MATH. SCAND. 60 (1987), 31-38

BOUNDARY REGULARITY FOR HOLOMORPHIC MAPS FROM THE DISC TO THE BALL

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In this paper, which is a sequel to [7], we study the boundary behavior of proper holomorphic maps from the open unit disc Δ in the complex plane to the open unit ball $\boldsymbol{B}_n$ in $\mathbf{C}^n$. Given such a map, call if f, the fact that f is proper implies that the image $f(\Delta)$ is an analytic subvariety in $\boldsymbol{B}_n$, whose closure in $\mathbf{C}^n$ consists of $f(\Delta) \cup \gamma$ where γ is some compact subset of the sphere $b\boldsymbol{B}_n$. The set γ is the *global cluster set of f*, that is to say, γ consists of all limit points of sequences of the form $\{f(z_j)\}_{j=1,2,\ldots}$ where $\{z_j\}_{j=1,2,\ldots}$ runs through all sequences in Δ that tend to $b\Delta$. In the sequel, we shall frequently denote this global cluster by $\mathscr{C}(f)$. In general, the set $\mathscr{C}(f)$ can be quite complicated, as examples constructed in [8] show. However, if the set γ is regular, it is natural to expect that the map f should behave in a regular way at $b\Delta$. In the case that γ is a smooth curve, this fact follows from work of Čirka [1], [2]. Here we study the case that γ is a rectifiable curve or more generally, that γ has finite length, i.e., $\Lambda^1(\gamma) < \infty$, where Λ^1 denotes one-dimensional Hausdorff measure.

Our main result is the following

THEOREM 1. *Let $f\colon \Delta \to \boldsymbol{B}_N$ be a proper holomorphic map with global cluster set of finite length. If $f(\Delta)$ has finite area, then f extends continuously to $\bar{\Delta}$. If, in addition, $\mathscr{C}(f)$ is a simple closed curve, then the derivative f' lies in the Hardy class $H^1(\Delta)$.*

Granted that the map f extends continuously to $\bar{\Delta}$ the conclusion that $f' \in H^1(\Delta)$ is a consequence of [7, Theorem 6]. The condition that $f(\Delta)$ have finite area is equivalent to the condition that $\int_\Delta |f'|^2$ be finite, so the first statement is a consequence of the following one variable result.

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[1]) Research supported in part by a grant from the Republic of Slovenia Science Foundation.

[2]) Research supported in part by the National Science Foundation through Grant DMS-8500357.

Received October 29, 1985.

THEOREM 2. *If $f\colon \Delta \to \mathbf{C}$ is a holomorphic map such that $\mathscr{C}(f)$ has finite length and whose Dirichlet integral $\int_\Delta |f'|^2$ is finite, then f extends continuously to $\bar{\Delta}$.*

If $\mathscr{C}(f)$ in Theorem 1 is a rectifiable simple closed curve, then to prove that f extends continuously to $\bar{\Delta}$, it suffices to assume that, if $f=(f_1,\dots f_N)$ with f_1 nonconstant, then $\int |f_1'|^2$ is finite. This follows from the proof of Theorem 3 below.

Given a bounded holomorphic function g on Δ and a point $\zeta \in b\Delta$, we denote by $\mathscr{C}(g;\zeta)$ the cluster set of g at ζ, i.e., the set of all limit points of sequences of the form $\{g(z_j)\}_{j=1,2,\dots}$ as the sequence $\{z_j\}_{j=1,2,\dots}$ runs through all sequences in Δ that converge to the point ζ; it is a connected compact set. If $\lambda \subset b\Delta$ is an arc, we define cluster set $\mathscr{C}(g;\lambda)$ in a similar way.

THEOREM 3. *Let $f\colon \Delta \to \mathbf{B}_N$ be a proper holomorphic map. If $\mathscr{C}(f)$ is a rectifiable simple closed curve and if for each $\zeta \in b\Delta$, $\mathscr{C}(f;\zeta) \neq \mathscr{C}(f)$, then f extends continuously to $\bar{\Delta}$, and $f' \in H^1(\Delta)$.*

The proof of Theorem 2 depends on the following preliminary:

LEMMA 1. *Let L be a connected compact set of finite length in $\mathbf{C}$, and let $p,q \in L$, $p \neq q$. Let P_n, $n=1,2,\dots$, be an open set in $\mathbf{C}$ with*

1) *P_n connected,*
2) *$P_1 \supset P_2 \supset \dots$,*
3) *$bP_n \subset L$,*
4) *$p,q \in \bar{P}_n$.*

Then $\bigcap_{n=1}^{\infty} P_n$ contains a nonempty open set.

