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CROSS SECTIONS FOR QUOTIENT MAPS OF LOCALLY COMPACT GROUPS

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1.

In this paper G denotes a locally compact group, H a closed subgroup and $p=p_H$ the quotient map of G on G/H.

It is shown that there exists a locally bounded Baire cross section for p, i.e. map q of G/H into G such that $q(C)$ is relatively compact, when C is compact, $q^{-1}(B)$ is a Baire set, when B is a Baire set, and $p(q(x))=x,\ x \in G/H$.

This has been proved by G. W. Mackey [11], when G has countable basis for the topology and by S. Graf and G. Mägerl [5], when G is compact, and follows easily from the results of J. Feldman and F. P. Greenleaf [3], when H is metrizable.

The stronger result that q can be obtained continuous in a neighbourhood of $p(e)$ has been proved, when G/H has finite dimension by P. S. Mostert [13], cf. D. Montgomery and L. Zippin [12, section 4.15], and when H is a Lie group by A. M. Gleason, cf. [4], [13].

The method used here combines a proof from [3] and a Zorn's lemma argument, much like the argument in [12], on the set of cross sections $G/H \to G/K$, K compact normal subgroup of H.

The result was wanted in [14], cf. [10].

It follows that the bijection $(h,x) \mapsto q(x)h$ of $H \times G/H$ on G and its inverse are locally bounded Baire maps and preserve measurability of sets.

Finally it is shown that some results, e.g. that $C^*(G, G/H)$ is isomorphic to $C^*(H) \otimes \mathscr{K}(L^2(G/H))$, proved by Ph. Green under an extra condition [6, section 2], holds generally.

2.

When K and L are closed subgroups of G with $K \subseteq L$, we let p_K denote the quotient map of G on G/K, and $p_{K,L}$ the quotient map of G/K on G/L with $p_{K,L} \cdot p_K = p_L$. A cross section for $p_{K,L}$ is a map $q_{L,K}$ of G/L into G/K with $p_{K,L}(q_{L,K}(x))=x,\ x \in G/L$.

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Let R and S be locally compact spaces and f a map of R into S. We say that f is locally bounded if $f(C)$ is relatively compact when C is compact; f is called a Baire map, if $f^{-1}(B)$ is a Baire set [8] in R for each Baire set B in S; f is called a local Baire map if $f^{-1}(B) \cap C$ is a Baire set when B and C are Baire sets.

Note that continuous maps are locally bounded local Baire maps and that composition of locally bounded local Baire maps gives a locally bounded loĉal Baire map.

If f is open and continuous (like p) and C is compact of type G_δ then so is $f(C)$; in fact C is by Urysohn's lemma intersection of a decreasing sequence $(C_n)_{n \in \mathbb{N}}$ of compact neighbourhoods and $f(C) = \bigcap_{n \in \mathbb{N}} f(C_n)$.

It is wellknown that Haar measures are completion regular [8]; so are quasi invariant measures on quotient spaces G/H [2]. In fact to a measurable relatively compact subset M of G/H, we can choose an open relatively compact subset U of G with $p(U) \supseteqq M$ and a sequence $(C_n)_{n \in \mathbb{N}}$ of compact G_δ subsets of $p^{-1}(M) \cap U$ with measures increasing to the measure of $p^{-1}(M) \cap U$; then $M \setminus \bigcup_{n \in \mathbb{N}} p(C_n)$ has measure zero because any compact subset of $p^{-1}(M \setminus \bigcup_{n \in \mathbb{N}} p(C_n))$ is covered by finitely many translates $(U \cap p^{-1}(M) \setminus \bigcup_{n \in \mathbb{N}} C_n H)h$, $h \in H$.

Let ν be a Radom measure on R. We call a map of R into a topological space measurable if for any compact set in R the restriction of the map to some compact subset with almost the same measure is continuous [1].

If f is a locally bounded local Baire map and φ is a continuous map of S into a metrizable space, then $\varphi \circ f$ is measurable [1].

