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THE C*-ALGEBRAS OF CODIMENSION ONE FOLIATIONS WITHOUT HOLONOMY

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1. Introduction.

In the theory of noncommutative differential geometry, which is one of the most interesting fields of C*-algebras, A. Connes canonically associates a certain groupoid C*-algebra to a foliated manifold [2]. Its prototypes are crossed products C*-algebras of abelian C*-algebras by actions of Lie groups. In [6] we showed that the C*-algebra of a foliated bundle is stably isomorphic to a reduced crossed product, by the action of the total holonomy group, of the C*-algebra of continuous functions on the fibre. The structure of bundle gives us a way to describe the C*-algebra of foliated bundle in term of total holonomy group.

In the qualitative theory of foliations, there is an important class of foliations, that is, codimension one foliations without holonomy. Some of them are structurally more complicated than foliated bundles. Nevertheless, they have nice structures similar to those of foliated bundles, where the total holonomy group is replaced by the Novikov transformation [4], [7].

In the present paper, we show (Theorem 2.1) that the C*-algebra of a codimension one foliation without holonomy on a closed manifold is stably isomorphic to the crossed product of the C*-algebra of continuous functions on the circle by a free abelian group of finite rank acting on the circle through the Novikov transformation.

In this paper, all manifolds and foliations are assumed to be smooth. All groupoids are also assumed to be locally compact Hausdorff spaces and to satisfy the second axiom of countability.

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2. Novikov transformations.

Let $\mathcal{F}$ be a codimension one foliation without holonomy on a closed manifold, namely a compact manifold X without boundary. In [7] S. P. Novikov proved, among other things, that the universal covering manifold $\tilde{X}$ of X is diffeomorphic to $\tilde{L} \times \mathbf{R}$, where $\tilde{L}$ is the universal covering manifold of a leaf L of $\mathcal{F}$, and that the leaves of $\tilde{\mathcal{F}}$ induced from $\mathcal{F}$ are given by $\tilde{L} \times \{t\}$, $t \in \mathbf{R}$. Let $p: \tilde{X} \to \mathbf{R}$ be the projection onto the second factor. Since the covering transformations preserve $\tilde{\mathcal{F}}$, they induce a homomorphism

$$\chi : \pi_1(X) \to \mathrm{Diff}_+(\mathbf{R})$$

by the rule $p(g(x)) = \chi(g)(p(x))$ for $g \in \pi_1(X)$ and $x \in X$, where $\mathrm{Diff}_+(\mathbf{R})$ is the group of all orientation preserving diffeomorphisms of $\mathbf{R}$. The group $\pi_1(L)$ is embedded in $\pi_1(X)$ and coincides with the kernel of χ. Further, $\chi(g)$ has no fixed points unless it is the identity. From this Novikov concluded that $\mathrm{Im}(\chi) \cong \pi_1(X)/\pi_1(L)$ is a free abelian group of finite rank r ($\geqq 1$). In particular, a simply connected manifold cannot have a codimension one foliation without holonomy.

We call χ the *Novikov transformation* of $\mathcal{F}$ and r the *rank* of $\mathcal{F}$.

Let $f_1, \ldots, f_r$ be generators of $\mathrm{Im}(\chi)$ ($\subset \mathrm{Diff}_+(\mathbf{R})$). Define a $\mathbf{Z}$-action on $\mathbf{R}$ by $n(t) = f_1^n(t)$ for $n \in \mathbf{Z}$, $t \in \mathbf{R}$. Since f_1 has no fixed points, this $\mathbf{Z}$-action is free, and the quotient manifold is identified with the circle S^1. As the diffeomorphisms $f_2, \ldots, f_r$ commute with f_1, they induce diffeomorphisms $f'_2, \ldots, f'_r$ of S^1, respectively. Thus we obtain a homomorphism

$$\chi' : \pi_1(X) \to \mathrm{Diff}_+(S^1).$$

Notice that the image of χ' is a free abelian group of rank $r-1$. We call χ' also the *Novikov transformation* of $\mathcal{F}$.

The main result of this paper is:

THEOREM 2.1. *Let $\mathcal{F}$ be a codimension one foliation without holonomy on a closed manifold X. Then the* C**-algebra* $C^*(X, \mathcal{F})$ *of* $(X, \mathcal{F})$ *is stably isomorphic to the crossed product* $C(S^1) \times_{\chi'} \mathbf{Z}^{r-1}$, *where χ' is the Novikov transformation of $\mathcal{F}$.*

3. Topological groupoids associated with $(X, \mathcal{F})$.

Let $(X, \mathcal{F})$ be as in the previous section, and let r be the rank of $\mathcal{F}$. For a groupoid H, let α (respectively β): $H \to H^0$ denote the map associating to each point of H its domain (respectively range).

