)$ space contains a subspace isometric to $C(K)$ with K the Cantor set, and hence contains a copy of every separable Banach space.

The only point in the above mentioned part of [6] which is 'real', is the proof of Edwards' theorem. However, Effros [4] has developed a systematic procedure for approximating general L_1-functions by step functions, which may be used as a substitute of the Riesz decomposition property in the complex case. Our proof is strongly influenced by Effros' proof of the complex analogy of the Choquet–Meyer uniqueness theorem for simplexes, in fact this result follows from our argument. Finally, a theorem of Hustad [5] gives that Edwards' separation theorem actually characterizes the complex Lindenstrauss spaces. This result was implicit in Lazar's work [8] in the real case, see also [9].

2. Preliminaries and Notations.

We use the following notations:

V is a complex Banach space.

K is the unit ball in V^* with w^*-topology.

$M(K)$ is the Banach space of complex Radon measures on K.

$M_1^+(K)$ is the set of probability measures in $M(K)$.

T is the unit circle in C.

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Let now W be a complex $L_1(X,S,\mu)$ space for some measure space (X,S,μ). Given $p \in W$, let

$$N(p) = \{x \in X ;\ p(x) \neq 0\} .$$

We say that a countable set $\mathscr{B} = \{B_1, B_2, \ldots\}$ of S is a *partition* for p if

$$\begin{aligned} 0 < \mu(B_j) &< \infty, \quad && \text{for all } j , \\ B_j \cap B_k &= \emptyset, && j \neq k , \\ N(p) &\subseteq \bigcup B_j . \end{aligned}$$

Given $p \in W$ and a partition $\mathscr{B} = \{B_1, B_2, \ldots\}$ for p, the *conditional expectation* of p with respect to $\mathscr{B}$ is the step function

$$E(p \mid \mathscr{B}) = \textstyle\sum_j [\mu(B_j)^{-1} \int_{B_j} p\, d\mu] \chi_{B_j}$$

where χ_{B_j} is the characteristic function of B_j. The following relations hold for $p,q \in W$, $c \in \mathbf{C}$ almost everywhere:

(a) $E(p+q \mid \mathscr{B}) = E(p \mid \mathscr{B}) + E(q \mid \mathscr{B})$
(b) $E(cp \mid \mathscr{B}) = cE(p \mid \mathscr{B})$
(c) $|E(p \mid \mathscr{B})| \leqq E(|p| \mid \mathscr{B})$
(d) $\int E(p \mid \mathscr{B})\, d\mu = \int p\, d\mu$.

In particular it follows from (c) and (d) that

$$\|E(p \mid \mathscr{B})\| \leqq \|p\| . \tag{2.1}$$

The following lemma, due to Effros, gives a systematic procedure for approximating general functions with step functions.

LEMMA 2.1. *Given $p_1, p_2, \ldots, p_m \in W$ and $\varepsilon > 0$, there is a common partition $\mathscr{B}$ for $p_1, p_2, \ldots p_m$ with*

$$\|p_k - E(p_k \mid \mathscr{B})\| < \varepsilon; \quad k = 1, 2, \ldots, m .$$

PROOF. See [4, p. 50–51].

The next lemma is obvious.

LEMMA 2.2. *Let $z_1, z_2, \ldots, z_m \in \mathbf{C}$ with $\sum_k z_k = 0$. Then for all $\varepsilon > 0$ there exist $a \in \mathbf{R}^+$, $\zeta_k \in \mathbf{T}$, $n_k \in \mathbf{Z}^+$ such that*

$$|\zeta_k n_k a - z_k| < \varepsilon, \quad \textstyle\sum_k \zeta_k n_k a = 0, \quad n_k a \leqq |z_k| .$$

3. The key lemma and Effros' theorem.

For $\nu \in M_1{}^+(K)$ let $r(\nu)$ denote the *barycenter* of ν (see [1, p. 12–13]). We now come to the key lemma of the paper. Its proof is just a modification of [4, proof of theorem 4.3a implies b] and we will get this result as a corollary.

