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RESULTS ON BANACH IDEALS AND SPACES OF MULTIPLIERS

HANS G. FEICHTINGER

0. Introduction.

There are many Segal algebras on a nondiscrete, locally compact abelian group which are defined by means of conditions on the Fourier transforms of their elements. The most important examples of this type are the Banach algebras

$$A_p(G) = \{f \mid f \in L^1(G), \hat{f} \in L^p(\hat{G})\}, \quad 1 \leqq p < \infty .$$

These spaces as well as several generalizations have obtained much interest in the literature (cf. [4], [5], [20], [23] and others). For a survey of results concerning the $A_p(G)$-algebras as well as for a great number of further references the reader is referred to the article of Larsen [21].

In this paper a class of Segal algebras including the spaces mentioned above is to be discussed (section 3). Earlier results in this direction are extended and some of the proofs are simplified. The treatment is based on a method (section 2) that also gives results for Banach algebras which are the intersection of a Beurling algebra and a Segal algebra (section 4). Furthermore we characterize several spaces of multipliers.

1. Notations and terminology.

A Banach space B is called a (*left*) *Banach module* over a Banach algebra A if it is a (left) module over A in the algebraic sense and satisfies $\|ab\|_B \leqq \|a\|_A \|b\|_B$ for all $a \in A$, $b \in B$. The closed linear span of $AB = \{ab \mid a \in A, b \in B\}$ in B is called *essential* part B_e of B. B is called *essential* if $B_e = B$. If the Banach module B is continuously embedded in A and the module operation is given by the multiplication in A we call B a (*left*) *Banach ideal* of A. In particular, B is a Banach algebra itself. Results on dense Banach ideals are to be found in the papers of Burnham ([1]–[3]). He calls them *abstract Segal algebras* and in [6] and [27] they are called *normal ideals.* A special type of Banach ideals of $L^1(G)$ are the *Segal algebras* in the sense of Reiter [24, section 4]. A dense Banach ideal of $L^1(G)$ is a Segal algebra iff it is essential (see

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[27]). For facts concerning Segal algebras and Beurling algebras the reader is referred to [23] and [24].

For two Banach modules B_1, B_2 over A we write simply (B_1, B_2) for the Banach space of module homomorphismus form B_1 into B_2 i.e. the space of all bounded linear operators satisfying $T(ab) = aT(b)$ for all $a \in A$, $b \in B_1$. These operators are often called *(right) multipliers* from B_1 into B_2. If B_1 and B_2 are Segal algebras or Beurling algebras it can be shown by standard methods that T is a right multiplier if and only if it commutes with left translations. For results concerning multipliers of Banach algebras on abelian groups see [20].

$C^b(G)$ $(C^0(G))$ denotes the space of all bounded continuous functions on a locally compact group (vanishing at infinity) with the supremum norm. The space $K(G)$ of continuous functions with compact support is dense in $C^0(G)$. For convenience we shall often write C^0 or L^p instead of $C^0(G)$ or $L^p(G)$ (Lebesgue space with respect to the left Haar measure on G). Further unexplained notation is taken from [23], [24] and [31], Chapter 15.

2. The main result.

For later reference we state the following assumptions:

(1) A is a Banach algebra, continuously embedded and dense in another Banach algebra A_1;
(2) A has bounded, two-sided approximate units;
(3) B is a proper Banach ideal of A_1;
(4) $A \cap B$ is dense in A.

THEOREM 2.1. *Suppose* A, A_1, B *satisfy* (1) – (4). *Then*

i) *$A \cap B$ is a proper, dense Banach ideal of A;*
ii) *$A \cap B$ is an essential Banach ideal of A if B is an essential Banach ideal of A_1.*

PROOF. Since A and B are both complete and continuously embedded in A_1 $A \cap B$ is a Banach space with respect to the norm $\|f\|_{A\cap B} := \|f\|_A + \|f\|_B$. That $A \cap B$ is a proper subspace of A will follow from Lemma 2.2. The density is a consequence of (4).