Consider the covering $X' = X/\pi_1(L) \to X$ corresponding to the normal subgroup $\pi_1(L)$ of $\pi_1(X)$. Then X' is diffeomorphic to $L \times \mathbf{R}$, and the leaves of the foliation $\mathcal{F}'$ induced from $\mathcal{F}$ are $L \times \{t\}$, $t \in \mathbf{R}$. Let $G(\mathcal{F})$ and $G(\mathcal{F}')$ be the holonomy groupoids of $\mathcal{F}$ and $\mathcal{F}'$ respectively. It is clear that they are locally compact Hausdorff spaces and satisfy the second axiom of countability.

LEMMA 3.1. *Let q: $G(\mathcal{F}') \to G(\mathcal{F})$ be the natural projection. Then the triple $(G(\mathcal{F}'), q, G(\mathcal{F}))$ is a Galois covering space such that the covering transformation group Π, which is isomorphic to $\mathbf{Z}^r$, is contained in the group* Aut $(G(\mathcal{F}'))$ *of all automorphisms of the groupoid $G(\mathcal{F}')$.*

PROOF. First notice that the action of $\mathbf{Z}^r \cong \pi_1(X)/\pi_1(L)$ on X' gives us an action as groupoid automorphisms.

By the definition of the topology of holonomy groupoid, the projection q is a covering map in a usual sense. It is evident that the action of $\mathbf{Z}^r$ on $G(\mathcal{F}')$ is free, and that, through this action, $\mathbf{Z}^r$ is contained in Π.

Let $\gamma, \gamma' \in G(\mathcal{F}')$ have the same image in $G(\mathcal{F})$. We only have to show that there exists a $g \in \mathbf{Z}^r$ such that $g(\gamma) = \gamma'$. Put $\alpha(\gamma) = x$, $\alpha(\gamma') = x'$, $\beta(\gamma) = y$, and $\beta(\gamma') = y'$. Notice that $q(x) = q(x')$, $q(y) = q(y')$. Since q: $X' \to X$ is a Galois covering space, there exists a $g \in \mathbf{Z}^r$ such that $g(x) = x'$. Since $g(y)$ and y' are on the same leaf, and $q(g(y)) = q(y')$, by the construction of X' we have $g(y) = y'$. This shows that $g(\gamma) = \gamma'$, because $\mathcal{F}'$ is without holonomy.

REMARK 3.2. Let $G(\tilde{\mathcal{F}})$ be the holonomy groupoid of $\tilde{\mathcal{F}}$. Then $G(\tilde{\mathcal{F}})$ is a Galois covering space of $G(\mathcal{F})$, but the covering transformation group is not contained in Aut $(G(\tilde{\mathcal{F}}))$ in general.

Suppose now that a discrete group Π is acting on a groupoid G' with space of units X' as groupoid automorphisms. We construct a new groupoid $G' \times \Pi$ as follows.

Consider the cartesian product $G' \times \Pi$. Define α, β: $G' \times \Pi \to X'$ by

$$\alpha(\gamma, g) = \alpha(g^{-1}(\gamma)) \quad \text{and}$$
$$\beta(\gamma, g) = \beta(\gamma)\,.$$

For (γ_1, g_1), $(\gamma_2, g_2) \in G' \times \Pi$, the composite $(\gamma_1, g_1) \cdot (\gamma_2, g_2)$ is defined by

$$(\gamma_1, g_1) \cdot (\gamma_2, g_2) = (\gamma_1(g_1(\gamma_2)), g_1 g_2)$$

if and only if $\alpha(\gamma_1, g_1) = \beta(\gamma_2, g_2)$. With these structures, $G' \times \Pi$ becomes a topological groupoid with space of units X'.

REMARK 3.3. To show that $G' \times \Pi$ is actually a groupoid, we must use the fact that Π acts on G' as groupoid automorphisms.