LEMMA 3.1. *Let V be a Lindenstrauss space and $g: K \to \langle -\infty, \infty]$ be a lower semicontinuous concave function satisfying*

$$(3.1) \qquad \textstyle\sum_{i=1}^{n} g(\zeta_i x) \geqq 0 \quad \text{whenever } \zeta_i \in \mathsf{T}, \sum_{i=1}^{n} \zeta_i = 0,\ x \in K\ .$$

If $\nu \in M_1{}^+(K)$ with $r(\nu) = 0$, then $\nu(g) \geqq 0$.

PROOF. We may select a net of atomic measures

$$\nu_\gamma = \textstyle\sum_{k=1}^{n(\gamma)} c_{\gamma k}\delta(p_{\gamma k}), \quad \sum_{n=1}^{n(\gamma)} c_{\gamma k} = 1,\ c_\gamma > 0\ ,$$
$$\textstyle\sum_{k=1}^{n(\gamma)} c_{\gamma k} p_{\gamma k} = r(\nu_\gamma) = 0\ ,$$

such that ν_γ converges to ν in the w^*-topology. Fix γ and $\varepsilon > 0$. By lemma 2.1 there is a common partition $\mathscr{B} = \{B_1, B_2, \ldots\}$ for $p_{\gamma 1}, p_{\gamma 2}, \ldots, p_{\gamma n(\gamma)}$ such that for all k

$$\|p_{\gamma k} - E(p_{\gamma k} \mid \mathscr{B})\| < \varepsilon\ .$$

It follows from (a), (b) of section 2 and (2.1) that

$$\textstyle\sum_k c_{\gamma k} E(p_{\gamma k} \mid \mathscr{B}) = 0\ .$$

Now by lemma 2.2 we get simple functions

$$p^\varepsilon{}_{\gamma k} = \textstyle\sum_{j=1}^{\infty} ((\zeta_{kj} n_{kj} a_j)/c_{\gamma k})\chi_{B_j}$$

where $\zeta_{kj} \in \mathsf{T}$, $n_{kj} \in \mathsf{Z}^+$, $a_j \in \mathsf{R}^+$ and

$$\textstyle\sum_k \zeta_{kj} n_{kj} = 0; \quad j = 1, 2, \ldots\ ,$$
$$\|(E(p_{\gamma k} \mid \mathscr{B}) - p^\varepsilon{}_{\gamma k})\chi_{B_j}\| < 2^{-j}\varepsilon; \quad j = 1, 2, \ldots\ ,$$
$$\|p^\varepsilon{}_{\gamma k}\| \leqq \|p_{\gamma k}\|.$$

Put $\nu^\varepsilon{}_\gamma = \sum_k c_{\gamma k}\delta(p^\varepsilon{}_{\gamma k})$. Then $\nu^\varepsilon{}_\gamma \to \nu$ as $\gamma \to \infty$, $\varepsilon \to 0$. Let $q_j = \mu(B_j)^{-1}\chi_{B_j}$, then the probability measure

$$\lambda^\varepsilon{}_{\gamma k} = \textstyle\sum_j ((n_{kj} a_j)/c_{\gamma k})\mu(B_j)\delta(\zeta_{kj} q_j) + (1 - \|p^\varepsilon{}_{\gamma k}\|)\delta(0)$$

has barycenter $p^\varepsilon{}_{\gamma k}$. Thus $\lambda^\varepsilon{}_\gamma = \sum_k c_{\gamma k}\lambda^\varepsilon{}_{\gamma k}$ dominates $\nu^\varepsilon{}_\gamma$ in the sense of Choquet. Now since g is concave and satisfies (3.1)

$$\nu^\varepsilon{}_\gamma(g) \geqq \lambda^\gamma{}_\varepsilon(g) \geqq \textstyle\sum_k c_{\gamma k} \sum_j ((n_{kj} a_j)/c_{\gamma k})\mu(B_j) g(\zeta_{kj} q_j)$$
$$= \textstyle\sum_j a_j \mu(B_j) \sum_k n_{kj} g(\zeta_{kj} q_j) \geqq 0$$

which completes the proof of lemma 3.1.