In order to prove ii) we observe that it follows from (1), (2) and (4) that A has bounded left approximate units $(u_\alpha) \subseteq A \cap B$ and that automatically $\|u_\alpha\|_{A_1} \leqq C_1$ and

$$\lim_\alpha \|u_\alpha f - f\|_{A_1} = 0 \quad \text{for all } f \in A_1 .$$

Since B is an essential Banach ideal this implies $\lim_\alpha \|u_\alpha g - g\|_B = 0$ for all $g \in B$. This shows that $A \cap B$ has left approximate units, in particular $A \cap B$ is an essential Banach ideal of A.

LEMMA 2.2. *Let A, A_1 satisfy* (1). *If A has bounded right approximate units it is not contained in any proper* (*algebraic*) *left ideal of A_1.*

PROOF. We may consider A_1 as an essential right Banach A-module with respect to multiplication. By the use of the factorization theorem for Banach modules ([15], 32.22) condition (2) implies $A_1 = A_1 A$. Therefore the inclusion $A \subseteqq I$ for a left ideal $I \subseteqq A_1$ implies $I \subseteqq A_1 = A_1 A \subseteqq A_1 I \subseteqq I$, that is $I = A_1$.

COROLLARY 2.3. (cf. [1]) *A proper dense Banach ideal B of a Banach algebra A_1 cannot have bounded right approximate units.*

It is now very difficult to characterize $(A, A \cap B)$ in terms of (A, A) and (A_1, B):

THEOREM 2.4. *Suppose* (1)–(3) *is satisfied. Then for every left A_1-module C, (A, C) is the restriction of (A_1, C). In particular,*

$$(A, A \cap B) = (A, A) \cap (A_1, B)|_A .$$

PROOF. It follows from (1) that a linear operator from A_1 to C gives an A-module homomorphism as restriction iff it is an A_1-module homomorphism. Thus no confusion arises if we don't specify what kind of multipliers we mean.

Since one inclusion is trivial the assertion follows from the fact that any $T \in (A, C)$ can be extended to an operator on A_1 due to condition (1) and the equality

$$\begin{aligned} \|T(f)\|_C &= \lim_\alpha \|T(fu_\alpha)\|_C = \lim_\alpha \|fT(u_\alpha)\|_C \\ &\leqq \|f\|_{A_1} \sup \|T(u_\alpha)\|_C \leqq \|f\|_{A_1} \; \|T\| C_1 . \end{aligned}$$

3. Applications to Segal algebras.

Among the various possible applications of the results of section 2 the construction of a class of Segal algebras on an abelian group G seems to be the most interesting one. We only mention here those facts which are either new or provide simple proofs for known results. In this section we shall use results on Banach function spaces in the sense of Zaanen [31], Chapter 15. A Banach space of (equivalence classes) of measurable functions on G is called a Banach function space (with respect to the Haar measure) iff it is an $L^\infty(G)$-module under pointwise multiplication.

THEOREM 3.1. *Let F be a Banach function space on $\hat{G}$ such that $K(\hat{G})$ is dense in F and $C^0 \cap F$ is a proper subspace of $C^0(\hat{G})$. Then we have*

i) $$S_F(G) = \{f \mid f \in L^1(G), \hat{f} \in F\}$$

is a proper Segal algebra on G with the norm

$$\|f\| = \|f\|_1 + \|\hat{f}\|_F$$

and (L^1, S_F) *can be identified with*

$$M_{(C^0,F)}(G) = \{\mu \mid \mu \in M(G), \hat{\mu} \in (C^0, F)\}$$

ii) *If F is reflexive as a Banach space the equality*

$$(L^1, S_F) \cong M_F(G) = \{\mu \mid \mu \in M(G), \hat{\mu} \in F\}$$

holds. If, furthermore, $F \cap C^0(\hat{G})$ *is contained in* $L^2(\hat{G})$, (L^1, S_F) *and* S_F *are isomorphic as Banach algebras.*