PROOF. Choose coordinates so that $p=0$, $q=i(=\sqrt{-1})$. The set L has finite length so [5, 2.10.11, p. 176] for almost all $y \in \mathbf{R}$, the set

$$L(y) = \{(x,y)\colon\ (x,y) \in L\}$$

is finite. Let $\# y$ denote its cardinality. Note that for $y \in (0,1)$, $\# y \geq 2$.

Fix $y_0 \in (0,1)$ with $\# y < \infty$. Denote by $\Omega_\infty, \Omega_1, \Omega_2, \dots$, components of $\mathbf{C}\setminus L$, Ω_∞ the unbounded component. We shall that at most finitely many of these components meet the line $l(y_0) = \{(x,y_0)\colon x \in \mathbf{R}\}$.

Each of the sets $\Omega_j, j=1,2,\dots,\infty$, is open, so $l(y_0) \cap \Omega_j$, if not empty, is an open subset of $l(y_0)$. As such, it is a countable union of open intervals $\lambda(j;k)$, $k=1,\dots$. The endpoints of $\lambda(j;k)$ lie in L, and a given point

$z \in l(y_0)$ that is an endpoint of a $\lambda(j,k)$ is the endpoint of at most one other $\lambda(j'; k')$, because the Ω_j's are disjoint. This implies that the number of Ω_j's meeting $l(y_0)$ is not in excess of $2\# y_0$. (This is very crude, but all we need is the finiteness of the set of Ω_j's with the property.)

Assume now that $\bigcap_{n=1}^{\infty} P_n$ does not contain an open set. We have

$$P_n = L_n \cup W_n$$

where L_n is a subset of L and W_n is a union of some collection of the Ω_j's. As $\cap P_n$ does not contain an open set, for each j, there is an $N(j)$ so that for $n \geqq N(j)$, P_n does not contain Ω_j. (Here, $j=1,\ldots,\infty$.)

Let $\Omega_{\alpha_1},\ldots,\Omega_{\alpha_r}$, $\alpha_1,\ldots,\alpha_r \in \{1,\ldots,\infty\}$ be the finitely many Ω's that meet $l(y_0)$. If $n \geqq \max(N(\alpha_1),\ldots N(\alpha_r))$, then P_n does not meet $l(y_0) \cap (\mathbf{C} \setminus L)$. That is to say, for n large, P_n is a connected open set such that $0, i \in \overline{P_n}$, but $P_n \cap l(y_0)$ is finite. Plainly no such connected open set exists. The lemma is proved.

Granted Lemma 1, Theorem 2 is proved as follows.

It is enough to prove that $\mathscr{C}(f; 1)$ contains only one point. Assume that $p, q \in \mathscr{C}(f; 1)$, $p \neq q$. As $\int_\Delta |f'|^2 < \infty$, a result of Tsuji [3, p. 47] yields a sequence $\{r_n\}_{n=1}^{\infty}$ decreasing to zero such that if

$$L_n = \{\zeta \in \Delta : |\zeta - 1| = r_n\},$$

then the length of $f(L_n)$ decreases to zero as $n \to \infty$. (In the statement of the result given in this reference it is assumed that f is injective, but this hypothesis is unneeded in the proof.) As f is bounded, we may suppose, by passing to a subsequence, that there is a point $z \in \mathscr{C}(f)$ such that $f(L_n) \to z$ in the sense that every neighborhood of z contains all but finitely many of the sets $f(L_n)$. We may also suppose that

$$\sum_{n=1}^{\infty} \Lambda' f(L_n) < \infty .$$

Let

$$L = \mathscr{C}(f) \cup \bigcup_{n=1}^{\infty} f(L_n) .$$

Since each $f(L_n)$ is a path with endpoints belonging to $\mathscr{C}(f)$ and since $f(L_n) \to z$, it follows that L is a compact connected set of finite length.

