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DELTA-PLURISUBHARMONIC FUNCTIONS

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1. Introduction.

Let K be a convex cone in a linear space F. We denote by δK the set of elements φ in F which have a representation $\varphi = \varphi_1 - \varphi_2$ where $\varphi_1, \varphi_2 \in K$. It is sometimes possible to give countably many seminorms on δK, turning it into a Fréchet space.

In Schaefer [7, p. 221] it is proved that if F is a Fréchet space and if K is a closed convex cone in F, then δK is a Fréchet space with topology defined by the seminorms

$$\|\varphi\|_j = \inf(|\varphi_1|_j + |\varphi_2|_j) ; \quad \varphi = \varphi_1 - \varphi_2, \ \varphi_1, \varphi_2 \in K), \qquad j \in \mathbf{N}$$

where $|\cdot|_j$ are a generating family of seminorms on F.

In this paper we will consider δK and its dual where K is the convex cone of plurisubharmonic functions. Function spaces of this type have been studied by Arsove [1], Kiselman [6] and Cegrell [4], [5]. The main result of this paper is to be foûnd in section 5. We prove that, on pseudoconvex sets, every continuous functional on δPSH which is carried by a compact pluripolar set can be written as a difference of two positive functionals.

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The following notation will be used. Let U be an open subset of $\mathbf{C}^n$. $C^p(U)$ are the p times continuously differentiable functions, $C_0^p(U)$ those with compact support in U. SH(U) and PSH(U) denote the subharmonic and the plurisubharmonic functions respectively. Ha(U) stands for the harmonic and Ph(U) for the plurisubharmonic functions. By $B(U)$ we mean the positive Borelmeasures on U. $B(U)$ is a closed convex cone in $\delta B(U)$ where $\delta B(U)$ carries the topology of total variation on compact subsets of U.

2. Positive linear operators.

Our general reference for this section is Schaefer [7]. Let F be a locally convex topological vector space over $\mathbf{R}$ and let K be a convex cone contained in F. Put

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$$K' = \{\mu \in F' \; ; \; \mu(x) \geqq 0 \; \forall x \in K\} .$$

THEOREM 2.1. *Assume that there is a generating family of seminorms $(P_i)_{i \in I}$ on F such that*

$$P_i(x+y) = P_i(x) + P_i(y) \qquad \forall x, y \in K, \; i \in I$$

Then $F' = \delta(K')$

PROOF. Indeed, if $\xi \in F'$ we can write $\xi = \eta + \frac{1}{2}\xi - (\eta - \frac{1}{2}\xi)$ where $\eta \in F'$ is an extension to all of F of $CP_i|_K$ for a suitable choice of $i \in I$ and $C \in \mathbf{R}^+$.

Consider now δK where K is a convex cone in a linear space F. Assume that we have a family $(P_i)_{i \in N}$ of seminorms on δK. Put

$$\|\varphi\|_i = \inf (P_i(\varphi_1) + P_i(\varphi_2) \; ; \; \varphi = \varphi_1 - \varphi_2, \; \varphi_1, \varphi_2 \in K), \qquad \varphi \in \delta K, \; i \in \mathbf{N} .$$

Then $(\|\cdot\|_i)_{i=1}^{\infty}$ is a family of seminorms turning δK into a metrizable locally convex topological vector space. Furthermore, if δK is Hausdorff for $(P_i)_{i=1}^{\infty}$ then δK is Hausdorff for $(\|\cdot\|_i)_{i=1}^{\infty}$. We call this topology a δ-topology defined by the seminorms $(P_i)_{i=1}^{\infty}$. Observe that $P_i(\varphi) = \|\varphi\|_i$ for all φ in K.

The following Theorem has the same proof as Lemma 2, p. 221 in Schaefer [7].

