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THE APPROXIMATION PROPERTY FOR A PAIR OF BANACH SPACES

ELMER BONDE

0. Introduction.

In this note we will treat Grothendieck's approximation property (a.p.) from a sort of weaker viewpoint. More precisely we will consider a fixed pair of Banach spaces (X, Y), for which operators from X into Y behave like one of the spaces have the a.p. After Enflo's famous example of a Banach space without the a.p., it seems that many spaces do not have the a.p. The motivation for this note should be seen in the light of this, because many theorems which assume that a space have the a.p., in this way can be extended to spaces, where a pair has the property above.

First we will define the a.p. for pairs of Banach spaces and show some simple equivalent formulations of it. Thereafter we define an analogue to the bounded approximation property for a pair and look at some factorization theorems. Finally we mention some examples of pairs, which have this property without any of the spaces having the usual a.p.

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1. Notation.

Let in the following X and Y be Banach spaces and $L(X, Y)$ be the space of all linear and bounded operators from X into Y. Further we let $K(X, Y)$ denote the subspace of $L(X, Y)$ consisting of the compact operators.

$X \otimes_\pi Y$ denotes the completion of the algebraic tensor product $X \otimes Y$ in the greatest cross norm $(\|\cdot\|_\pi)$. Similarly $X \otimes_\varepsilon Y$ denotes the completion of $X \otimes Y$ in the cross norm $\|\cdot\|_\varepsilon$, defined by

$$\|\xi\|_\varepsilon = \sup\left\{\left|\sum_n x^*(x_n)y^*(y_n)\right| \;:\; \xi = \sum_n x_n \otimes y_n\,,\right.$$
$$\left. x^* \in X^*,\ \|x^*\| \leqq 1,\ y^* \in Y,\ \|y^*\| \leqq 1\right\}.$$

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We will identify $Y^*\otimes X$ with the finite dimensional operators from Y into X. If this identification is extended, we get a continuous map

$$i\colon Y^*\otimes_\pi X \to L(Y,X),$$

so that $i(Y^*\otimes_\pi X)=N(Y,X)$; the nuclear operators from Y into X. For the ε-tensor product, we have

$$Y^*\otimes_\varepsilon X = \overline{Y^*\otimes X}^{\|\cdot\|} \subseteq L(Y,X)$$

and since clearly $N(Y,X)\subseteq Y^*\otimes_\varepsilon X$, we write

$$i\colon Y^*\otimes_\pi X \to Y^*\otimes_\varepsilon X .$$

Further, we will let τ denote the topology of uniform convergence on compact sets in X on the space $L(X,Y)$.

A T in $L(X,Y)$ is called *approximable*, if it can be approximated uniformly on every compact subset of X by finite dimensional operators, i.e. $T\in \overline{X^*\otimes Y}^\tau$.

The general notation will closely follows [5]. Let us recall that X has the approximation property (abbreviated a.p.), if every operator T in $L(X,X)$ is approximable. Finally we will mention one of Grothendieck's famous equivalent formulations of a.p. (see [2]), namely that X has the a.p. if and only if

$$i\colon X^*\otimes_\pi X \to X^*\otimes_\varepsilon X$$

is $1-1$.

2. The approximation property of (X, Y).

Definition 2.1. Let X and Y be Banach spaces. The pair (X,Y) is said to have the approximation property (abbreviated A.P.), if every operator T in $L(X,Y)$ is approximable.

We clearly have that (X,Y) has the A.P. if the space X or Y has the a.p. The converse is not true, as we shall see in Section 4.

Let $j\colon Y^*\otimes_\pi X \to (L(X,Y),\tau)^*$ be defined by

$$j\Big(\sum_n y_n^*\otimes x_n\Big)(T) = \sum_n y_n^*(Tx_n) \quad \text{for all } T\in L(X,Y).$$

By [5, (Proposition 1.e.3)], j is a linear map *onto* $(L(X,Y),\tau)^*$.

We have the following

Proposition 2.2. *(X,Y) has the* A.P. *if and only if* $i^{-1}(0)=j^{-1}(0)$.