For each n, let $\Omega_n = \{\zeta \in \Delta : |\zeta - 1| < r_n\}$. Then $P_n = f(\Omega_n)$ is a decreasing sequence of open sets satisfying the conditions (1)–(4) of Lemma 1. By

Lemma 1, $\bigcap_{n=1}^{\infty} P_n$ must contain a nonempty open set. This contradicts the fact that $\bigcap_{n=1}^{\infty} \overline{P_n}=\mathscr{C}(f;1)\subset\mathscr{C}(f)$ has empty interior.

This completes the proof of Theorem 2.

LEMMA 2. *Let $f\colon \Delta \to \boldsymbol{B}_N$ be a proper holomorphic map such that $\mathscr{C}(f;\lambda)$ is a rectifiable arc for some arc $\lambda\subset b\Delta$. Let λ_i, $i=1,2$ be arcs in $\bar{\Delta}$, λ_i with one endpoint, q_i, in λ and otherwise contained in Δ. If $\Delta\cap\lambda_1\cap\lambda_2=\varnothing$ and if for some $w\in b\boldsymbol{B}_N$,*

$$\lim_{\substack{\zeta\in\lambda_1\\ \zeta\to b\Delta}} f(\zeta) = \lim_{\substack{\zeta\in\lambda_2\\ \zeta\to b\Delta}} f(\zeta) = w$$

then $q_1=q_2$.

PROOF. Let p_i, $i=1,2$, be the endpoint of λ_i that lies in Δ. With no loss of generality, we can suppose that $p_1=p_2$.

Assume that $q_1\neq q_2$. Denote by f^* the radial (or equivalently, nontangential) boundary function of f. By [3, p. 19] our assumptions imply that $f^*(q_1)=f^*(q_2)=w$. Thus, it is enough to get a contradiction in the special case when λ_1 and λ_2 are radii. Denote by λ_0 the arc in λ with q_1 and q_2 as endpoints. Then $\lambda_0\cup\lambda_1\cup\lambda_2$ is a simple closed curve; let its interior be denoted by Ω. Let L be the cluster set of f at λ_0 with respect to the domain Ω. By hypothesis, L is a rectifable arc. The set J defined by $J=f(\lambda_1\cup\lambda_2)\cup\{w\}$ is a closed subset of $\boldsymbol{B}_N\cup\{w\}$, and its polynomially convex hull, $\hat{J}$, is also contained in $\boldsymbol{B}_N\cup\{w\}$. We shall show below that the nontangential limits of f along λ_0 are dense in L. This implies that there exist $w_0\in L\setminus\{w\}$ and $\theta\in\mathrm{R}$ such that $e^{i\theta}\in\lambda_0\setminus\{q_1,q_2\}$, and such that the nontangential limit $f^*(e^{i\theta})$ exists and is w_0. Thus, if γ is a short open radial segment in Δ terminating at $e^{i\theta}$, then w_0 is a limit point of $f(\gamma)$. Moreover, if γ is short enough, then since f is proper, $f(\gamma)$ will be disjoint from the compact subset $\hat{J}$ of $\boldsymbol{B}_N\cup\{w\}$.

This is impossible, however: By the maximum principle, $f(\Omega)\subset(L\cup J)\hat{}$, and, as noted in [7, Lemma 7], $(L\cup J)\hat{}=L\cup\hat{J}$. As $f(\gamma)\subset f(\Omega)$, we have reached a contradiction.

It remains to see that, as claimed, the nontangential limits of f along λ_0 are dense in L. If not, let w_1 be a point of L at positive distance from the set $f^*(\lambda_0)$. Then w_1 is also at positive distance from the set $f(\lambda_1\cup\lambda_2)$. Consequently, there is some $\delta>0$ such that if Φ is a conformal map from Δ to Ω, the nontangential boundary function g^* of $g=f\circ\Phi$ satisfies $\operatorname{Re}\langle g^*|w_1\rangle\leqq 1-\delta$, and the Poisson integral representation shows that this implies $\operatorname{Re}\langle g|w_1\rangle\leqq 1-\delta$ on Δ, that is $\operatorname{Re}\langle f|w_1\rangle\leqq 1-\delta$ on Ω which contradicts the fact that $w_1\in L$. The lemma is proved.