If h is a measurable map of R into a Banach space E and ψ is a bounded map of R into the Banach space of bounded linear operators on E with $\psi(\cdot)e$ measurable for each $e \in E$, then $r \mapsto \psi(r)(h(r))$ is measurable. In fact any compact set in R has a compact subset K with almost the same measure, such that $h(K)$ has a dense countable subset F; making K a little smaller we may assume that h and $\psi(\cdot)e$, $e \in F$, are continuous on K; then $\psi(\cdot)e$ is continuous on K for $e \in h(K)$ and $r \mapsto \psi(r)(h(r))$ is continuous on K.

3.

LEMMA 1. *Assume G is σ-compact; let $(U_n)_{n \in \mathbb{N}}$ be a sequence of neighbourhoods of e in G. There exists a Baire subset T of G with the properties:*

Each coset gH, $g \in G$, intersets T in a non-empty compact set,

$T \cap CH$ is relatively compact for each compact subset C of G,

$T^{-1}T \cap H \subseteqq \bigcap_{n \in \mathbb{N}} U_n$, and

$p(B \cap T)$ is a Borel set, when B is closed, and a Baire set, when B is a closed Baire set.

PROOF. The proof closely follows Feldman and Greenleaf [3]; for the reader's convenience I reproduce it here.

We may assume that U_n is a compact Baire set and that $U_{n+1}^{-1}U_{n+1} \subseteq U_n$, $n \in \mathbf{N}$.

Choose a sequence $(g(i))_{i \in \mathbf{N}}$ from G such that the sets $g(i)p(U_2)$, $i \in \mathbf{N}$, give a locally finite covering of G/H, and define $V(i)=g(i)U_2$, $U(i)=g(i)U_1$, $i \in \mathbf{N}$.

Now assume that for some $n \in \mathbf{N}$ and $m=1,2,\ldots,n$ we have chosen elements $g(i_1,i_2,\ldots,i_m) \in G$ and compact G_δ sets $U(i_1,i_2,\ldots,i_m)$ and $V(i_1,i_2,\ldots,i_m)$, $(i_1,i_2,\ldots,i_m) \in \mathbf{N}^m$, such that

$$G/H = \bigcup_{i \in \mathbf{N}} p(V(i)) ,$$

$$p(V(i_1,i_2,\ldots,i_{m-1})) \subseteq \bigcup_{i \in \mathbf{N}} p(V(i_1,i_2,\ldots,i_{m-1},i)), \qquad m=2,3,\ldots,n ,$$

$U(i_1,i_2,\ldots,i_m)$ is contained in the interior of

$$U(i_1,i_2,\ldots,i_{m-1}) \cap g(i_1,i_2,\ldots,i_m)U_m, \qquad m=2,3,\ldots,n ,$$

and $V(i_1,i_2,\ldots,i_m)$ is contained in the interior of $U(i_1,i_2,\ldots,i_m)$, $m=1,2,\ldots,n$.

Let $(i_1,i_2,\ldots,i_n) \in \mathbf{N}^n$. Choose for $g \in V(i_1,i_2,\ldots,i_n)$ two compact G_δ neighbourhoods $U(i_1,i_2,\ldots,i_n)(g)$ and $V(i_1,i_2,\ldots,i_n)(g)$ of g such that $U(i_1,i_2,\ldots,i_n)(g)$ is contained in the interior of $U(i_1,i_2,\ldots,i_n) \cap gU_{n+1}$ and $V(i_1,i_2,\ldots,i_n)(g)$ is contained in the interior of $U(i_1,i_2,\ldots,i_n)(g)$.

Choose a sequence $(g(i_1,i_2,\ldots,i_n,i))_{i \in \mathbf{N}}$ from $V(i_1,i_2,\ldots,i_n)$ such that, with the notation

$$V(i_1,i_2,\ldots,i_n)(g(i_1,i_2,\ldots,i_n,i_{n+1})) = V(i_1,i_2,\ldots,i_{n+1})$$

and correspondingly for $U(i_1,i_2,\ldots,i_{n+1})$, we have

$$p(V(i_1,i_2,\ldots,i_n)) \subseteq \bigcup_{i \in \mathbf{N}} p(V(i_1,i_2,\ldots,i_n,i)) .$$

So we can recursively choose such elements and sets for each $n \in \mathbf{N}$.