Proof. By definition of i and j, we have

$$(*)\qquad i^{-1}(0) = \left\{ u \in Y^*\otimes_\pi X \;\middle|\; \sum_n y_n^*(Tx_n) = 0 \right.$$

$$\left. \text{for all } T\in X^*\otimes Y,\ u \in \sum_n y_n^*\otimes x_n \right\}$$

$$j^{-1}(0) = \left\{ u \in Y^*\otimes_\pi X \;\middle|\; \sum_n y_n^*(Tx_n) = 0 \right.$$

$$\left. \text{for all } T\in L(X,Y),\ u \in \sum_n y_n^*\otimes x_n \right\}.$$

Therefore $j^{-1}(0)\subseteq i^{-1}(0)$.

a. If $L(X,Y)=\overline{X^*\otimes Y}^{\tau}$ we get by (*), that $i^{-1}(0)\subseteq j^{-1}(0)$.

b. Suppose $i^{-1}(0)=j^{-1}(0)$. Let $\varphi \in (L(X,Y),\tau)^*$ so that $\varphi(S)=0$ for all $S \in X^*\otimes Y$. Because j is onto $(L(X,Y),\tau)^*$, there exists an $u \in Y^*\otimes_\pi X$, such that $j(u)=\varphi$. If

$$u = \sum_n y_n^*\otimes x_n ,$$

especially

$$\sum_n y_n^*(y)x_n = 0 \qquad \text{for all } y \in Y,$$

that is $u \in i^{-1}(0)=j^{-1}(0)$. Therefore $j(u)=\varphi=0$.

We also have

PROPOSITION 2.3. *Let X and Y be Banach spaces, then*

(a) (X, Y^{**}) *has the* A.P.

$\Downarrow$

(b) $i\colon Y^*\otimes_\pi X \to Y^*\otimes_\varepsilon X$ *is* $1-1$.

$\Downarrow$

(c) (X,Y) *has the* A.P.

Especially we have the following corollary, which should be compared with Grothendieck's equivalent formulation of the a.p. mentioned last in Section 1.

COROLLARY 2.4. *If Y is a reflexive Banach space, then (X,Y) has the* A.P. *if and only if*

$$i\colon Y^*\otimes_\pi X \to Y^*\otimes_\varepsilon X$$

is $1-1$.

PROOF OF 2.3. (a) $\Rightarrow$ (b): Suppose $\overline{X^*\otimes Y^{**}}^{\tau} = L(X, Y^{**})$. Let

$$u = \sum_n y_n^* \otimes x_n \in Y^* \otimes_\pi X$$

such that $i(u)(y)=0$ for all $y \in Y$. Then by looking at the double dual of $i(u)$, we get

$$\sum_n y_n^*(Sx_n) = 0 \qquad \text{for all } S \in X^* \otimes Y^{**} .$$

The assumption therefore gives that

$$\sum_n y_n^*(Tx_n) = 0 \qquad \text{for all } T \in L(X, Y^{**}) .$$

The implication then follows from the well known fact that $(Y^* \otimes_\pi X)^* = L(X, Y^{**})$.

(b) $\Rightarrow$ (c): If i: $Y^* \otimes_\pi X \to Y^* \otimes_\varepsilon X$ is $1-1$, then $i^{-1}(0)=j^{-1}(0)$ and the implication is obtained by Proposition 2.2.

In general the two implications in Proposition 2.3 cannot be reversed, as the following example shows. Let Z be a Banach space with the a.p., so that Z^* fails to have the a.p. Then the pair (Z^{**}, Z) has the A.P. Because Z^* does not have the a.p.,

$$(*) \qquad i_1 : Z^{**} \otimes_\pi Z^* \to Z^{**} \otimes_\varepsilon Z^*$$

is *not* $1-1$. By a simple argument, this is seen to imply that

$$i : Z^* \otimes_\pi Z^{**} \to Z^* \otimes_\varepsilon Z^{**}$$

is *not* $1-1$, hence (c) does not imply (b) in Proposition 2.3.

We could have chosen Z^* with the a.p., so that Z^{**} does not have the a.p. Again (Z^{**}, Z) have the A.P., but now (*) gives that i: $Z^* \otimes_\pi Z^{**} \to Z^* \otimes_\varepsilon Z^{**}$ is $1-1$. Because (Z^{**}, Z^{**}) fails to have the A.P., we get that (b) does not imply (a) in Proposition 2.3.