REMARK. Knowing now that $q_1=q_2$, call it q, we know that $\lambda_1\cup\lambda_2$ is a simple closed curve, which is the boundary of a bounded domain, Ω. The function f is bounded, and is continuous on $(\Omega\cup\lambda_1\cup\lambda_2)\setminus\{q\}$. Moreover, $f|((\lambda_1\cup\lambda_2)\setminus\{q\})$ extends continuously to $\lambda_1\cup\lambda_2$. It follows that f extends continuously to all of $\bar{\Omega}$: A bounded holomorphic function on a domain bounded by a simple closed curve that has continuous boundary values is continuous on the closed domain.

We shall need the following simple geometric fact:

LEMMA 3. *If $\Gamma\subset\mathbf{C}^N$ is an arc with endpoints w, z, if $v\in\Gamma\setminus\{w,z\}$, and if $\varepsilon>0$, there is $\delta>0$ small enough that if $L\subset\Gamma+\delta\boldsymbol{B}_N$ is a connected set containing z and w, then L meets $v+\varepsilon\boldsymbol{B}_N$.*

The point is that for given ε, if δ is sufficiently small, the set

$$(\Gamma+\delta\boldsymbol{B}_N)\setminus(v+\varepsilon\boldsymbol{B}_N)$$

has at least two components, and z and w lie in distinct components.

LEMMA 4. *Let $r>0$, let $\lambda=\{e^{i\theta}\colon |\theta|\leqq r\}$, and let $f\colon \Delta\to\boldsymbol{B}_N$ be a proper holomorphic map such that the cluster set $\mathscr{C}(f;\lambda)$ is a rectifiable arc. Let $-r<\theta_1<\theta_2<\theta_3<r$. If the radial limits $f^*(e^{i\theta_1})$, $f^*(e^{i\theta_2})$ and $f^*(e^{i\theta_3})$ exists they are distinct, and $f^*(e^{i\theta_2})$ lies in the subarc, L, of $\mathscr{C}(f;\lambda)$ with endpoints $f^*(e^{i\theta_1})$ and $f^*(e^{i\theta_3})$.*

That the given radial limits are distinct is the content of Lemma 2; the main point of the present lemma is the given order-preserving property of f^*.

PROOF. Suppose that $f^*((e^{i\theta_2})\notin L$. To fix notation, suppose that $f^*(e^{i\theta_1})$ belongs to the arc in $\mathscr{C}(f;\lambda)$ connecting $f^*(e^{i\theta_2})$ and $f^*(e^{i\theta_3})$. Let w be a point in $\mathscr{C}(f;\lambda)$ between $f^*(e^{i\theta_1})$ and $f^*(e^{i\theta_2})$.

For $\varepsilon\in(0,1)$, put

$$D_\varepsilon(w) \;=\; \{z\in\boldsymbol{B}_N : \; 1-\varepsilon<\operatorname{Re}(z,w)\}\;.$$

Fix $r_0\in(0,1)$ and $\varepsilon_0>0$ so that $f(\zeta)\notin D_{\varepsilon_0}(w)$ when $\zeta=re^{i\theta_1}$, $\zeta=re^{i\theta_2}$ or $\zeta=re^{i\theta_3}$ with $r\in(r_0,1)$. Having fixed r_0, we can choose ε_0^1, $0<\varepsilon_0^1<\varepsilon_0$, so that $f(r_0e^{i\theta})\notin D_{\varepsilon_0^1}(w)$ for all $\theta\in[\theta_1,\theta_3]$.

Given $\delta>0$, there is ϱ, $0<\varrho<1$, large enough that $f(re^{i\theta})\in\mathscr{C}(f;\lambda)+\delta\boldsymbol{B}_N$ when $r\in(\varrho,1)$, $\theta\in[\theta_1,\theta_3]$.