Define $B(1)=p(V(1))$ and

$$B(i) = p(V(i)) \setminus \bigcup_{j<i} p(V(j)), \qquad i>1 ,$$

and recursively

$$B(i_1,i_2,\ldots,i_n,1) = p(V(i_1,i_2,\ldots,i_n,1)) \cap B(i_1,i_2,\ldots,i_n)$$

and

$$B(i_1,i_2,\ldots,i_n,i)$$
$$= B(i_1,i_2,\ldots,i_n) \cap p(V(i_1,i_2,\ldots,i_n,i)) \setminus \bigcup_{j<i} p(V(i_1,i_2,\ldots,i_n,j)) , \qquad i>1 .$$

This gives us Baire sets in G/H, pairwide disjoint for fixed $n \in \mathbf{N}$, with $G/H = \bigcup_{n \in \mathbf{N}} B(i)$ and

$$B(i_1, i_2, \ldots, i_n) = \bigcup_{i \in \mathbf{N}} B(i_1, i_2, \ldots, i_n, i), \qquad n \in \mathbf{N}\,.$$

The sets $U(i_1, i_2, \ldots, i_n) \cap p^{-1}(B(i_1, i_2, \ldots, i_n))$ are Baire sets in G, pairwise disjoint for fixed $n \in \mathbf{N}$, with

$$p\big(U(i_1, i_2, \ldots, i_n) \cap p^{-1}(B(i_1, i_2, \ldots, i_n))\big) = B(i_1, i_2, \ldots, i_n)\,.$$

Define

$$T_n = \bigcup_{i_1} \bigcup_{i_2} \cdots \bigcup_{i_n} U(i_1, i_2, \ldots, i_n) \cap p^{-1}(B(i_1, i_2, \ldots, i_n)), \qquad n \in \mathbf{N}\,,$$

and $T = \bigcap_{n \in \mathbf{N}} T_n$. For $g \in G$ there is a unique sequence $(i_1, i_2 \ldots)$ such that $p(g) \in \bigcap_{n \in \mathbf{N}} B(i_1, i_2, \ldots, i_n)$; we find

$$gH \cap T_n = gH \cap U(i_1, i_2, \ldots, i_n) \subseteqq g(i_1, i_2, \ldots, i_n) U_n, \qquad n \in \mathbf{N};$$

as $(gH \cap U(i_1, i_2, \ldots, i_n))_{n \in \mathbf{N}}$ is a decreasing sequence of non-empty compact sets, $gH \cap T = \bigcap_{n \in \mathbf{N}} gH \cap T_n$ is a non-empty compact set. For any $y \in gH \cap T$ we have $y \in g(i_1, i_2, \ldots, i_n) U_n$ and

$$y^{-1} g(i_1, i_2, \ldots, i_n) U_n \subseteqq U_n^{-1} U_n \subseteqq U_{n-1}\,,$$

so

$$gH \cap T \subseteqq yH \cap g(i_1, i_2, \ldots, i_n) U_n \subseteqq y U_{n-1}, \qquad n > 1\,,$$

and

$$gH \cap T \subseteqq y \bigcap_{n \in \mathbf{N}} U_n\,.$$

Equivalently $T^{-1} T \cap H \subseteqq \bigcap_{n \in \mathbf{N}} U_n$.

If C is compact in G, $p(C)$ is covered by finitely many of the sets $p(V(i))$ and $T_1 \cap CH$ is covered by the corresponding sets $U(i)$. Thus $T \cap CH$ is relatively compact.

Now assume B is a closed Baire set in G. By a compactness argument

$$p(B \cap T) = \bigcap_{n \in \mathbf{N}} p(B \cap T_n)\,.$$

As $B \cap U(i_1, i_2, \ldots, i_n)$ is a compact Baire set, and

$$p(B \cap T_n) = \bigcup_{i_2} \bigcup_{i_2} \cdots \bigcup_{i_n} p(B \cap U(i_1, i_2, \ldots, i_n)) \cap B(i_1, i_2, \ldots, i_n)\,,$$

we see that $p(B \cap T)$ is a Baire set.

In the same way it is seen that $p(B \cap T)$ is a Borel set when B is closed.

LEMMA 2. *Assume G is σ-compact; let K be a closed normal subgroup of H with H/K metrizable. There exists a locally bounded Baire and Borel cross section for $p_{K,H}$.*

PROOF. Choose a sequence $(U_n)_{n\in\mathbf{N}}$ of neighbourhoods of e in G with $\bigcap_{n\in\mathbf{N}} U_n \cap H = K$, and to this a Baire set T in G as in Lemma 1.