3. The bounded approximation property of (X, Y) and factorization.

Corresponding to the bounded approximation property (b.a.p.) for one Banach space, we introduce

DEFINITION 3.1. Let X and Y be Banach spaces. The pair (X, Y) is said to have the λ-bounded approximation property (abbreviated λ-B.A.P.), if every operator $T \in L(X, Y)$ can be approximated in the topology τ with finite dimensional operators, which is uniformly bounded by $\lambda\|T\|$.

In this terminology we simply have that X has the λ-b.a.p. if and only if (X, Y) (or (Y, X)) has the λ-B.A.P. for all Banach spaces Y.

It is well known that $K(Y, X) = Y^* \otimes_\pi X$ for all Y if and only if X has the a.p. This does not hold for a single pair of Banach spaces, since (Y, X) has the B.A.P. (or A.P.) does not imply that $K(Y, X) = Y^* \otimes_\pi X$. Indeed, let Y be a Banach space with a basis, such that Y^* does not have the a.p. Then there exists a space X for which $K(Y, X) \neq Y^* \otimes_\varepsilon X$ (see [5], Theorem 1.e.5), but (Y, X) has the B.A.P.

The converse we leave as an open problem.

Problem 3.2. Does there exist Banach spaces X and Y, so that $K(X, Y) = Y^* \otimes_\varepsilon X$ but (Y, X) does not have the B.A.P. (or A.P.).

Notice that this is related to problem 1.e.9 of [5].

We have the following theorem, the proof of which is essentially the same as A. Pelczynski's proof of Theorem 1 in [8].

Theorem 3.3. *Let X and Y be Banach spaces. An operator T in $L(X, Y)$ can be approximated with finite dimensional operators pointwise if and only if T can be factored through a space Z_T with a basis; that is, there are $W \in L(X, Z_T)$ and $U \in L(Z_T, Y)$ such that $UW = T$.*

Proof. First we note that if E is a Banach space with $\dim E = n$ ($n \in \mathbb{N}$), then there are $\{U_i\}_{i=1}^{n^2} \subseteq L(E, E)$ such that

$$(1)\qquad \begin{cases} \dim U_k E = 1, \quad \left\| \sum_{i=1}^{k} U_i \right\| \leqq 2 \quad 1 \leqq k \leqq n^2 \text{ and} \\ \sum_{i=1}^{n2} U_i x = x \qquad \text{for all } x \in E . \end{cases}$$

To see this, define for $i = rn + j$ $(0 \leqq r < n,\ 1 \leqq j \leqq n)$, $U_i x = x_j^*(x) x_j / n$, where $\{x_j\}_{j=1}^n$, $\{x_j^*\}_{j=1}^n$ is an Auerbach system for E.

The "if" part of the theorem is trivial.

For the "only if" part, we assume that there exists $\{T_n\}_{n=1}^\infty \subseteq X^* \otimes Y$, so that

$$T_n x \xrightarrow{n} Tx \qquad \text{for all } x \in X .$$

If $S_1 = T_1$ and $S_n = T_n - T_{n-1}$ for $n > 1$, then

$$(2)\qquad \sum_{n=1}^{\infty} S_n x = Tx \qquad \text{for all } x \in X .$$

Set $(\dim S_n X)^2 = m_n$. Hence for every n there are $\{U_{in}\}_{i=1}^{m_n} \subseteq L(S_n X)$, as in (1). Set

$W_j = U_{in} S_n$, where $j = m_1 + m_2 + \ldots + m_{n-1} + i$; $1 \leqq i \leqq m_n$, $n = 1, 2, \ldots$. Then $W_j \colon X \to Y$ is a 1-dimensional operator. The last property in (1) and (2) gives that

$$\sum_{j=1}^{\infty} W_j x = Tx \quad \text{for all } x \in X .$$

Let $w_j \in W_j X$ with $\|w_j\| = 1$. Define

$$Z_T = \left\{ (a_j)_{j=1}^{\infty} \;\middle|\; \sum_{j=1}^{\infty} a_j w_j \in Y \text{ converging} \right\}$$

and set

$$|||(a_j)_{j=1}^{\infty}||| = \sup_k \left\| \sum_{j=1}^{k} a_j w_j \right\| .$$