To see the connection with Effros' uniqueness theorem we need more notation. A function $f: K \to \mathsf{C}$ is called T-homogeneous if $f(\zeta p) = \zeta f(p)$ for all $\zeta \in \mathsf{T}$, $p \in K$. If $f \in C(K)$ then function

$$(\mathrm{hom}_{\mathsf{T}} f)(p) = \int \zeta^{-1} f(\zeta p)\, d\zeta$$

where $d\zeta$ is the unit Haar measure on T, is continuous and T-homogeneous. When $\mu \in M(K)$ then $\mathrm{hom}_{\mathsf{T}} \mu = \mu \circ \mathrm{hom}_{\mathsf{T}}$.

Let $\sigma: K \to K$ be the homeomorphism $\sigma(k) = -k$. Then

$$\mathrm{hom}_{\mathsf{T}} \sigma\mu = -\mathrm{hom}_{\mathsf{T}} \mu .$$

We can now state the complex analogy of the Choquet–Meyer uniqueness theorem for simplexes. The corresponding real result is due to Lazar [8].

THEOREM 3.2. *The following statements are equivalent*

(a) *V is a Lindenstrauss space.*

(b) *If $\nu_1, \nu_2 \in M_1{}^+(K)$ are maximal with $r(\nu_1) = r(\nu_2)$ then* $\mathrm{hom}_{\mathsf{T}} \nu_1 = \mathrm{hom}_{\mathsf{T}} \nu_2$.

PROOF. (a) $\Rightarrow$ (b). Put $\nu = \frac{1}{2}(\nu_1 + \sigma\nu_2)$. Then ν is a maximal probability measure with $r(\nu) = 0$. It suffices to prove $\mathrm{hom}_{\mathsf{T}} \nu = 0$, that is, $\nu(f) = 0$ for all T-homogeneous functions in $C(K)$. Choose a net of purely atomic measures $\{\nu_\gamma\}$ with $r(\nu_\gamma) = 0$ which converges to ν in the w^*-topology. In the same way as in lemma 3.1 construct for each γ and $\varepsilon > 0$ $\lambda^\varepsilon{}_\gamma$. By w^*-compactness $\{\lambda^\varepsilon{}_\gamma\}$ has a convergent subset, which by maximality must converge to ν. But $\lambda^\varepsilon{}_\gamma(\mathrm{hom}_{\mathsf{T}} f) = 0$ for all $f \in C_{\mathsf{C}}(K)$, so (b) follows.

(b) $\Rightarrow$ (a). See [4, p. 57–58].

4. Edwards' separation theorem with applications.

Following Cunningham [3] we define an *L-projection* e on a Banach space W to be a linear map on W into itself such that

$$e \text{ is a projection, that is, } e^2 = e ,$$
$$\|x\| = \|ex\| + \|x - ex\| \quad \text{for all } x \in W .$$

A linear subspace of W is called an *L-ideal* if it is the range of an L-projection. We refer to [2] for the outline of these concepts.

We can now state and prove Edwards' separation theorem.

THEOREM 4.1. *The following statements are equivalent.*

(i) *V is a Lindenstrauss space.*

(ii) *Let g be a lower semicontinuous concave function such that*

$$\sum_{i=1}^n g(\zeta_i x) \geqq 0 \quad \textit{whenever } \zeta_i \in \mathsf{T},\ \sum_{i=1}^n \zeta_i = 0,\ x \in K . \tag{4.1}$$

If $N \subseteq V^*$ is a w^*-closed L-ideal, and $f: N \cap K \to \mathsf{C}$ is an affine T-homogeneous w^*-continuous function such that $\operatorname{Re} f \leqq g \mid N \cap K$, *then there exist* $h \in V$ *such that* $\operatorname{Re} h \mid K \leqq g$ *and* $h \mid N \cap K = f$.