For a continuous map p from a topological space Y to the space G^0 of units of G, we denote by $Y \times_{G^0} G$ the space of all pairs $(y, \gamma) \in Y \times G$ such that $p(y) = \beta(\gamma)$. By an action of G on the right on Y with respect to p, we mean a continuous map from $Y \times_{G^0} G$ to Y such that, if the image of (y, γ) is denoted by $y \cdot \gamma$, $(y \cdot \gamma) \cdot \gamma' = y \cdot (\gamma\gamma')$, $y \cdot u = y$ for $u \in G^0$ and $p(y, \gamma) = \alpha(\gamma)$.

In a similar way, an action on the left is defined.

LEMMA 3.4. *Let G' be a topological groupoid furnished with an action of a discrete group Π as groupoid automorphisms. Assume that the action is free. Then the quotient space G is a topological groupoid, which is equivalent to the groupoid $G' \times \Pi$ in the sense of* [3, 2.3.2].

PROOF. It is obvious that G is a topological groupoid. Let X, X' be the spaces of units of G, G' respectively. Define maps $p: G' \to X$ and $p': G' \to X'$ by

$$p(\gamma) = \pi(\alpha(\gamma)) \quad \text{and}$$
$$p'(\gamma) = \beta(\gamma),$$

where π is the quotient map from G' onto G.

For any $x \in X'$ and $\gamma \in G$ with $\pi(x) = \beta(\gamma)$, there exists a unique $\gamma' \in G'$ such that $\beta(\gamma') = x$ and $\pi(\gamma') = \gamma$. By $\tilde{\gamma}(x, \gamma)$ we denote this unique lift of γ with $\beta(\tilde{\gamma}(x, \gamma)) = x$.

With respect to p,the groupoid G acts on the right on G' by

$$\gamma' \cdot \gamma = \gamma' \tilde{\gamma}(\alpha(\gamma'), \gamma)$$

whenever $p(\gamma') = \beta(\gamma)$. Similarly, with respect to p', the groupoid $G' \times \Pi$ acts on the left on G' by

$$(\gamma, g) \cdot \gamma' = \gamma(g(\gamma'))$$

whenever $\alpha(\gamma, g) = p'(\gamma')$. These actions commute with each other.

For $x \in X'$, the action of G preserves $p'^{-1}(x)$ and is free and transitive. Let $i: X' \to G'$ be the natural inclusion. The map $p \circ i$ gives rise to a space $X' \times_X G$, which is a right G-space. Then there exists a G-equivariant homeomorphism

$$h: G' \to X' \times_X G$$

which covers the identity mapping of X'. This shows that $p': G' \to X'$ is a principal G-bundle in the sense of [3, Definition 2. 2.2].

For $x \in X$, the action of $G' \times \Pi$ on G' preserves $p^{-1}(x)$ and is free and transitive. Let U be an open subset of X evenly covered by $\pi^0: X' \to X$, and let x be a continuous section for π^0 defined on U. The section s is regarded as a local section for $p': G' \to X$. Using the map $p' \circ s$, define a space $(G' \times \Pi) \times_{X'} U$, which is a left $(G' \times \Pi)$-space. Define a map $h: (G' \times \Pi) \times_{X'} U \to p^{-1}(U)$ by

$$h((\gamma, g), x) = \gamma .$$

Then h is a $(G' \times \Pi)$-equivariant homeomorphism. This means that $p\colon G' \to X$ is locally trivial. Hence, $p\colon G' \to X$ is a principal $(G' \times \Pi)$-bundle. Consequently, the space G' defines an equivalence between $G' \times \Pi$ and G as defined in 2.3 of [3].

From Lemma 3.4, it follows that the groupoid $G(\mathscr{F}')$ (respectively $\mathsf{R} \times \mathsf{Z}^r$) is equivalent to $G(\mathscr{F})$ (respectively $S^1 \times \mathsf{Z}^{r-1}$).

LEMMA 3.5. *The groupoid* $G(\mathscr{F}') \times \Pi$ *is equivalent to* $\mathsf{R} \times \mathsf{Z}^r$.