It is straightforward to show that $(Z_T, |||\cdot|||)$ is a Banach space and that the unit vector basis form a monotone, normalized basis for Z_T. Let $W \colon X \to Z_T$ be given by $Wx = (a_j)_{j=1}^{\infty}$, where $W_j x = a_j w_j$. If $k = m_1 + m_2 + \ldots + m_{n-1} + j$, we have

$$\left\| \sum_{l=1}^{k} a_l w_l \right\| = \left\| \sum_{l=1}^{k} W_l x \right\| \leqq \left\| \sum_{i=1}^{n-1} S_i x \right\| + \left\| \sum_{i=1}^{j} U_{in} \right\| \|S_n x\|$$
$$\leqq \|T_{n-1} x\| + 2\|(T_n - T_{n-1})x\| \leqq 5K\|T\| \, \|x\| ,$$

where K is a constant such that $\|T_n\| \leqq K\|T\|$ for all $n \in \mathrm{N}$ (the principle of uniform boundedness). Therefore $\|W\| \leqq 5K\|T\|$. Moreover let $U \colon Z_T \to Y$ be given by

$$U\big((a_j)_{j=1}^{\infty}\big) = \sum_{j=1}^{\infty} a_j w_j .$$

Then clearly $\|U\| \leqq 1$ and $Tx = UWx$ for all $x \in X$. Hence the "only if" part is shown.

This theorem has an interesting collary, which should be compared with Pelczynski's man theorem of [8].

Corollary 3.4. *Let Y be a Banach space and X a separable Banach space, then (X, Y) has the* B.A.P. *if and only if all operators T in $L(X, Y)$ can be factored through a space with a basis.*

To see this, we need an observation

Lemma 3.5. *Suppose for all $T \in L(X, Y)$, there is $\{T_n\} \subseteqq X^* \otimes Y$ such that*

$$T_n x \xrightarrow{n} Tx \quad \text{for all } x \in X .$$

Then there exists a constant C, such that for all $T \in L(X, Y)$, *we can choose* $\{T'_n\} \subseteq X^* \otimes Y$ *with*

$$\|T'_n\| \leqq C\|T\| \quad \text{for all } n \in \mathbf{N} \quad \text{and}$$

$$T'_n x \xrightarrow{n} Tx \quad \text{for all } x \in X .$$

PROOF OF 3.5. Let

$$\chi = \{(T_n)_{n=1}^{\infty} \mid T_n \in X^* \otimes_\varepsilon Y,\ n \in \mathbf{N} \text{ and } \lim_n T_n x \text{ exists for all } x \in X\} .$$

By the uniform boundedness principle we can define

$$|||(T_n)||| = \sup_n \|T_n\| \quad \text{for all } (T_n) \in \chi .$$

In a natural way χ is a vector space and it is easily seen that $(\chi, |||\cdot|||)$ is a Banach space. The map $\Phi\colon \chi \to L(X, Y)$ given by

$$\Phi : (T_n)_{n=1}^{\infty} \to \lim_n T_n$$

is clearly linear and bounded. By the assumption Φ is *onto* and hence the open mapping theorem gives a constant C', such that for all $T \in L(X, Y)$ there exists $(T'_n) \subseteq \chi$, which comply with $\Phi((T'_n)) = T$ and $|||(T'_n)||| \leqq C'\|T\|$. Then there are $(S_n) \subseteq X^* \otimes Y$ such that

$$S_n x \xrightarrow{n} Tx \quad \text{for all } x \in X \quad \text{and}$$

$$\|S_n\| \leqq (1 + C')\|T\| \quad \text{for } n \in \mathbf{N} .$$

PROOF OF 3.4. By Lemma 3.5 we see that if all operators in $L(X, Y)$ can be approximated pointwise with finite dimensional operators, then (X, Y) has the B.A.P. It is easily checked that the reverse implication is true if X is separable.

We can in a way generalize the "if" part of Corollary 3.4, that is

PROPOSITION 3.6. *Let X and Y be Banach spaces and* $\lambda \geqq 1$. *If every operator T in* $L(X, Y)$ *can be factored through a space* Z_T *with the* λ-b.a.p., *then* (X, Y) *has the* B.A.P.

