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ACKNOWLEDGMENT OF PRIORITY: SECOND DERIVATIVES OF CONVEX FUNCTIONS

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I. Ya. Bakel'man (1965) proved that for every convex function f on $\mathbf{R}^n$, its second partial derivatives in the sense of distributions (generalized functions), $\partial^2[f]/\partial x_i \partial x_j$, are signed Radon measures (finite on compact sets) ν_{ij}. Yu. G. Reshetnyak (1968) proved that a locally integrable function f on U, a convex open subset of $\mathbf{R}^n$, is (equal almost everywhere to) a convex function if and only if for every real $\xi_1, \ldots, \xi_n$, $\sum_{i,j=1}^n \xi_i \xi_j \partial^2[f]/\partial x_i \partial x_j$ is a nonnegative Radon measure. This is close to the theorems in section 3 of [3]: a distribution T on U is of the form $T=[f]$ for f convex if and only if $\{\partial^2 T/\partial x_i \partial x_j\}_{i,j=1}^n$ is a nonnegative matrix-valued Radon measure ν. Also, Reshetnyak (1968, Theorem 3) obtains a proof of the theorem of A. D. Alexandrov (1939): for any convex f on $\mathbf{R}^n$, the pointwise second derivative tensor $D^2 f$, taken through the set where there is a first derivative vector Df, exists Lebesgue almost everywhere. Thus a question at the end of section 5 of [3] is answered.

I have not seen in the literature the other results in sections 2 and 4–10 of [3], in particular Theorem 6.1: for f convex, the measure $D^2[f]=\nu$ is absolutely continuous with respect to $(n-1)$-dimensional Hausdorff measure.

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