THEOREM 2.2. *Assume that we are given a family $(P_i)_{i=1}^{\infty}$ of seminorms on δK turning it into a Hausdorff space. If every Cauchy sequence of the special form $s_n = \sum_{v=1}^{n} \varphi_v$, $\varphi_v \in K$ is convergent with limit in K (when δK is provided with the $(P_i)_{i=1}^{\infty}$ topology), then the δ-topology turns δK into a Fréchet space.*

THEOREM 2.3. *Assume that δK and δL have been equipped with δ-topologies which turn δK into a Fréchet space and δL into a Hausdorff space such that $(\delta L)' = \delta(L')$. Let $u \colon \delta K \to \delta L$ be a linear map such that $u(K) \subset L$. Consider*

$$\begin{array}{ccc} \delta K & \xrightarrow{\;u\;} & \delta u(K) \subset \delta L \\ {\scriptstyle \Phi}\downarrow & \nearrow {\scriptstyle \tilde{u}} & \\ \delta K/\operatorname{Ker} u & & \end{array}$$

where Φ is the canonical map and $\tilde{u}$ the map which makes the diagram commutative.

Then:

1. *u is continuous.*
2. *$\tilde{u}^{-1}$ is continuous if and only if $\delta u(K)$ is a Fréchet space for the topology induced by δL.*

3. *If* $\tilde{u}^{-1}$ *is continuous then every element in* $(\delta K)'$ *which vanishes on* $\operatorname{Ker} u$ *is in* $\delta(K')$.

COROLLARY 2.4. *If* $(\delta u(K))' = \delta(u(K)')$ *when* $\delta u(K)$ *carries the* δ*-topology defined by the seminorms on* δL *then*

$$u\colon \delta K \to \delta u(K)$$

is continuous when $\delta u(K)$ *carries the* δ*-topology defined by the seminorms on* δL.

PROOF OF THE THEOREM. 1. follows from Chapter V, Theorem 5.5 and Theorem 5.6 in Schaefer [7].

2. $\operatorname{Ker} u$ is a closed subspace of δK since u is continuous. Thus $\delta K/\operatorname{Ker} u$ is a Fréchet space. Furthermore, $\tilde{u}$ is continuous so if $\delta u(K)$ is a Fréchet space it follows from the closed graph theorem that $\tilde{u}^{-1}$ is continuous.

On the other hand, if $\tilde{u}^{-1}$ is continuous, it is clear that $\delta u(K)$ must be a Fréchet space.

3. Given $\mu \in (\delta K)'$ vanishing on $\operatorname{Ker} u$. Put

$$\tilde{\mu}(\tilde{\varphi}) = \mu(\Phi^{-1}(\tilde{\varphi})); \qquad \tilde{\varphi} \in \delta K/\operatorname{Ker} u\,.$$

It is clear that $\tilde{\mu}$ is well-defined and continuous. Since $\tilde{u}^{-1}$ is continuous $\tilde{\mu}\circ\tilde{u}^{-1}$ is continuous so by the Hahn–Banach theorem there is a $\nu \in (\delta L)'$ extending $\tilde{\mu}\circ\tilde{u}^{-1}$. By assumption $\nu = \nu_1 - \nu_2$ where $\nu_1, \nu_2 \in L'$. Since u is continuous and since $u(K) \subset L$ we have $\nu_1 \circ u$, $\nu_2 \circ u \in K'$ and

$$\begin{aligned}\nu_1\circ u - \nu_2\circ u &= \nu_1\circ\tilde{u}\circ\Phi - \nu_2\circ\tilde{u}\circ\Phi \\ &= (\nu_1-\nu_2)\circ\tilde{u}\circ\Phi = \tilde{\mu}\circ\tilde{u}^{-1}\circ\tilde{u}\circ\Phi = \tilde{\mu}\circ\Phi = u\end{aligned}$$

and the proof is complete.

PROOF OF THE COROLLARY. Take $L = u(K)$ in 1.

3. Delta-plurisubharmonic functions and currents.

Let U be an open subset of $\mathbf{C}^n$. Then PSH (U) is a closed convex cone of $L_{\text{loc}}(U)$ and we can form the space δPSH (U) which is a Fréchet space with the δ-topology defined by the seminorms on $L^1_{\text{loc}}(U)$. We put

$$\|\varphi\|_A = \inf\left(\int_A |\varphi_1| + |\varphi_2|\ ;\ \varphi = \varphi_1 - \varphi_2, \varphi_1, \varphi_2 \in \text{PSH}\,(U)\right), \qquad \varphi \in \delta\text{PSH}\,(U)$$

for $A \subset\subset U$.