It follows that given ε, $0<\varepsilon<\varepsilon_0$, there is ϱ, $0<\varrho<1$, such that if $r\in(\varrho,1)$ and

$$\lambda = \{re^{i\theta} : \theta_1 < \theta < \theta_2\} \quad \text{and} \quad \mu = \{re^{i\theta} : \theta_2 < \theta < \theta_3\} ,$$

then both $f(\lambda)$ and $f(\mu)$ meet $D_\varepsilon(w)$.

Let $\{r'_n\}_{n=1}^{\infty}$ be an increasing sequence in $(r_0, 1)$ with limit 1. There is then a decreasing sequence $\{\varepsilon_n\}_{n=1}^{\infty}$, $\varepsilon_1 < \varepsilon_0$, with limit zero so that if

$$\lambda'_n = \{r'_n e^{i\theta} : \theta_1 < \theta < \theta_2\} \quad \text{and} \quad \mu'_n = \{r'_n e^{i\theta} : \theta_2 < \theta < \theta_3\} ,$$

then both $f(\lambda'_n)$ and $f(\mu'_n)$ miss $D_{\varepsilon_n}(w)$.

Also there is an increasing sequence $\{r_n\}_{n=1}^{\infty}$, $r_n < r'_n < 1$ such that if

$$\lambda_n = \{r_n e^{in\theta} : \theta_1 < \theta < \theta_2\} \quad \text{and} \quad \mu_n = \{r_n e^{i\theta} : \theta_2 < \theta < \theta_3\} ,$$

then $f(\lambda_n)$ and $f(\mu_n)$ both meet $D_{\varepsilon_n}(w)$.

For $n = 1, 2, \ldots$, defined sets A_n, B_n, P_n, and Q_n by

$$P_n = \{re^{i\theta} : r'_n < r < 1,\ \theta_1 < \theta < \theta_2\}$$
$$Q_n = \{re^{i\theta} : r'_n < r < 1,\ \theta_2 < \theta < \theta_3\}$$

and

$$A_n = \{\zeta \in P_n : f(\zeta) \in D_{\varepsilon_n}(w)\}$$
$$B_n = \{\zeta \in Q_n : f(\zeta) \in D_{\varepsilon_n}(w)\} .$$

The set A_n is open, and at each point of bA_n (boundary taken with respect to C), which is in Δ, $\operatorname{Re}\langle f(\zeta), w\rangle = 1 - \varepsilon_n$. Thus, bA_n does not meet the set

$$T_n = \lambda'_n \cup \{re^{i\theta_1} : r'_n \leqq r < 1\} \cup \{re^{i\theta_2} : r'_n \leqq r < 1\} ,$$

as follows by the choice of r_0 and λ'_n and μ'_n. However, by the choice of λ_n and μ_n, the set A_n meets λ_n, say at ζ_n. If A'_n denotes the component of A_n containing ζ_n, then A'_n meets λ_{n+1}. If not, then as $\overline{A'_n}$ (closure in C) does not meet T_n, the set $\overline{A'_n}$ is a compact subset of Δ. This would imply that the function $\operatorname{Re}\langle f, w\rangle$ is equal to $1 - \varepsilon_n$ on bA'_n but greater than $1 - \varepsilon_n$ on A'_n. Thus, A'_n meets λ_{n+1}, say at ζ_{n+1}.

Consequently, there is a curve, γ_n, in A_n connecting $\zeta_n \in \lambda_n$ to $\zeta_{n+1} \in \lambda_{n+1}$. Iterating this construction yields a continuous map $\gamma: [0,1) \to P_1$ such that $\lim_{t \to 1^-} \operatorname{dist}(\gamma(t), b\Delta) = 0$ and such that

$$\lim_{t \to 1^-} \operatorname{Re}\langle f(\gamma(t)), w\rangle = 1 ,$$

that is $\lim_{t \to 1^-} f(\gamma(t)) = w$.