(iii) *If g is above then there exist $h \in V$ such that* $\operatorname{Re} h \mid K \leqq g$

PROOF. (i) $\Rightarrow$ (ii). We shall first assume that

$$\textstyle\sum_{i=1}^n g(\zeta_i x) \geqq 0 \quad \text{whenever } \zeta_i \in \mathsf{T},\ \sum_{i=1}^n \zeta_i = 0,\ x \in K$$

and that for some $\varepsilon > 0$

$$\operatorname{Re} f \leqq g \mid N \cap K - \varepsilon .$$

Let $x \in N \cap K$ and assume

$$x = \textstyle\sum_{i=1}^n \alpha_i x_i, \quad \sum \alpha_i = 1,\ \alpha_i \geqq 0,\ x_i \in K .$$

Let e be the L-projection onto N and put

$$y_i = e x_i / \|e x_i\|, \quad z_i = (x_i - e x_i)/\|x_i - e x_i\| .$$

Then the measure

$$\nu = \textstyle\sum_{i=1}^n \alpha_i \|e x_i\| \delta(y_i) + \sum_{i=1}^n \alpha_i \|x_i - e x_i\| \delta(z_i) + \sum_{i=1}^n \alpha_i (1 - \|x_i\|)\delta(0)$$

is a probability measure with $r(\nu) = x$ (if $\|e x_i\| = 0$ or $\|x_i - e x_i\| = 0$, just delete this term). Since the last two terms have resultant zero we get from lemma 3.1

$$\begin{aligned} \nu(g) &\geqq \textstyle\sum \alpha_i \|e x_i\| g(y_i) \\ &\geqq \textstyle\sum \alpha_i \|e x_i\| \operatorname{Re} f(y_i) + \varepsilon = \operatorname{Re} f(x) + \varepsilon . \end{aligned}$$

Since ν dominates $\sum \alpha_i \delta(x_i)$ and g is concave we get

$$\textstyle\sum \alpha_i g(x_i) \geqq \nu(g) \geqq \operatorname{Re} f(x) + \varepsilon .$$

Now [1, corollary 3.6] gives $\check{g} \geqq \operatorname{Re} f(x) + \varepsilon$ (here $\check{g}$ denotes the lower envelope of g, see [1, p. 4]).

By Hahn–Banach in product space [1, p. 2], [6, p. 170] there exist $h \in V$ such that

$$h \mid N \cap K = f \quad \text{and} \quad \operatorname{Re} h \leqq g + \varepsilon .$$

Let now f and g be as in the theorem and observe that the requirement (4.1) is equivalent to $\check{g}(0) \geqq 0$. (Use lemma 3.1 and [1, corollary 3.6] as above.) By the preceeding argument there exist $h_1 \in V$ such that

$$h \mid N \cap K = f \quad \text{and} \quad \operatorname{Re} h \leqq g + 1 .$$

From [2, lemma 1.2] and the compactness of the convex hull of two compact convex sets, we have

$$\mathrm{Sup}_K{}^C(g \wedge \mathrm{Re}\, h_1)^\vee = \mathrm{conv}(\mathrm{Sup}_K{}^C \check{g} \cup \mathrm{Sup}_K{}^C \,\mathrm{Re}\, h_1)$$

where $\mathrm{Sup}_K{}^C$ denotes the supergraph and C is a constant $\geqq \|g\|+1$. Thus there exist $p_1, q_1 \in K$, $\lambda_1 \in [0,1]$ such that

$$\begin{aligned} 0 &= \lambda_1 p_1 + (1-\lambda_1) q_1 , \\ 0 &\geqq (g \wedge \mathrm{Re}\, h_1)^\vee(0) \geqq \lambda_1 \check{g}(p_1) + (1-\lambda_1)\, \mathrm{Re}\, h_1(q_1) \\ &= \lambda_1(\check{g}(p_1) - \mathrm{Re}\, h_1(p_1)) \geqq -\lambda_1 . \end{aligned}$$

Put

$$\delta_1 = \max\{\tfrac{1}{2}, -(g \wedge \mathrm{Re}\, h_1)^\vee(0)\} .$$

Then the function $g_1 = g \wedge \mathrm{Re}\, h_1 + \delta_1$ is lower semicontinuous and concave and $\check{g}_1(0) \geqq 0$. By the preceeding argument there exist $h_2 \in V$ such that

$$h_2 | N \cap K = f, \quad \mathrm{Re}\, h_2 \leqq g + \delta_1, \quad \mathrm{Re}\, h_2 \leqq \mathrm{Re}\, h_1 + \delta_1 .$$