DEFINITION. A compact subset K of U is said to be a *carrier* for $\mu \in (\delta\mathrm{PSH}(U))'$ if to any open subset O containing K there is a constant c so that

$$|\mu(\varphi)| \leqq c\|\varphi\|_O, \quad \forall \varphi \in \delta\mathrm{PSH}(U).$$

DEFINITION. A subset K of U is said to be a *support* for $\mu \in (\delta\mathrm{PSH}(U))'$ if, for any open O with $K \subset\subset O$, $\mu(\varphi)=0$ for every $\varphi \in \delta\mathrm{PSH}(U)$ which vanishes on O.

The following lemma will be used later on. The proof is similar to that of Lemma 3.4 in Kiselman [6] and Lemma 1.6 in Cegrell [4].

LEMMA 3.5. *Let U be pseudoconvex. Assume that $\varphi, \psi \in \mathrm{PSH}(U)$ are continuous and equal in a neighborhood of a compact holomorphically convex set K. Then $\|\varphi-\psi\|_K=0$.*

DEFINITION. Denote by $S(U)$ the convex cone of closed and positive $(1,1)$-currents,

$$t = i\sum_{i,j} t_{ij} dz_i \wedge d\bar{z}_j$$

and by $\delta S(U)$ differences of such elements. The coefficients t_{ij} are measures on U and by use of Theorem 2.2 it is easily seen that $\delta S(U)$ is a Fréchet space with topology defined by the seminorms

$$\|t\|_K = \inf \int_K \sum_{i=1}^{n} t_{ii}^1 + t_{ii}^2, \qquad K \subset\subset U$$

where the inf is taken over

$$t^1 = i\sum_{i,j} t_{ij}^1 dz_i \wedge d\bar{z}_j \in S(U)$$

$$t^2 = i\sum_{i,j} t_{ij}^2 dz_i \wedge d\bar{z}_j \in S(U)$$

with $t=t^1-t^2$.

REMARK. It follows from Theorem 2.1 that $(\delta S(U))' = \delta(S(U)')$.

4. Operators vanishing on Ph.

Denote by j the map

$$j = i\partial\bar{\partial}\colon \delta\mathrm{PSH}(U)/\mathrm{Ph}(U) \to \delta S(U)$$

LEMMA 4.6. *The map j is continuous. Furthermore, if U is pseudoconvex with $H^2(U,\mathbf{C})=0$, then j^{-1} is continuous.*

($H^2(U,\mathbf{C})$ denotes the second cohomology group with complex coefficients.)

PROOF. By Theorem 2.3.1 j is continuous. Now, if U is pseudoconvex with $H^2(U,\mathbf{C})=0$ then j is a bijection so by Theorem 2.3.2 j^{-1} is continuous.

THEOREM 4.7. *Assume that $\mu \in (\delta\mathrm{PSH}\,(U))'$ where U is pseudoconvex with $H^2(U,\mathbf{C})=0$. Then the following conditions are equivalent.*

1) $\mu=\mu_1-\mu_2$ *where* $\mu_1,\mu_2 \in (\mathrm{PSH}'\,(U))$,

2) *there is a compact set K in U and a constant c so that*

$$|\mu(\varphi)| \leqq c\int_K \Delta\varphi, \qquad \forall\, \varphi \in \mathrm{PSH}\,(U)\,,$$

(Δ is the Laplace operator)

3) *μ vanishes on the pluriharmonic functions.*

PROOF. It is clear that 2) $\Rightarrow$ 1) $\Rightarrow$ 3). That 3) $\Rightarrow$ 2) follows from Lemma 4.6. (It follows directly from Lemma 4.6 and Theorem 2.3,3 that 1) $\Leftrightarrow$ 3).)

DEFINITON. Let $M(U)$ denote the positive measures μ on U which can be written $\mu=\Delta\varphi$ for a $\varphi \in \mathrm{PSH}\,(U)$. $\delta M(U)$ is the set of differences of such elements.