PROOF. Recall that $X' \cong \mathsf{R} \times L$, and that $G(\mathscr{F}') \cong \mathsf{R} \times L \times L$ with $\alpha(t, x, y) = (t, y)$, $\beta(t, x, y) = (t, x)$. The space $X' \times \mathsf{Z}^r$ is a right $(\mathsf{R} \times \mathsf{Z}^r)$, left $(G(\mathscr{F}') \times \Pi)$-space by the following actions:

$$(t, x, g) \cdot (g^{-1}(t), h) = (t, x, gh) ,$$

$$(t', x', y', g') \cdot (t, x, g) = (t', x', y', g'g) \quad \text{if } g'(t, x) = (t', y') .$$

Define maps $q_1\colon X' \times \mathsf{Z}^r \to \mathsf{R}$ and $q_2\colon X' \times \mathsf{Z}^r \to X'$ by

$$q_1(t, x, g) = g^{-1}(t) ,$$

$$q_2(t, x, g) = (t, x) .$$

It is easy to see that the space $X' \times \mathsf{Z}^r$ together with the maps q_1 and q_2 defines an equivalence between $G' \times \Pi$ and $\mathsf{R} \times \mathsf{Z}^r$.

4. Proof of the theorem.

A smooth density for $\mathscr{F}$ gives rise to a left Haar system $\{\mu^y\}_{y \in X}$ for $G(\mathscr{F})$. The C*-algebra $C^*(X, \mathscr{F})$ is nothing but the reduced groupoid C*-algebra $C_r^*(G(\mathscr{F}))$ with respect to this left Haar system.

Canonical invariant measures on the discrete groups Z^r and Z^{r-1} induce left Haar systems for the groupoids $\mathsf{R} \times \mathsf{Z}^r$ and $S^1 \times \mathsf{Z}^{r-1}$, respectively. The corresponding reduced groupoid C*-algebras are nothing but the reduced crossed products $C_0(\mathsf{R}) \times_\chi \mathsf{Z}^r$ and $C(S^1) \times_{\chi'} \mathsf{Z}^{r-1}$, respectively.

Let (G', π, G) be as in Lemma 3.4, and let $\{\mu^y\}_{y \in X}$ be a left Haar system for G. Then $\{\mu^y\}$ induces a left Haar system $\{\lambda^x\}_{x \in X'}$ for G', which is invariant under the action of Π. Since $(G' \times \Pi)^x = G'^x \times \Pi$ for $x \in X'$, the Haar system $\{\lambda^x\}$ with a canonical invariant measure on Π defines a left Haar system $\{\bar{\lambda}^x\}_{x \in X'}$ for G'.

Since the foliation $\mathscr{F}$ is without holonomy, the C*-algebras associated to $G(\mathscr{F})$, $G(\mathscr{F}') \times \Pi$, $\mathsf{R} \times \mathsf{Z}^r$, and $S^1 \times \mathsf{Z}^{r-1}$ are all separable. Consequently, to show that they are stably isomorphic, it suffices to show that they are strongly Morita equivalent [1]. Then the main theorem follows from the following propositions.

PROPOSITION 4.1. *Let* (G', π, G) *be as in Lemma* 3.4 *together with left Haar systems considered above. Then* $C_r^*(G' \times \Pi)$ *is strongly Morita equivalent to* $C_r^*(G)$.

PROPOSITION 4.2. *The* C*-*algebra* $C_r^*(G' \times \Pi)$ *is strongly Morita equivalent to* $C_r^*(\mathbf{R} \times \mathbf{Z}^r)$.

PROOF OF PROPOSITION 4.1. We show that the equivalence G' given in the proof of Lemma 3.4 actually gives us a strong Morita equivalence. First recall that G' itself is a topological groupoid with a left Haar system $\{\lambda^x; x \in X'\}$. Thus $C_c(G')$ becomes a pre-C*-algebra.

Define a right action of $C_c(G)$ on $C_c(G')$ by

$$(\xi f)(\gamma) = \int_{G'^{\beta(\gamma)}} \xi(\gamma') f(\pi(\gamma'^{-1}\gamma))\, d\lambda^{\beta(\gamma)}(\gamma')$$

for $f \in C_c(G)$, $\xi \in C_c(G')$. Define a linear map P from $C_c(G')$ onto $C_c(G)$ by

$$(P\xi)(\gamma) = \sum_{\pi(\gamma') = \gamma} \xi(\gamma') .$$

The map P induces a $C_c(G)$-valued inner product $\langle \cdot, \cdot \rangle$ on $C_c(G')$ by

$$\langle \xi, \eta \rangle = P(\xi^* \eta) .$$

or, more explicitly

$$\langle \xi, \eta \rangle(\gamma) = \sum_{x \in \pi_0^{-1}(\beta(\gamma))} \int_{G'^x} \overline{\xi(\gamma'^{-1})}\, \eta(\gamma'^{-1}\tilde{\gamma}(x, \gamma))\, d\lambda^x(\gamma')$$

for $\xi, \eta \in C_c(G')$ and $\gamma \in G$. The linear map P is continuous for the inductive limit topology. Since $C_c(G')$ contains an approximate identity for the inductive limit topology [8, 1.9. Proposition, p. 56], the linear span of the range of $\langle \cdot, \cdot \rangle$ is dense in $C_c(G)$.