PROOF. First we will show the assertion if $Z_T = Z$ fixed for all $T \in L(X, Y)$ and $Z \cong (\sum \oplus Z)_2$. Let $W \in L(X, Z)$ and $U \in L(Z, Y)$ such that $T = UW$. Define

$$\gamma(T) = \inf\{\|U\|\,\|W\|\} ,$$

where infimum is taken over all factorizations of T through Z. By an argument similar to Johnson's in [4, Section 2] (in fact because we do not want a "compact factorization", it can be done much more easily), it can be shown that γ is a norm on the space $\Gamma_Z(X, Y)$ of all operators $T \in L(X, Y)$, which can be factored through Z, and that $(\Gamma_Z(X, Y), \gamma)$ is a Banach space. The assumption gives that $\Gamma_Z(X, Y) = L(X, Y)$, hence γ and $\|\cdot\|$ are equivalent norms, especially there exists a constant K, such that $\gamma(T) \leq K\|T\|$ for all $T \in L(X, Y)$. By this and the fact that (X, Z) has the B.A.P. it is easy to show that (X, Y) has the B.A.P.

If the Z_T's are not equal, we look at

$$Z \overset{\text{def}}{=} \left(\sum_{T \in L(X, Y)} \oplus \; Z_T \right)_2 .$$

It is clear that Z comply with $Z \cong (\sum \oplus Z)_2$. Further, a standard argument gives that Z has the λ-b.a.p. T has a factorization through Z_T, T that is $T = UW$ where $W \in L(X, Z_T)$ and $U \in L(Z_T, Y)$. If $i: Z_T \to Z$ and $P_T: Z \to Z_T$ is respectively the natural inclusion and projection, then $T = (UP_T)(iW)$ gives a factorization of T through Z. Hence we are back in the first part of the argument.

The idea of the proof of Theorem 3.3 can be used to show:

THEOREM 3.7. *Let X and Y be Banach spaces. Then $T \in X^* \otimes_\varepsilon Y$ if and only if T has a factorization through a space Z_T with a basis, such that $T = UW$, where $W \in X^* \otimes_\varepsilon Z_T$ and $U \in L(Z_T, Y)$.*

PROOF. The "if" part is trivial.

For the "only if" part we refer to the same notation as in the proof of Theorem 3.3. Let $T \in X^* \otimes_\varepsilon Y$. We can clearly assume that there exists $\{T_n\}_{n=1}^\infty \subseteq X^* \otimes Y$ with

$$T_n \xrightarrow{n} T \quad \text{and} \quad \|T_n - T_m\| < 2^{-m} \text{ for } n \leq m .$$

As before we let $S_1 = T_1$ and $S_n = T_n - T_{n-1}$ $(n > 1)$. Then $\sum_{n=1}^\infty S_n$ converges uniformly on B_X to T. W_j is defined by $\{U_{in}\}_{i=1}^{m_n}$ as in Theorem 3.3, such that

$$\sum_{j=1}^{\infty} W_j x = Tx \quad \text{for all } x \in X .$$

Now we will show that $\sum_{j=1}^\infty W_j$ converges uniformly on B_X to T.

Let $x \in B_X$ and $\varepsilon > 0$. Choose $k_0 = \sum_{j=1}^{n_0 - 1} m_j$, where $n_0 \in \mathrm{N}$ such that $2^{-n_0} < \varepsilon/4$. For $k \geq k_0$ there exists $n \geq n_0$ complying with

$$d_{n-1} \stackrel{\text{def}}{=} \sum_{j=1}^{n-1} m_j \leqq k \leqq \sum_{j=1}^{n} m_j \stackrel{\text{def}}{=} d_n .$$

Then

$$\begin{aligned} \left\|Tx - \sum_{j=1}^{k} W_j x\right\| &= \left\|\sum_{j=d_{n-1}+1}^{\infty} W_j x - \sum_{j=d_{n-1}+1}^{k} W_j x\right\| \\ &\leqq \left\|\sum_{j=n}^{\infty} \sum_{i=1}^{m_j} U_{ij} S_i x\right\| + \left\|\sum_{i=1}^{k'} U_{in} S_n x\right\| \\ &= \left\|\sum_{j=n}^{\infty} S_j x\right\| + \left\|\sum_{i=1}^{k'} U_{in}\right\| \|S_n x\| < \varepsilon \qquad (0 \leqq k' = k - d_{n-1} \leqq m_n) . \end{aligned}$$