Proof. We set $A = F^1(\hat{G}) = \{\hat{f} \mid f \in L^1(\hat{G})\}$ $\|\hat{f}\|_{F^1} = \|f\|_1$, $A^1 = C^0(\hat{G})$ and $B = C^0(\hat{G}) \cap F$ with

$$\|h\|_B = \|h\|_\infty + \|h\|_F.$$

Then conditions (1)–(4) are fullfilled, because $F^1(\hat{G}) \cap K(\hat{G})$ is dense in $F^1(\hat{G})$ as well as in $K(\hat{G})$ and $F^1(\hat{G})$ has bounded approximate units. Theorem 2.1 implies that $F^1(\hat{G}) \cap F$ is a proper, dense, essential Banach ideal of $F^1(\hat{G})$. Since the norm given by $\|f\|_1 + \|\hat{f}\|_\infty + \|\hat{f}\|_F$ and $\|f\| = \|f\|_1 + \|\hat{f}\|_F$ are equivalent, $(S_F(G), \|.\|)$ is a proper Segal algebra on G. By the use of the equations

$$(C^0, C^0 \cap F) = C^b \cap (C^0, F)$$

and $(L^1, L^1) = M(G)$ (Wendel's theorem, [20], Theorem 0.1.1) the characterization of (L^1, S_F) follows from Theorem 2.4.

In order to prove ii) we observe that we have $(C_0, F) = F$ for reflexive Banach function spaces (considered as C^0-modules, cf. [26, p. 474]). The last assertion follows from the fact that $\mu \in M(G)$, $\hat{\mu} \in L^2(\hat{G})$ implies $\mu = f \in L^1 \cap L^2(G)$.

It is obvious that $S_F(G)$ is [strongly] character invariant ($f \in S_F$ implies $\chi f \in S_F$ for all $\chi \in \hat{G}$ [and $\|\chi f\| = \|f\|$]; cf. [11, section 3] for these definitions) if F is translation invariant ($h \in F$ implies $L_y h \in F$ [and $\|L_y h\|_F = \|h\|_F$] for all $y \in \hat{G}$). In view of Theorem 3.1 it is of interest to have a characterization of (C^0, F).

Lemma 3.2. (C^0, F) *can be identified with the Banach space*

$$\tilde{F} = \{f \mid kf \in F \text{ for all } k \in K(\hat{G}) \text{ and}$$
$$\|f\|^{\sim} = \sup[\|kf\|_B;\ k \in K(\hat{G}), \|k\|_\infty \leqq 1] < \infty\}.$$

The proof is left to the reader. It is not difficult to see that $\tilde{F}$ is just the local closure $\mathrm{lc}\,F$ of F in the sense of Schaeffer [22, section 22]. Thus it coincides with the second associate space (=second Köthe dual) of F. Consequently (C^0, F) coincides with F if F has the weak Fatou property (see [31, section 65]). For example $L^p(\hat{G})$, $1 \leqq p \leqq \infty$ has this property. Since any reflexive Banach function space has this property (see [31, section 73]) we have another proof for the equality $(C^0, F) = F$ for reflexive spaces.

Using remark B of [11, section 4] it is easy to characterize the L^1-relative completion

$$\tilde{S_F}^1(G) := \{f \mid f = L^1 - \lim f_n, \ \sup \|f_n\| < \infty\}$$

of $S_F(G)$.

COROLLARY 3.3. $\tilde{S_F}^1(G) = S_{\tilde{F}}(G)$.

EXAMPLES. The most important examples are of course obtained by using $F = L^p(\hat{G})$, $1 \leqq p < \infty$, or more generally the Lorentz spaces $L(p,q)(\hat{G})$. It is well known that these spaces are reflexive for $1 < p, q < \infty$ (see [16, pp. 259–262]). Therefore Theorem 3.1 extends and simplifies the proof of known results concerning the algebras $A(p,q)(G)$ and $A^p(G)$ (cf. [5, 1.12–3.14], [19], [21]). Corollary 3.3 extends Theorem 4 of [4].