From $T^{-1}T \cap H \subseteq K$ we get that $p^{-1}(x) \cap T$ is contained in one K coset $q_K(x)$, i.e. $q_K(x)$ is determined by $\{q_K(x)\} = p_K(p^{-1}(x) \cap T)$.

If M is compact in G/H, then $p^{-1}(M) \cap T$ and $q_K(M) = p_K(p^{-1}(M) \cap T)$ are relatively compact.

If B is a compact G_δ set in G/K, then $p_K^{-1}(B)$ is a closed Baire set in G so $q_K^{-1}(B) = p(p_K^{-1}(B) \cap T)$ is a Baire set.

In the same way it is shown that q_K is a Borel map.

THEOREM. *Let G be a locally compact group and H a closed subgroup. There exists a locally bounded Baire cross section for the quotient map of G on G/H.*

PROOF. Assume first that G is σ-compact. Let $\mathscr{M}$ denote the set of pairs (K, q_K) where K is a compact normal subgroup of H and q_K is a locally bounded Baire cross section for $p_{K,H}$.

Let $\mathscr{K}$ denote the set of compact normal subgroups K of H with H/K metrizable; then $\bigcap_{K\in\mathscr{K}} K = \{e\}$ [7, Corollary A [10] and for each $K \in \mathscr{K}$ there exists by Lemma 2 a locally bounded Baire cross section for $p_{K,H}$. Thus $\mathscr{M}$ is not empty.

Define when (K, q_K) and (L, q_L) belong to $\mathscr{M}$ that $(K, q_K) \leqq (L, q_L)$ if $L \subseteq K$ and $p_{L,K} \circ q_L = q_K$. This gives a partial ordering of $\mathscr{M}$. We show that $\mathscr{M}$ is inductively ordered.

Let $\tilde{\mathscr{L}}$ be a completely ordered non-empty subset of $\mathscr{M}$, let $\mathscr{L}$ denote the set of first components of the pairs (L, q_L) in $\tilde{\mathscr{L}}$, and set $K = \bigcap_{L\in\mathscr{L}} L$; K is a compact normal subgroup of H.

Now G/K is projective limit of the spaces G/L, $L \in \mathscr{L}$, that is $x \mapsto (p_{K,L}(x))_{L\in\mathscr{L}}$ defines a homeomorphism φ of G/K onto the closed subset

$$\left\{ (x_L)_{L\in\mathscr{L}} \in \prod_{L\in\mathscr{L}} G/L \;\middle|\; L, M \in \mathscr{L},\ L \subseteq M \Rightarrow p_{L,M} x_L = x_M \right\}$$

of $\prod_{L\in\mathscr{L}} G/L$.

Therefore $x \mapsto \varphi^{-1}(q_L(x))_{L\in\mathscr{L}}$ defines a map q_K of G/H into G/K. If $q_K(x) = y$, then for $(L, q_L) \in \tilde{\mathscr{L}}$ we have $p_{K,L}(y) = q_L(x)$ and

$$p_{K,H}(y) = p_{L,H} \circ p_{K,L}(y) = p_{L,H} \circ q_L(x) = x\,,$$

so $p_{K,L}\circ q_K = q_L$ and $p_{K,H}\circ q_K(x) = x$, $x \in G/H$.

When C is a compact subset of G/H, then $\varphi(G/K) \cap \prod_{L \in \mathscr{L}} \overline{q_L(C)}$ is compact, so q_K is locally bounded.

To show that $q_K^{-1}(B)$ is a Baire set, when B is a Baire set in G/K, it is enough to show that $f \circ q_K$ is a Baire function on G/H for each function f in a dense subalgebra of $C_\infty(G/K)$. By the Stone-Weierstrass Theorem it is enough to observe that $F \circ p_{K,L} \circ q_K = F \circ q_L$ is a Baire function for $L \in \mathscr{L}$, $F \in C_\infty(G/L)$.

Thus (K, q_K) is a majorant for $\tilde{\mathscr{L}}$, and $\mathscr{M}$ is inductively ordered.

Next let (K, q_K) denote some maximal element in $\mathscr{M}$. It only rests to show that $K = \{e\}$, and for this it is enough to show that $K \cap L = K$ for all $L \in \mathscr{K}$.