Let $(\mu, \mathscr{H}, L)$ be a representation of the groupoid G, and let $(\mu', \mathscr{H}', L')$ be the representation of G' induced from $(\mu, \mathscr{H}, L)$ by $\pi: G' \to G$ ([8, 1.6 Definition, p. 52], [2, Definition IV 1, p. 68]). Let $f \in C_c(G')$ be fixed. Let K be the support of f, and let φ be the characteristic function of the compact set $\alpha(K)$ in X'. For $\xi, \eta \in \Gamma(\mathscr{H})$, define $\xi', \eta' \in \Gamma(\mathscr{H}')$ by

$$\xi'(x) = \varphi(x)\xi(\pi(x)) ,$$

$$\eta'(x) = \varphi(x)\eta(\pi(x)) .$$

Then we have $(L(P(f))\xi, \eta) = (L'(f)\xi', \eta')$, in particular

$$(L(P(f))\xi, \xi) = (L'(f)\xi', \xi') .$$

Therefore, for any $f \in C_c(G')$ and $\xi \in \Gamma(\mathscr{H})$, we have

$$(L(\langle f, f\rangle)\xi, \xi) \geqq 0 .$$

This means that $\langle f, f\rangle$ is positive (cf. [8, p. 84]). It is not difficult to see that the inner product $\langle \cdot, \cdot \rangle$ is definite. With this inner product, $C_c(G')$ becomes a right-$C_c(G)$-rigged space [9, Definition 2.8, p. 197].

Let E be the Hilbert C*-module, over $C_r^*(G)$, obtained by completing $C_c(G')$ with respect to the norm:

$$\|\xi\| = \|\langle \xi, \xi\rangle\|^{\frac{1}{2}} \quad \text{for } \xi \in C_c(G'),$$

and let $\mathscr{L}(E)$ be the space of all bounded linear operators on the Hilbert C*-module E ([5, 1.13], [9, Definition 2.3]).

Define a left action of $C_c(G' \times \Pi)$ on $C_c(G')$ by

$$(f\xi)(\gamma) = \sum_{g \in \Pi} \int_{G'^{\beta(\gamma)}} f(\gamma', g)\xi(g^{-1}(\gamma'^{-1}\gamma))\, d\lambda^{\beta(\gamma)}(\gamma') .$$

Then, by a direct computation, we see that this action gives rise to a (continuous) homomorphism

$$j : C_r^*(G' \times \Pi) \to \mathscr{L}(E) .$$

Since $C_r^*(G)$ acts on $L^2(G^y, \mu^y)$ for $y \in X$, we can define a Hilbert space

$$E \otimes_{C_r^*(G)} L^2(G^y, \mu^y)$$

(cf. [5, p. 522]). For any $x \in X'$, there exists an isometry from $E \otimes_{C_r^*(G)} L^2(G^{\pi(x)}, \mu^{\pi(x)})$ onto $L^2(G'^x \times \Pi, \bar{\lambda}^x)$ such that $j(f) \otimes 1 = \bar{R}_x(f)$ for $f \in C_c(G' \times \Pi)$, where $\bar{R}_x$ is the right regular representation of $C_c(G' \times \Pi)$ on $L^2((G' \times \Pi)^x, \bar{\lambda}^x)$. Indeed, for

$$f \otimes \xi \in C_c(G') \otimes L^2(G^{\pi(x)}, \mu^{\pi(x)}) ,$$

define $\psi(f \otimes \xi)$ by

$$(f \otimes \xi)(\gamma, g) = R_x(f^g)(\xi)(\gamma) ,$$

where $f^g(\gamma') = f(g^{-1}(\gamma'))$, R_x is the right regular representation of $C_c(G')$ on $L^2(G'^x, \lambda^x)$, and ξ is regarded in a natural way as an element of $L^2(G'^x, \lambda^x)$. Then ψ extends to the required isometry. Consequently, we have that

$$\|j(f)\| \geqq \|j(f) \otimes 1\| \geqq \|f\| .$$

Therefore j is an isometric embedding of $C_r^*(G' \times \Pi)$ into $\mathscr{L}(E)$.

