MATH. SCAND. 46 (1980), 332-333

ADDENDUM TO "HYPERFINITE STOCHASTIC INTEGRATION III"

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One could think of other ways to represent standard martingales as nonstandard martingales than the method described in [3]. Hoover and Perkins [2] have suggested to use the *-version of the original martingale; a detailed discussion of this representation is announced to appear in Hoover and Keisler [1]. Restricting time to a hyperfinite time-line, and using the weak Loeb-space representation constructed in Theorem 3 of [3], on easily verifies that the *-version is a hyperfinite representation of the original martingale, except that it does not live on a hyperfinite probability space. Changing the *-martingale a little, and passing to a quotient space, this is easily remedied. This gives a much simpler proof of [3, Theorem 3], and also shows that the Hoover-Perkins representation is equivalent to our much more cumbersome construction.

But the more complicated proof given in [3] has its advantages; it contains more information and can be used to obtain more general results: If our nonstandard model is K-saturated, an inspection of the proof of [3, Theorem 7] gives us the following result:

THEOREM 1. *Let $\langle \Omega, \mathscr{G}, P \rangle$ be a hyperfinite probability space, and let $\{\mathscr{F}_t\}$ be a family of sub-σ-algebras of $L(\mathscr{G})$ such that $\mathscr{F}_\infty$ has cardinality less than K. Let M be an L^2-martingale adapted to this family. Then there exists a family $\{\mathscr{G}_t\}$ of internal subalgebras of $\mathscr{G}$ such that M is a martingale adapted to $\{L(\mathscr{G}_t)\}$. Moreover, we may take $\mathscr{F}_t \subset \sigma(L(\mathscr{G}_t) \cup \mathscr{N})$, where $\mathscr{N}$ is the null-sets of $L(P)$.*

If we replace the condition that $\mathscr{F}'_t \subset L(\mathscr{G}_t)$ in Definition 4 of [3] by the slightly weaker condition that $\mathscr{F}'_t \subset \sigma(L(\mathscr{G}_t) \cup \mathscr{N})$ we obtain as a corollary the following stronger version of [3, Theorem 7]:

THEOREM 2. *Let $\{\langle \Omega, \mathscr{G}, P \rangle, \mathscr{F}', \theta\}$ be a weak Loeb-space representation of the probability space $\langle Z, \mathscr{F}, \mu \rangle$, and let M be an L^2-martingale adapted to a family $\{\mathscr{F}_t\}$ of sub-σ-algebras of $\mathscr{F}$. Then there exists a weak Loeb-space representation $\{\langle \Omega, \{\mathscr{G}_t\}, P \rangle, \mathscr{F}'_t, \theta, M^\theta\}$ of M.*

Thus as long as we can represent the probability space by a mapping θ, we can represent the martingale by the *same* mapping. If e.g. Ω has reasonable topological structure and can be represented by a standard part map, so can M. (However, the last example can also be obtained by the Hoover-Perkins approach using the representation theorems for Radon-spaces proved by R. M. Anderson.) We also get corresponding stronger versions of [3, Theorems 13 and 14].

REFERENCES

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