We shall show that $\lim_{t \to 1^-} \gamma(t)$ exists. Otherwise, there are points $\zeta, \zeta' \in b\Delta, \zeta \neq \zeta'$ and there are points $t_n, t'_n \in (0,1)$, $t_n, t'_n \to 1$ with $\gamma(t_n) \to \zeta$, $\gamma(t'_n) \to \zeta'$. For large n, $\gamma([t_n, t'_n])$ and $\gamma([t_n, t_{n+1}])$ both meet the radial segment $|0,\xi|$, where we choose $\xi \in b\Delta$ so that $f^*(\xi)$ exists, ξ in the arc $\{e^{i\theta}: \theta_1 \leqq \theta \leqq \theta_2\}$ between ζ and ζ'. It follows then that $f^*(\xi)=w$. This is true for every choice of ξ, so f is a constant. Contradiction. Thus, $\lim_{t\to 1^-} \gamma(t)$ exists.

If we invoke [9, p. 208], we find that there is an arc $L_1 \subset \gamma([0,1]) \cup \lim_{t\to 1^-} \gamma(t)$ that is contained in $P_1 \cup \{\lim_{t\to 1^-} \gamma(t)\}$ and that has $\lim_{t\to 1^-} \gamma(t)$ as an endpoint such that

$$\lim_{\substack{\zeta \in L_1 \\ \zeta \to b\Delta}} f(\zeta) = w .$$

A parallel construction yields an arc L_2 contained, except for an endpoint in $b\Delta$, in Q such that

$$\lim_{\substack{\zeta \in L_2 \\ \zeta \to b\Delta}} f(\zeta) = w .$$

Lemma 2 shows that $L_1 \cap b\Delta = L_2 \cap b\Delta$ whence this common point must be $e^{i\theta_2}$. However, this is impossible, for w was chosen not to be $f^*(e^{i\theta_2})$.

This completes the proof of Lemma 4.

The following is a well-known fact in the theory of real functions:

LEMMA 5. *If A is a dense subset of $(0,1)$ and if f: $A \to \mathsf{R}$ is an increasing function, then there is F: $(0,1) \to \mathsf{R}$, an increasing function, that agrees on A with f.*

A suitable choice for the function F is given by

$$f(x) = \tfrac{1}{2}\left\{\sup_{\substack{t\in A\\ t\leqq\alpha}} f(t) + \inf_{\substack{t\in A\\ t\geqq\alpha}} f(t)\right\} .$$

PROOF OF THEOREM 3. The hypothesis that $\mathscr{C}(f) \neq \mathscr{C}(f;\, 1)$ implies that for some $\delta > 0$, if

$$\lambda = \{e^{i\theta} : |\theta| \leqq \delta\} ,$$

then $L = \mathscr{C}(f;\, \lambda)$ is a rectifiable arc. Let A be the set of points θ in $(-\delta,\delta)$ such that $f^*(e^{i\theta})$ exists. It is of full measure in $(-\delta,\delta)$ and so, certainly, dense. We know, moreover, that $\theta \mapsto f^*(e^{i\theta})$ is order-preserving from A into L. Lemma 5 yields a function F: $(-\delta,\delta) \to L$ that extends $\theta \mapsto f^*(e^{i\theta})$ and that is order-preserving. As L is rectifiable, F is of bounded variation.

As the remarks of the preceding paragraph apply *mutatis mutandis* when

$\mathscr{C}(f;1)$ is replaced by $\mathscr{C}(f;\zeta)$ for every $\zeta \in b\Delta$, we conclude that there exists a function F on $b\Delta$ that is of bounded variation and that agrees almost everywhere with f^*. This implies that f extends continuously to $\bar{\Delta}$ and, moreover, that $f' \in H^1(\Delta)$ (cf. [4, p. 42].)

It will be observed that although we have stated Theorems 1 and 3 for maps from the disc to the ball, the arguments work equally well for maps from the disc to any strictly convex domain and hence, by way of the embedding theorem of Fornaess and Henkin [6; 10, p. 668 of the English translation], the results are valid for maps from the disc to a strongly pseudoconvex domain.

It is possible that our main result is a special case of a more general one. Perhaps the hypothesis that $f(\Delta)$ have finite area is redundant: If E is a compact connected set of finite length, then presumably the polynomially convex hull of E has finite area. We do not have a proof of this fact. Also, it seems likely to us that if f is a bounded holomorphic function on the unit disc whose global cluster set has finite length, then f extends continuously to $\bar{\Delta}$. Some care is required here though: Simple examples show that the condition is not strong enough to yield $f' \in H^1$.

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