As above there exist $p_2, q_2 \in K$, $\lambda_2 \in [0,1]$ such that

$$\begin{aligned} 0 &= \lambda_2 p_2 + (1-\lambda_2) q_2 , \\ 0 &\geqq (g \wedge \mathrm{Re}\, h_2)^\vee(0) \geqq \lambda_2(\check{g}(p_2) - \mathrm{Re}\, h_2(p_2)) \geqq -\lambda_2 \delta_1 . \end{aligned}$$

Put

$$\delta_2 = \max\{2^{-2}, -(g \wedge \mathrm{Re}\, h_2)^\vee(0)\} .$$

Then the function $g_2 = g \wedge \mathrm{Re}\, h_2 + \delta_2$ is lower semicontinuous and concave and $\check{g}_2(0) \geqq 0$. Thus, we may proceed by induction to get a sequence $\{h_n\}$ from V such that

$$h_n | N \cap K = f, \quad \mathrm{Re}\, h_n \leqq g + \delta_{n-1}, \quad \mathrm{Re}\, h_n \leqq \mathrm{Re}\, h_{n-1} + \delta_{n-1} , \tag{4.2}$$

where

$$\delta_{n-1} = \max\{2^{-(n-1)}, -(g \wedge \mathrm{Re}\, h_{n-1})^\vee(0)\} .$$

As above we get $p_n, q_n \in K$, $\lambda_n \in [0,1]$ such that

$$0 = \lambda_n p_n + (1-\lambda_n) q_n , \tag{4.3}$$

$$0 \geqq (g \wedge \mathrm{Re}\, h_n)^\vee(0) \geqq \lambda_n(\check{g}(p_2) - \mathrm{Re}\, h_n(p_n)) \geqq -\lambda_n \delta_{n-1} . \tag{4.4}$$

If there exist $\delta \in \langle 0,1 \rangle$ and a positive integer N such that $\lambda_n \leqq \delta$ for all $n \geqq N$ then it follows from (4.4) that $\lim_{n\to\infty} \delta_n = 0$. From (4.2) we then get that the sequence $\{h_n\}$ converges in norm to an element $h \in V$ which has all the required properties. Now assume there is a subsequence $\{\lambda_{n_k}\} \subseteq \{\lambda_n\}$ such that $\lim_{k\to\infty} \lambda_{n_k} = 1$.

Then from (4.3) it follows that $\lim_{k\to\infty}\|p_{n_k}\|=0$ in particular $\lim_{k\to\infty}p_{n_k}=0$ in the w^*-topology. Now it follows from (4.4) that

$$\begin{aligned}0 &\geqq \limsup\,(g\wedge \mathrm{Re}\,h_{n_k})^{\vee}(0) \geqq \liminf\,(g\wedge \mathrm{Re}\,h_{n_k})^{\vee}(0)\\ &\geqq \liminf \lambda_{n_k}(\check{g}(p_{n_k})-\mathrm{Re}\,h_{n_k}(p_{n_k}))\\ &\geqq \liminf\,(\check{g}(p_{n_k})-\mathrm{Re}\,h_{n_k}(p_{n_k}))\\ &\geqq \liminf \check{g}(p_{n_k})-\limsup \mathrm{Re}\,h_{n_k}(p_{n_k})\\ &\geqq \check{g}(0)-\limsup\,(\|g\|+2)\|p_{n_k}\| = \check{g}(0) \geqq 0\,.\end{aligned}$$

This gives $\lim\delta_{n_k}=0$ so by (4.2) we get that the sequence $\{h_{n_k}\}$ converges in norm to an element h in V which has all the desired properties. The proof of (ii) is complete.

Remark. Part of the above proof borrows ideas from [2, proof of lemma 5.3].