Let $W_j x = a_x(j) w_j$ and $Wx = \sum_{i=1}^{\infty} a_x(i) e_i$, where $\{e_i\}_{i=1}^{\infty}$ is the unit vector basis in Z_T. Next, set

$$P_k W x = \sum_{j=1}^{k} a_x(j) e_j; \qquad \text{rank}\,(P_k W) = k$$

$$\begin{aligned} |||Wx - P_k Wx||| &= \sup_{n \geqq k+1} \left\|\sum_{j=k+1}^{n} a_x(j) w_j\right\| \\ &= \sup_{n \geqq k+1} \left\|\sum_{j=k+1}^{\infty} W_j x - \sum_{j=n+1}^{\infty} W_j x\right\| < 2\varepsilon \end{aligned}$$

for all $x \in B_X$ (if $k \geqq k_0 = k_0(\varepsilon)$). That is $P_k W$ converges in operator norm to W. Hence $W \in X^* \otimes_\varepsilon Z_T$.

4. Examples.

Now we will look at some examples of pairs (X, Y) with the B.A.P., such that either X of Y has the a.p.

Maurey have shown that if X and Y are Banach spaces respectively of type 2 and cotype 2, then every operator T in $L(X, Y)$ can be factored through a Hilbert space [7]. (See [6] for the definition of type and cotype.) By this and Proposition 3.6, we get

PROPOSITION 4.1. *If X is of type 2 and Y of cotype 2, then (X, Y) have the* B.A.P.

EXAMPLE 4.2. It is well-known that l_p for $1 \leqq p < \infty$ is of type $\min(p, 2)$ and cotype $\max(2, 9)$. By results of respectively Szankowski and Enflo, [11], [1], we have: For every $1 \leqq p < \infty$, $p \neq 2$, there exists a subspace X of l_p without the a.p.

Hence for $1 \leqq p_1 < 2$ and $2 < p_2 < \infty$, we can choose $Y \subseteq l_{p_1}$ and $X \subseteq l_{p_2}$ without the a.p. The pair (X, Y) has by Proposition 4.1 the B.A.P. Note that if X is of type 2, then X^* is of cotype 2. Hence if X is chosen as above, then (X, X^*) has the B.A.P.

EXAMPLE 4.3. Pisier has constructed an infinite dimensional Banach space X such that

(a) X and X^* is of cotype 2,

(b) $X \otimes_\pi X \cong X \otimes_\varepsilon X$,

(c) $X^* \otimes_\pi X \xrightarrow{i} X^* \otimes_\varepsilon X$ is *onto*

(see [10]).

The space X does not have the a.p. but (X, X^*) has the B.A.P.

To see this, recall the following well-known facts

(I) $$(X \otimes_\pi Y)^* = L(Y, X^*)$$

(II) $$(X \otimes_\varepsilon Y)^* = I_1(Y, X^*)$$

($I_1(Y, X^*)$ is the 1-integral operators from Y into X^*, see [9]).

It follows easily from (c) and (II) that X does not have the a.p.

(b) implies because of (I) and (II) that $L(X, X^*) = I_1(X, X^*)$ and by the definition of integral operators, we get that every operator in $L(X, X^*)$ has a factorization through a L_1-space. Hence Proposition 3.6 gives that (X, X^*) has the B.A.P.

EXAMPLE 4.4. Let A and B be C*-algebras. U. Haagerup have shown that every operator T in $L(A, B^*)$ can be factored through a Hilbert space [3]. Especially we get that (A, B^*) have the B.A.P. for arbitrary C*-algebras A and B. We recall that there exists C*-algebras withput the a.p. (see [12]). Hence we can get an example of the prescribed type.

We have now seen various examples of pairs (X, X^*) with the B.A.P., such that X does not have the a.p. That this is not true for every Banach space X, can easily be seen, if se let $X = Y \oplus_2 Y^*$, where $Y$is a Banach space with the a.p. such that Y^* does not have the a.p.

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MATEMATISK INSITUT
ODENSE UNIVERSITET
CAMPUSVEJ 55
5230 ODENSE M
DENMARK