DEFINITION. Denote by $m(U)$ the positive measures on U which are in $M(U^1)$ for every pseudoconvex $U^1 \subset\subset U$ with $H^2(U^1,\mathbf{C})=0$. $\delta m(U)$ is the set of differences of such measures.

THEOREM 4.8. *$\delta m(U)$ is a Fréchet space with topology defined by the seminorms*

$$|||t|||_K = \inf\left(\int_K t_1+t_2\ ;\ t=t_1-t_2,\ t_1,t_2 \in m(U)\right); \qquad K\subset\subset U\,.$$

Furthermore, if U is pseudoconvex then $\delta m(U)=\delta M(U)$, Ker $\Delta =$ Ha (U) *and the operators in the following diagram are continuous.*

$$\begin{array}{ccc} \delta\mathrm{PSH}\,(U) & \xrightarrow{\ \Delta\ } & \delta m(U) \\ \Phi \downarrow & \tilde{\Delta}^{-1} \swarrow\!\nearrow \tilde{\Delta} & \\ \delta\mathrm{PSH}\,(U)/\mathrm{Ker}\,\Delta & & \end{array}$$

PROOF. By Theorem 2.2 it is enough to prove that every Cauchy sequence $(s_n)_{n=1}^{\infty}$ of the form

$$s_n = \sum_{\nu=1}^{n} t_\nu, \quad t_\nu \in m(U)$$

is convergent with limit in $m(U)$. It is clear that $\sum_{\nu=1}^{\infty} t_\nu$ is a positive measure on U and we claim that $\sum_{\nu=}^{\infty} t_\nu \in m(U)$.

Given a psedoconvex set $U^1 \subset\subset U$ with $H^2(U,\mathbf{C})=0$, we can find $\varphi_n \in \mathrm{PSH}\,(U^1)$ with

$$\Delta\varphi_n = s_n = \sum_{\nu=1}^{n} t_\nu .$$

This means that $\lim_{n\to\infty} i\partial\bar{\partial}\varphi_n$ exists as a positive, closed (1,1)-current on U^1. So there is a $\varphi \in \mathrm{PSH}\,(U^1)$ with

$$i\partial\bar{\partial}\varphi = \lim_{n\to\infty} i\partial\bar{\partial}\varphi_n .$$

In particular $\Delta\varphi = \sum_{\nu=1}^{\infty} t_\nu$ hence $\lim_{n\to+\infty} s_n = \sum_{\nu=1}^{\infty} t_\nu \in m(U)$.

Assume now that U is pseudoconvex. It is clear that $\delta M(U) \subset \delta m(U)$ and Theorem 5.3 in Kiselman [6] proves that $m(U) \subset \delta M(U)$, hence $\delta m(U) = \delta M(U)$.

Furthermore, Ha$\,(U)$ is a linear subspace of δPSH$\,(U)$ by Kiselman [6, Proposition 5.1] and it is closed since the topology on δPSH$\,(U)$ is stronger than that induced by $L^1_{\mathrm{loc}}(U)$. Thus δPSH$\,(U)/$Ha$\,(U)$ is a Fréchet space. The continuity of Δ and $\tilde{\Delta}^{-1}$ follows now from Theorem 2.3, 1–2.

THEOREM 4.9. *Let U be pseudoconvex. Assume that $\mu \in (\delta\mathrm{PSH}\,(U))'$ and that μ vanishes on* Ha$\,(U)$. *Then $\mu = \mu_1 - \mu_2$ where $\mu_1, \mu_2 \in$* PSH$'\,(U)$.

PROOF. Theorem 4.8 and Theorem 2.3,3.

5. Delta-plurisubharmonic functionals with a small carrier.

THEOREM 5.10. *Let U be pseudoconvex and assume that $\mu \in (\delta\mathrm{PSH}(U))'$. If μ is carried by a compact pluripolar set then $\mu(\varphi)=0$ for every continuous plurisubharmonic function φ.*

PROOF. Let P be a compact pluripolar set which carries μ. There is a $\psi \in \mathrm{PSH}\,(U)$, $\psi \not\equiv -\infty$ such that

$$P \subset \{z \in U \;;\; \psi(z) = -\infty\} = P_1 .$$

Choose K_n, $n \in \mathbf{N}$, a fundamental sequence of compact in U. Given φ, continuous and plurisubharmonic on U.