For $L \in \mathscr{K}$ we have that $KL = LK$ is a compact normal subgroup of H and that KL/L is metrizable.

Let q_{KL} denote $p_{K,KL}\circ q_K$; then q_{KL} is a locally bounded Baire cross section for $p_{KL,H}$.

Choose by Lemma 2 a locally bounded Baire cross section $q_{KL,L}$ for $p_{L,KL}$, and set $q_L = q_{KL,L}\circ q_{KL}$; then q_L is a locally bounded Baire map of G/H into G/L with

$$\begin{aligned} p_{L,H}\circ q_L(x) &= p_{KL,H}\circ p_{L,KL}\circ q_{KL,L}\circ q_{KL}(x) \\ &= p_{KL,H}\circ q_{KL}(x) = x, \quad x \in G/H\,. \end{aligned}$$

Note also that $p_{K,KL}\circ q_K = q_{KL} = p_{L,KL}\circ q_L$.

Now $x \mapsto (p_{K\cap L,K}(x), p_{K\cap L,L}(x))$ defines a homeomorphism ψ of $G/K \cap L$ onto the closed subspace

$$\{(y,z) \in (G/K)\times(G/L) \mid p_{K,KL}(y) = p_{L,KL}(z)\}$$

of $(G/K)\times(G/L)$, so $x \mapsto \psi^{-1}(q_K(x), q_L(x))$ defines a map $q_{K\cap L}$ of G/H into $G/K\cap L$. If $q_{K\cap L}(x) = y$, then $p_{K\cap L,K}(y) = q_K(x)$ and

$$p_{K\cap L,H}(y) = p_{K,H}\circ p_{K\cap L,K}(y) = p_{K,H}\circ q_K(x) = x\,,$$

so $p_{K\cap L,K}\circ q_{K\cap L} = q_K$ and $p_{K\cap L,H}\circ q_{K\cap L}(x) = x$, $x \in G/H$.

It is easily seen that $q_{K\cap L}$ is a locally bounded Baire map. Thus $(K\cap L, q_{K\cap L}) \in \mathscr{M}$ and $(K, q_K) \leqq (K\cap L, q_{K\cap L})$. Maximality of (K, q_K) gives the wanted relation $K = K \cap L$.

Now drop the assumption that G is σ-compact. Since G/H is paracompact we can choose a family $(M_i)_{i\in I}$ of pairwise disjoint open σ-compact (hence Baire) subsets with $G/H = \bigcup_{i\in I} M_i$.

To $i \in I$ we can choose an open σ-compact subgroup G_i of G with $M_i \subseteqq p(G_i)$; set $H_i = H \cap G_i$. Since G_i is σ-compact and G_iH is closed $p \mid G_i$ defines a homeomorphism of G_i/H_i onto $p(G_i)$, so there exists a locally bounded Baire map q_i of $p(G_i)$ into G_i with $p\circ q_i(x) = x$, $x \in p(G_i)$.

Define $q\colon G/H \to G$ by $q(x)=q_i(x)$, $x \in M_i$, $i \in I$. Since a compact set in G/H has empty intersection with all but finitely many M_i, $i \in I$, q is locally bounded. Since a compact Baire set in G has empty intersection with all but finitely many of the open and closed sets $p^{-1}(M_i)$, $i \in I$, q is a Baire map.

4.

Let q be a locally bounded Baire cross section for p. We may assume, substituting $qq(p(e))^{-1}$ for q, that $q(p(e))=e$.

Define a map $P\colon G \to H$ by $P(g)=q\circ p(g)^{-1}g$; then P is a locally bounded local Baire map with $P(e)=e$ and $P(gh)=P(g)h$, $g \in G$, $h \in H$. This gives a generalization of a result of M. Takesaki and N. Tatsuuma, cf. [9].

The map $\varphi\colon G \to H \times G/H$ given by $\varphi(g)=(P(g),p(g))$ and the inverse map $(h,x) \mapsto q(x)h$ are both locally bounded local Baire maps, hence they are both Baire maps.

Let μ and β be left Haar measures on G and H respectively and let λ be a quasi invariant measure on G/H, so $\lambda=\varrho\mu$ for some measurable function $\varrho\colon G \to]0,\infty[$; we may assume that ϱ and $1/\varrho$ are bounded on compact sets [2].