(i) $\Rightarrow$ (iii). Take $N=\{0\}$

(iii) $\Rightarrow$ (i) When W is a Banach space $z\in W$, $r>0$, then

$$B(z,r) = \{x\in W \mid \|x-z\|\leqq r\}\,.$$

Let now $\{B(x_i,r_i)\}$ be n balls in V with the *weak intersection property*, that is

$$\bigcap_{k=1}^{n} B(x^*(x_k),r_k) \neq \emptyset \quad \text{for } x^*\in K\,.$$

Define $g\colon K\to \mathsf{R}$, by

$$g(x^*) = (\mathrm{Re}\,x^*(x)-r_1)\vee \ldots \vee (\mathrm{Re}\,x^*(x_n)-r_n)\,.$$

Then g is continuous and convex. For each $x^*\in K$, choose

$$z\in \bigcap_{i=1}^{n} B(x^*(x_i),r_i)\,.$$

Then we have

$$\textstyle\sum_{i=1}^{m} g(\zeta_i x^*) \leqq \sum_{i=1}^{m}\mathrm{Re}\,(\zeta_i z) = 0; \quad \text{whenever } \zeta_i\in\mathsf{T},\ \sum_{i=1}^{m}\zeta_i=0\,.$$

Now by (iii) there is $x\in V$ such that for all $x^*\in K$

$$g(x^*) \leqq \mathrm{Re}\,x^*(x)\,,$$

or equivalently

$$\mathrm{Re}\,x^*(x_k-x) \leqq r_k, \quad k=1,\ldots,n\,.$$

So by the Hahn–Banach theorem

$$x\in \bigcap_{k=1}^{n} B(x^*(x_k),r_k)\,.$$

Now a theorem of Hustad [5, theorem 4.9] gives that V is a Lindenstrauss space.

REMARK. The concept of biface used in [6] and L-ideal coincides when the L-ideals are restricted to K in the case V is a real Lindenstrauss space, see [2, p. 168].

Let now E be a locally convex space. Denote by $c(E)$ the convex non-empty subsets in E and by $\bar{c}(E)$ the closed sets in $c(E)$. A map φ from a convex set C into $c(E)$ is called convex if

$$\lambda\varphi(x_1)+(1-\lambda)\varphi(x_2) \subseteqq \varphi(\lambda x_1+(1-\lambda)x_2), \quad 0 \leqq \lambda \leqq 1,\ x_1,x_2 \in C .$$

The map φ is said to be lower semicontinuous if

$$\{x\ ;\ \varphi(x) \cap V \neq \emptyset\} \quad \text{is open for every open } U \text{ in } E .$$

We say that φ is T-*symmetric* if $\varphi(\zeta x)=\zeta\varphi(x)$, $\zeta \in \mathsf{T}$, $\zeta x, x \in C$. By a *selection* for φ we mean a map $f\colon C \to E$ such that $f(x) \in \varphi(x)$ for all $x \in C$.

THEOREM 4.2. *Let V be a Lindenstrauss space and E a Frechét space. Let $\varphi\colon K \to \bar{c}(E)$ be a convex T-symmetric w^*-continuous map. Then φ admits a w^*-continuous affine T-symmetric selection h. Moreover, if N is a w^*-closed L-ideal and $f\colon N \cap K \to E$ a selection for $\varphi | N \cap K$, then h can be chosen such that $h | N \cap K = f$.*

PROOF. Assume first $E=\mathsf{C}$ and let U be an open disk in C and define $g\colon K \to \mathsf{R}$ by

$$g(x) = \sup \operatorname{Re}(\varphi(x)+U) \quad x \in K .$$

Then g is a lower semicontinuous and concave, and the T-symmetry of φ ensures $\sum_{i=1}^{n} g(\zeta_i x) > 0$ whenever $\sum_{i=1}^{n} \zeta_i = 0$, $\zeta_i \in \mathsf{T}$, $x \in K$.

By theorem 4.1 we get $h \in V$ such that

$$\operatorname{Re} h(x) < g(x), \quad \forall x \in K .$$

Now the T-symmetry ensures $h(x) \in \varphi(x)+U$ for all $x \in K$. The rest of the proof is now as in [6, p. 172–173].

The above selection theorem has interesting applications. Among them we should like to mention the following theorem due to Lazar and Lindenstrauss. The proof is now similar to the real case [6, p. 174].

THEOREM 4.3. *If V is a separable Lindenstrauss space with nonseparable dual space, then V contains a subspace isometric to $C(K)$ with K the Cantor set, on which there is a contractive projection.*

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UNIVERSITY OF ODENSE, DENMARK
AND
UNIVERSITY OF OSLO, NORWAY