There is a number of further examples, e.g. weighted L^p-spaces, Lorentz and Orlicz spaces or amalgams of such spaces with a sequence space, such as the spaces $A(A, X)$ considered in [10]. Using these spaces one obtains among others the algebras S_p studied by Unni ([30]). The Segal algebras treated in [27] and Example 16 of [3] are included in our considerations as well. It is left to the reader to write down further examples.

It is remarkable that the Segal algebras defined in this section cannot have (weak) factorization nor may they contain other Segal algebras with factorization (cf. Corollary 2.5 of [13]). We conclude this section with characterizations of several spaces of multipliers.

THEOREM 3.4. *Let G be a noncompact abelian group and let $S_F(G)$ be as in* 3.2. *Then*

$$(S_F, L^1) \cong M(G)$$

if F has absolutely continuous norm (with respect to the Haar measure on $\hat{G}$), in particular if F is reflexive as a Banach space.

PROOF. The proof is not stated explicitely here because it is merely a slight modification of the proof of Theorem 6.3.1 given in [20]. The only remarkable property of $L_p(\hat{G})$ that has been used in this proof is Lebesgue's theorem on dominated convergence, but an equivalent theorem holds just for Banach function spaces with absolutely continuous norm (see [31, section 72, Theorem 2]). That any reflexive Banach function space F has this property follows from section 73 of [31].

COROLLARY 3.5. *Let G be a noncompact, abelian group and let F be a reflexive Banach function space on $\hat{G}$. Then we have*

$$(S_F, S^2) \cong M(G) \quad \text{and} \quad (S^2, L^1) \cong (S^2, S^2) \cong M(G)$$

for any Segal algebra $S^2 \supseteqq S_F(G)$, in particular (S_F, S_F) can be identified with $M(G)$.

PROOF. The assertions follow from the inclusions

$$M(G) \subseteqq (S_F, S^2) \subseteqq (S_F, L^1) = M(G)$$

$$M(G) \subseteqq (S^2, S^2) \subseteqq (S^2, L^1) \subseteqq (S_F, L^1) = M(G)\,.$$

Corollary 3.5 shows that the results of section 3.5 of [20] may be considered as a consequence of Theorem 3.4, because we have

$$A_1(G) = S_{L^1(G)} \subseteqq L^1 \cap C^0(G) \subseteqq L^1 \cap L^p(G) \quad \text{for all } p \geqq 1\,.$$

Corollary 3.5 can also be used to prove that $S_F(G)$ is never a subspace of Wiener's algebra $W(G)$ which can be defined for any locally compact group (cf. [12]). It follows from the arguments of [7, p. 264] that any pseudomeasure with compact support $\sigma \in P_c(G)$ defines a multiplier from $W(G)$ to $L^1(G)$. On the other hand $P_c(G)$ is not contained in $M(G)$ for any nondiscrete, locally compact abelian group (cf. [18, Proposition 4.1]). Thus we have

COROLLARY 3.6. *Let S be a Segal algebra with $(S, L^1) \cong M(G)$ (for example $S \supseteqq S_F(G)$, $S_F(G)$ as in 3.5). Then S is not contained in $W(G)$.*

We mention that this result can be extended to $W^p(G)$, $1 < p < \infty$, as defined by Krogstad (cf. [18, Corollary 3.8]). Corollary 3.5 together with 3.6 gives a partial answer to a question raised by Larsen [21, p. 231].

4. Applications to Banach ideals of Beurling algebras.

In this section G denotes a general noncompact, locally compact group if not otherwise stated. The methods of Section 2 are applied to generate dense Banach ideals of a Beurling algebra $L^1_w(G)$.

THEOREM 4.1. *Let B be a pseudosymmetric Segal algebra on G and let $L^1_w(G)$ be a Beurling algebra. Then $L^1_w(G) \cap B$ is a proper, dense, essential left Banach ideal of $L^1_w(G)$ with two-sided approximate units.*

PROOF. If B is pseudosymmetric, $B \cap K(G)$ is dense in B as well as in $L^1_w(G)$ (cf. [24]). Therefore Theorem 2.1 is applicable with $A = L^1_w(G)$ and $A_1 = L^1(G)$. The proof that $L^1_w \cap B$ has right (two-sided) approximate units (which are necessarily unbounded in B) is the same as for pseudosymmetric Segal algebras, using the continuity of $y \to R_y f$ and of $y \to L_y f$ from G into $L^1_w \cap B$ for all $f \in L^1_w \cap B$.