From

$$\int_G f\varrho\, d\mu = \int_{G/H}\int_H f(q(x)h)\, d\beta(h)\, d\lambda(x)$$

$$= \int_{H\times G/H} f\circ\varphi^{-1}\, d\beta\times\lambda, \qquad f \in C_c(G)\,,$$

we get that $\varrho\mu(B)=\beta\times\lambda(\varphi(B))$ for any relatively compact Baire set B in G.

The measures $\varrho\mu$ and $\beta\times\lambda$ are quasi invariant on the quotient spaces $G/\{e\}$ and $(H\times G)/(\{e\}\times H)$, therefore completion regular. It follows that a subset B of G is μ measurable if and only if $\varphi(B)$ is $\beta\times\lambda$ measurable, and $\varrho\mu(B) =\beta\times\lambda(\varphi(B))$ when B is measurable.

For any map f of $H\times G/H$ into a metrizable space, f and $f\circ\varphi$ are simultaneously measurable. So for any Banach space E the spaces $\mathscr{B}_c(H\times G/H,E)$ and $\mathscr{B}_c(G,E)$ of bounded measurable maps of compact support are isomorphic, and isometric in any L^p norm, so $L^p(\beta\times\lambda,E)$ is linearly isometric to $L^p(\varrho\mu,E)$; a further multiplication with $\varrho^{1/p}$ gives a linear isometry onto $L^p(\mu,E)$ (when $p=2$ this is known from the theorem on induction in stages).

In the same way $f \mapsto f\circ\varphi^{-1}$ defines a linear homeomorphism between the spaces $L_c^p(\beta\times\lambda,E)$ and $L_c^p(\varrho\mu,E)$ with inductive limit topologies and, since ϱ and $1/\varrho$ are bounded on compact sets, between $L_c^p(\beta\times\lambda,E)$ and $L_c^p(\mu,E)$.

5.

Let A be a C*-algebra and α a homeomorphism of G into the group of *-automorphisms of A with $g \mapsto \alpha(g)a$ continuous for each $a \in A$.

The following theorem was proved in [6] under assumption of the existence of a measurable locally bounded cross section.

THEOREM (Ph. Green). *The* C*-*algebra* $C^*(G, C_\infty(G/H) \otimes A)$ *of the diagonal action of* G *on* $C_\infty(G/H) \otimes A$ *is isomorphic to the* C*-*tensor product of the* C*-*algebra* $C^*(H, A)$ *of the action of* H *on* A *with the compact operators on* $L^2(G/H)$.

PROOF. Let q be a locally bounded Baire cross section with $q(p(e)) = e$. Define P and φ as in section 4.

Define a function $T(f \otimes g)$ on G when $f \in \mathscr{B}_c(H, A)$ and $g \in \mathscr{B}_c(G/H, \mathbf{C})$ by

$$T(f \otimes g)(s) = g(p(s))\alpha(q(p(s)))(f(P(s))), \qquad s \in G;$$

thus

$$T(f \otimes g) \circ \varphi^{-1}(h, x) = g(x)\alpha(q(x))(f(h))$$

and

$$T(f \otimes g) \circ \varphi^{-1} \in \mathscr{B}_c(H \times G/H, A)$$

(cf. section 2), and so $T(f \otimes g) \in \mathscr{B}_c(G, A)$.

Extend T by linearity to a map of $\mathscr{B}_c(H, A) \otimes \mathscr{B}_c(G/H, \mathbf{C})$ into $\mathscr{B}_c(G, A)$.

The only part of Green's proof in doubt under the altered assumption on q is the proof that the range of T is dense in $L_c^2(\mu, A)$.

Now $(UF)(h, x) = \alpha(q(x))(F(h, x))$ defines a linear homeomorphism U of $L_c^2(\beta \times \lambda, A)$. Since $C_c(H, A) \otimes C_c(G/H)$ is dense in $L_c^2(\beta \times \lambda, A)$, so is the subspace spanned by the functions $U(f \otimes g) = T(f \otimes g) \circ \varphi^{-1}$; it follows that the range of $T \mid C_c(H, A) \otimes C_c(G/H)$ is dense in $L_c^2(\mu, A)$.

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