Put
$$\theta_n = \sup(\varphi - \sup_{K_n} \varphi, \psi/n^2).$$

Then $\theta_n = \varphi - \sup_{K_n} \varphi$ near P_1 so $\mu(\theta_n) = \mu(\varphi - \sup_{K_n} \varphi)$ by Lemma 3.5. Furthermore, $(\sum_{\nu=1}^{n} \theta_\nu)_{n=1}^{\infty}$ is a Cauchy sequence in δPSH (U). Indeed, given $K \subset\subset U$ we choose n_0 so that $K \subset K_{n_0}$. If $s > t > n_0$ then

$$\left\| \sum_{\nu=1}^{s} \theta_\nu - \sum_{\nu=1}^{t} \theta_\nu \right\|_K \leqq \left\| \sum_{\nu=t+1}^{s} \theta_\nu \right\|_{K_{n_0}} = \int_{K_{n_0}} \left| \sum_{\nu=t+1}^{s} \theta_\nu \right| \leqq \int_{K_{n_0}} \sum_{\nu=t+1}^{s} |\theta_\nu|$$
$$\leqq \sum_{\nu=t+1}^{s} \frac{1}{\nu^2} \int_{K_{n_0}} |\psi| \to 0, \quad t \to +\infty .$$

Thus $(\mu(\sum_{\nu=1}^{n} \theta_\nu))_{n=1}^{\infty}$ is a bounded set. Now, μ vanishes on constants since

$$N\mu(-1) = \mu\left(\sum_{\nu=1}^{N} \sup(-1, \psi/\nu^2) \right)$$

and since

$$\left(\sum_{\nu=1}^{n} \sup(-1, \psi/\nu^2) \right)_{n=1}^{\infty}$$

is a Cauchy sequence in δPSH (U). Hence

$$\left(\mu\left(\sum_{\nu=1}^{N} \theta_\nu \right) \right)_{N=1}^{\infty} = (N\mu(\varphi))_{N=1}^{\infty}$$

is a bounded sequence and it follows that $\mu(\varphi) = 0$.

THEOREM 5.11. *Let U be pseudoconvex and assume that $\mu \in (\delta\text{PSH}(U))'$. If μ is carried by a compact pluripolar set then*

$$\mu = \mu_1 - \mu_2 \textit{ where } \mu_1, \mu_2 \in \text{PSH}'(U).$$

PROOF. Since any $h \in \text{Ha}(U)$ can be written as a difference of two continuous plurisubharmonic functions it follows from Theorem 5.10 that vanishes on Ha (U). The Theorem follows now from Theorem 4.9.

Let $\nu(z, \varphi)$ denote the Lelong number at z.

LEMMA 5.12. *Let σ be a positive measure with compact support in U. Then*

$$\delta\text{PSH}(U) \ni \varphi \to \int \nu(z, \varphi)\, d\sigma(z) \in \mathbf{R}$$

defines an element in PSH$'$ (U).

PROOF. Theorem 2.3,1.

It has been proved by Kiselman [6, Theorem 6.2] that if $\mu \in (\delta\mathrm{PSH}(\mathbf{C}^n))'$ is carried by zero and if $\mu(\varphi\circ\alpha)=\mu(\varphi)$ for all $\varphi \in \delta\mathrm{PSH}(\mathbf{C}^n)$ and all unitary transformations α then μ is a constant times the Lelong number at zero. The following example shows that, in contradistinction to the convex and subharmonic cases, there are functionals in $\mathrm{PSH}'(U)$ with disjoint carriers and supports.

EXAMPLE. ($\mathbf{C}^2$) Let D denote the set $\{(z_1,z_2)\in\mathbf{C}^2 ; |z_1|<1, |z_2|<1\}$ and put

$$A(\varphi) = \int_{|z_2|<1/2} \nu((0,z_2),\varphi)\,dm$$

where m is the Lebesgue measure in $\mathbf{C}^1$ and $\varphi \in \delta\mathrm{PSH}(D)$. By Lemma 5.12 $A \in \mathrm{PSH}'(D)$ and since $A(\log|z_1|)>0$, A is not identically zero. By Siu [8, p. 89] $\nu((0,z_2),\varphi)$ is constant a.e. on $|z_2|<1$ so A is carried and supported by every point in $\{z_1=0, |z_2|<1\}$.