To finish the proof, it suffices to show that the image $j(C_r^*(G' \times \Pi))$ coincides with the two sided ideal $\mathscr{K}(E)$ of all compact operators on E. For $\xi, \eta \in C_c(G')$, define $f \in C_c(G' \times \Pi)$ by

$$f(\gamma, g) = \int_{G'^{\beta(\gamma)}} \xi(\gamma')\overline{\eta(g^{-1}(\gamma^{-1}\gamma'))}\, d\lambda^{\beta(\gamma)}(\gamma') .$$

A simple calculation shows that $j(f)=\theta_{\xi,\eta}$. Thus $\mathscr{K}(E)$ is contained in $j(C_r^*(G' \times \Pi))$.

To see that these two spaces coincide, it is sufficient to show that the image of a function of the form $\varphi\psi$ ($\varphi, \psi \in C_c(G' \times \Pi)$) is contained in $\mathscr{K}(E)$, because $C_c(G' \times \Pi)$ has an approximate identity. We may assume that the supports of φ and ψ are contained in $G' \times \{g\}$ and $G' \times \{h\}$ respectively. Moreover, using a partition of unity, we may further assume that the support of ψ is contained in an open subset $U \times \{h\}$ of $G' \times \{h\}$ such that

$$(k(U) \times \{h\}) \cap (U \times \{h\}) = \emptyset \quad \text{if } k \neq e .$$

Then, put $\xi(\gamma)=\varphi(\gamma, g)$ and $\eta(\gamma)=\overline{\psi(h(\gamma), h)}$ to obtain

$$j(\varphi\psi) = \theta_{\xi,\eta} .$$

Therefore $j(C_r^*(G' \times \Pi))=\mathscr{K}(E)$. This shows that E is a $C_r^*(G' \times \Pi)-C_r^*(G)$-imprimitivity bimodule.

Proof of Proposition 4.2. As in the proof of Proposition 4.1, we show that the equivalence given in the proof of Lemma 3.5 gives rise to an imprimitivity bimodule establishing a strong Morita equivalence.

Let H, H' denote the groupoids $\mathbf{R} \times \mathbf{Z}^r$, $X' \times \Pi$ respectively. Define a right action of $C_c(H)$ on $C_c(H')$ by

$$(\xi f)(t, x, g) = \sum_{h \in \Pi} \xi(t, x, gh) f((gh)^{-1}(t), h^{-1}) .$$

Recall that the space X' is diffeomorphic to $\mathbf{R} \times L$, and that for $t \in \mathbf{R}$ there exists a measure ν^t on $\{t\} \times L$ induced from the smooth density for the foliation $\mathscr{F}$. Notice that the family of these measures is invariant under the action of the covering transformation group.

Define a linear map $P\colon C_c(H') \to C_c(H)$ by

$$(P\xi)(t, g) = \int_{\{t\} \times L} \xi(t, x, g)\, d\nu^t(x) ,$$

and define a $C_c(H)$-valued inner product on $C_c(H')$ by

$$\langle \xi, \eta \rangle = P(\xi^* \eta) .$$

More explicitly,

$$\langle \xi, \eta \rangle(t, g) = \sum_{h \in \Pi} \int_{\{t\} \times L} \overline{\xi(h^{-1}(t, x), h^{-1})}\, \eta(h^{-1}(t, x), h^{-1}g)\, d\nu^t(x) .$$

We can show that, by an argument similar to the one used in the proof of Proposition 4.1, that $C_c(H')$ becomes a right-$C_c(H)$-rigged space.

Define a left action of $C_c(G' \times \Pi)$ on $C_c(H')$ by

$$(f\xi)(t,x,g) = \int_{(G' \times \Pi)^{(t,x)}} f(\gamma',h)\xi((\gamma',h)^{-1} \cdot (t,x,g))\, d\bar{\lambda}^{(t,x)}(\gamma') .$$

Then this action induces an injection

$$j : C_r^*(G' \times \Pi) \to \mathscr{L}(E')$$

with $j(C_r^*(G' \times \Pi)) = \mathscr{K}(E')$, where E' is the Hilbert C*-module obtained by completing $C_c(H')$. Therefore $C_r^*(G' \times \Pi)$ and $C_r^*(H)$ are strongly Morita equivalent.

Thus Theorem 2.1 is proved.

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