THEOREM 4.2. *Let B be a Segal algebra on a locally compact abelian group. Then $L^1_w(G) \cap B$ is a proper, dense, essential Banach ideal of $L^1_w(G)$, if the Beurling algebra $L^1_w(G)$ satisfies the condition of Beurling-Domar or if B is a strongly character invariant Segal algebra.*

PROOF. If $L^1_w(G)$ satisfies the condition of Beurling-Domar [23, Chapter 6], then the set

$$\{f \mid f \in L^1_w(G),\ \hat{f} \in K(\hat{G})\}$$

is dense in $L^1_w(G)$ ($F^1_w(\hat{G})$ is a Wiener algebra on $\hat{G}$ in this case) as well as in B (cf. [23]). Thus Theorem 2.1 is applicable. If B is strongly character invariant, B is pseudosymmetric, because it is Banach module over $F^1(G)$ with pointwise multiplication (cf. [11, Lemma 3.7 and 3.8]). Therefore Theorem 4.1 gives the result in this case.

REMARKS. 1) 4.2 is not a special case of 4.1 because a Segal algebra on an abelian group need not be pseudosymmetric (cf. [25, Example 4]).

2) It would have been sufficient to suppose that B is character invariant ($M\chi f = \chi f \in B$ for all $f \in B$, $\chi \in \hat{G}$) and satisfies

$$\text{BD}\hat{)} \qquad \sum_{n=1}^{\infty} \frac{\log \|M_{\chi^n}\|_B}{n^2} < \infty \qquad \text{for all } \chi \in \hat{G}\,,$$

with $\|M_\chi\|_B$ being the operator norm of M_χ on B.

3) We mention that the ideal theorem (see [24, section 9, Theorem 1], [1], and [9]) is applicable in the situation of Theorem 4.1 and 4.2. Thus the ideal structure of $L^1_w \cap B$ is much the same as of $L^1_w(G)$. In particular,

$$F^1_w(\hat{G}) = \{\hat{f}, f \in L^1_w] \quad \text{and} \quad F^1_w(\hat{G}) \cap \hat{B}$$

have the same Wiener sets (see [23, Chapter 2] for the definitions) and the same Wiener-Ditkin sets (by the results of [2] or [9]). For example, it follows from a result of Stegeman [29] that any closed subgroup of $\mathbf{R}^n$ is a Wiener-Ditkin set for $F^1_{w_\alpha} \cap \hat{B}(\mathbf{R}^n)$ with $w_\alpha(x) = (1+|x|)^\alpha$, $0 \leqq \alpha < 1$.

4) Using Lemma III. 1.5. of [28] it is easily shown that a proper inclusion

$$L^1_{w1}(G) \subset L^1_{w2}(G)$$

leads to a proper inclusion $L^1_{w1} \cap B \subset L^1_{w2} \cap B$ in the situation of 4.1 or 4.2.

At the end of this section we discuss the problem of identifying $(L^1_{w1}, L^1_{w2} \cap B)$. Let us first assume that we have $L^1_{w1}(G) \subseteqq L^1_{w2}(G)$. Then by Theorem 2.4 the problem is reduced to the determination of

$$(L^1_{w1}, L^1_{w2}) = (L^1_{w2}, L^1_{w2})$$

and of (L^1, B). The first problem has been solved by Gaudry [14, Theorem 4]. He showed that

$$L^1_w, L^1_w) \cong M^1_{\bar{w}}(G) := \left\{\mu \,\middle|\, \mu \in M(G), \int \bar{w}\, d|\mu| < \infty\right\}$$

with $\bar{w}(y) = \sup_x w(y^{-1}x)/w(x)$. For many concrete examples $\bar{w}$ is equivalent with w, e.g. in case

$$w_\alpha(x) = (1+|x|)^\alpha, \qquad \alpha \geqq 0,$$

on $G = \mathbf{R}^n$. Characterizations of (L^1, B) can be found in [11], cf. also [21], and Theorem 3.2 of this paper. Instead of stating the corresponding theorems let us illustrate the result by a typical example:

Let $L^1_\alpha(\mathbf{R}^n)$ be the Beurling algebra on $\mathbf{R}^n$ defined by means of the weight function w_α, $\alpha \geqq 0$. Then we have for $B_{p,\beta} := \{f \mid f \in L^1(\mathbf{R}^n),\ \hat{f}w_\beta \in L^p(\mathbf{R}^n)\}$, $\beta \geqq 0$, $1 \leqq p < \infty$, $(L^1_\gamma, L^1_\alpha \cap B_{p,\beta}) \cong \{\mu \mid \mu w_\alpha \in M(\mathbf{R}^n),\ \hat{\mu} w_\beta \in L^p(\mathbf{R}^n)\}$ for $\gamma \geqq \alpha$. Moreover for $\beta > n/2p$ this space can be identified with $L^1_\alpha \cap B_{p,\beta}$ itself, because $\hat{\mu} w_\beta \in L^p(\mathbf{R}^n)$ implies $\hat{\mu} \in L^2(\mathbf{R}^n)$ for $\beta > n/2p$. For $p = \infty$, $L^\infty(\mathbf{R}^n)$ has to be replaced by $C^0(\mathbf{R}^n)$ in the definition of $B_{\infty,\beta}$, but not in the identification of the multiplier space. We mention again that similar results can be proved for Segal algebras such as

$$B_{p,\delta} := \{f \mid f \in L^1(\mathbf{R}^n),\ \hat{f}w_\delta/\mathbf{Z}^n \in l^p(\mathbf{Z}_n)\}, \qquad \delta \geqq 0,\ 1 \leqq p < \infty\,.$$

If $L^1_{w1}(G)$ is not a subspace of $L^1_{w2}(G)$, the situation changes completely, because (L^1_{w1}, L^1_{w2}) is trivial in this case. We do not give the proof for the general result here, because it is elementary, but somewhat lengthly. The following result is sufficient for most concrete situations.

LEMMA 4.3. *Let $L^1_{w2}(G)$ be a proper subspace of $L^1_{w1}(G)$. Then $(L^1_{w1}, L^1_{w2}) = \{0\}$.*

PROOF. Let $T \in (L^1_{w_1}, L^1_{w_2})$ be given. Then Lemma III. 1.5. of [28] implies

$$w_2(x) \leqq K_2 \|L_y T f\|_{1,w_2} = K_2 \|T L_y f\|_{1,w_2} \leqq K \|T\| \, \|L_y f\|_{1,w_1}$$
$$\leqq K_2 \|T\| K_1 w_1(x) \qquad \text{for all } f \in L^1_{w_1}(G).$$

This is not possible if $L^1_{w_2}$ is a proper subspace of $L^1_{w_1}$ (cf. [23]).

5. Further applications.

A natural generalization of the results of Section 3 can be obtained by replacing $\hat{G}$ by a noncompact, locally compact space X and $F^1(\hat{G})$ by a Wiener algebra A on X. If A has bounded approximate units essentially the situation of Section 2 is given and the main result applies. There is a great number of such Wiener algebras, for example spaces of functions in $C^0(\mathbf{R}^n)$ satisfying several kinds of differentiability properties or Lipschitz conditions.

We do not give further details here. We only state a theorem that can be considered as a direct generalization of Theorem 3.1 to non abelian groups. $A(G)$ ($B(G)$) denotes Eymard's Fourier (Stieltjes) algebra on G. (See [8].)

THEOREM 5.1. *Let G be an amenable group, and let F be a Banach function space on G containing $K(G)$ as a dense subspace. Then $A(G) \cap F$ is a dense, essential Banach ideal of $A(G)$ and the multiplier algebra $(A(G), A(G) \cap F)$ is isometrically isomorphic with $B(G) \cap \tilde{F}$.*

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