6. An application.

On $\mathbf{C}^2$ we write $d=\partial+\bar\partial$, $d^c=\partial-\bar\partial$ so that $dd^c\varphi=2i\partial\bar\partial\varphi$. The operator

$$(dd^c\varphi)^2 = dd^c\varphi \wedge dd^c\varphi ,$$

φ plurisubharmonic and continuous, has been studied by Bedford and Taylor in [2] and [3]. (In particular, see Section 5 in [3].)

Consider the bilinear map

$$C^2(U)\times C^2(U) \xrightarrow{t_U} \delta B(U)$$

defined by

$$t(\varphi,\psi)(\varphi) = \int \Phi dd^c\varphi \wedge dd^c\psi, \qquad \Phi \in C_0^\infty(U) .$$

THEOREM 6.13. *Consider* PSH $(U)\cap C(U)$ *as a closed convex cone in* $C(U)$ *and form* $\delta-(\mathrm{PSH}(U)\cap C(U))$. *The map* t_U *has an extension* T_U

$$[\delta-(\mathrm{PSH}(U)\cap C(U))]\times\delta\mathrm{PSH}(U) \xrightarrow{T_U} \delta B(U) .$$

T_U *is continuous and* $T(\varphi,\psi)$ *is a positive measure for* $\varphi \in \mathrm{PSH}(U)\cap C(U)$, $\psi \in \mathrm{PSH}(U)$.

PROOF. Define

$$T_U(\varphi, \psi)(\Phi) = \int \varphi \, dd^c\Phi \wedge dd^c\psi$$

for $\varphi \in \delta - (\mathrm{PSH}(U) \cap C(U))$, $\psi \in \delta\mathrm{PSH}(U)$, $\Phi \in C_0^\infty(U)$. T_U is then well-defined since $\varphi dd^c\Phi$ has continuous coefficients with compact support and since the coefficients of $dd^c\psi$ are Borel measures.

If $\varphi, \psi \in C^2(U)$ then by Proposition 2.1 in Bedford and Taylor [2] we have

$$T_U(\varphi, \psi)(\Phi) = t_U(\varphi, \psi)(\Phi)$$

which proves that T_U extends t_U.

Furthermore, by means of a regularisation of φ and ψ it is easy to see that $T_U(\varphi, \psi)$ is a positive measure for $\varphi \in \mathrm{PSH}(U) \cap C(U)$, $\psi \in \mathrm{PSH}(U)$. It follows now from Theorem 2.3,1 that T_U is separately continuous and therefore continuous since both $\delta - (\mathrm{PSH}(U) \cap C(U))$ and $\delta\mathrm{PSH}(U)$ are Fréchet spaces.

REMARK. There is no continuous bilinear form on $\delta\mathrm{PSH}(U) \times \delta\mathrm{PSH}(U)$ which extends t_U.

This follows from an example of Shiffman and Taylor which can be found in Siu [9].

THEOREM 6.14. *If* $\varphi \in \mathrm{PSH}(U) \cap C(U)$ *then* $T_U(\varphi, \varphi)$ *has no mass concentrated on a pluripolar set.*

PROOF. Given $\varphi \in \mathrm{PSH}(U) \cap C(U)$ and P, a compact pluripolar subset of U. We have to prove that

$$\int_P dT_U(\varphi, \varphi) = 0 .$$

Consider the linear form

$$\delta\mathrm{PSH}(U) \ni \psi \to \int_P dT_U(\varphi, \psi) .$$

This form is carried by P so, by Theorem 5.10 it vanishes on the continuous plurisubharmonic functions. In particular,

$$\int_P dT_U(\varphi, \varphi) = 0$$

and the proof is complete.

REMARK. For $n=2$, Theorem 6.14 is a sharper version of Corollary 2.5 in Bedford and Taylor [2].

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