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ON THE SPECTRUM OF INNER DERIVATIONS IN PARTIAL JORDAN TRIPLES

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1. Introduction.

Let D be a bounded balanced domain in a complex Banach space E. In contrast with the fact that the complete holomorphic classification of bounded domains of general type seems to be hopeless, Kaup-Upmeier [9] proved that for bounded balanced domains holomorphic equivalence is the same as linear equivalence. They achieved this result by a systematic study of the group G of all biholomorphic automorphisms of D, which makes it possible to give further refinements of this statement. They showed there exists a closed complex subspace E_0 and a continuous real trilinear map

$$E \times E_0 \times E \to E \quad (x, a, y) \mapsto \{xa^*y\}$$

symmetric complex bilinear in x, y and conjugate linear in a such that, regarding holomorphic vector fields as differential operators [7], for every $a \in E_0$ the vector field $(a - \{xa^*x\})\partial/\partial x$ is complete in D and that furthermore

$$\mathsf{G} = \mathrm{GL}(D) \cdot \{\exp[(a - \{xa^*x\})\partial/\partial x]: a \in E_0\}, \quad \mathsf{G}(0) = D \cap E_0$$

where $\mathrm{GL}(D) := \{\alpha \in \mathrm{GL}(E): \alpha(D) = D\}$. It would be a remarkable step, also with a possible independent interest in theoretical physics, characterizing those triple products which arise from the biholomorphic automorphism group of some bounded balanced domain in the above way. It is well-known [4] that the triple product $\{*\}$ satisfies the following topological algebraic postulates

(J1) $\{E_0 E_0^* E_0\} \subset E_0$

(J2) $\{ab^*\{xy^*z\}\} = \{\{ab^*x\}y^*z\} - \{x\{ba^*y\}^*z\} + \{xy^*\{ab^*z\}\}$
$(a, b, y \in E_0, x, z \in E)$

(J3) $a \,\square\, a^* \in \mathrm{Her}(E) \quad (a \in E_0)$

where $a \,\square\, b^*$ is the operator $x \mapsto \{ab^*x\}$ and $\mathrm{Her}(E)$ stands for the family of all E-Hermitian operators [2]. Such algebraic structures are called partial her-

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mitian Jordan triple systems or *partial* J*-*triples* (resp. J*-*triples* if $E = E_0$) for short in the following. We say that a partial J*-triple $(E, E_0, \{*\})$ is *positive* if for every $a \in E_0$ the spectrum $\mathrm{Sp}(a \,\square\, a^*)$ is non-negative and *geometric* if all vector fiels $(a - \{xa^*x\})\partial/\partial x$ $(a \in E_0)$ are complete in some bounded balanced domain in E. In 1983 Kaup [8] settled the case $E = E_0$ completely: A J*-triple is geometric if and only if $\inf_{\|a\|=1} \|\{aa^*a\| \neq 0$ and

$$(1.1) \qquad 0 \leqq \mathrm{Sp}(a \,\square\, a^*) \subset \tfrac{1}{2}\Omega_a + \tfrac{1}{2}\Omega_a \quad (a \in E = E_0)$$

where $\Omega_a := \{0\} \cup \mathrm{Sp}(a \,\square\, a^* \mid C_0(a))$ and $C_0(a)$ is the smallest $a \,\square\, a^*$-invariant subspace containing a. It was a far-reaching consequence of (1.1) that the Harish-Chandra realization of a bounded symmetric domain in a Banach space is always convex [7], [8].

The proof of (1.1) uses some properties of the quadratic representation which are not available for arbitrary geometric partial J*-triples. The aim of this paper is to develop a technique based on the ultrapower imbedding due to Dineen [5] to the study of the spectrum of the inner derivations $a \,\square\, a^*$. As main result we prove the following:

THEOREM 1.2. *Every geometric partial* J*-*triple is positive.*

The idea of the proof is the observation that a suitable ultrapower extension [5] of the abelian family $\{b \,\square\, b^* \colon b \in \mathscr{C}_0(a)\}$ admits convenient joint eigenvectors and its span is linearly homeomorphic to $\mathscr{C}_0(\Omega_a)$ by a mapping which can be factorized through the tensor square of the Gelfand representation of $\mathscr{C}_0(a)$. With this method we give also a new and Jordan theoretically very simple proof for Kaup's spectral estimate (1.1) for geometric J*-triples.

The analog of (1.1) for arbitrary geometric partial J*-triples is false: To every $p > 0$ the space $\mathbf{C}^2$ endowed with the triple product $\{(\zeta_1, \xi_1)(\alpha, 0)^* (\zeta_2, \xi_2)\} := \bar{\alpha}(\zeta_1\zeta_2, (\zeta_1\xi_2 + \zeta_2\xi_1)\cdot p)$ defined on $\mathbf{C}^2 \times (\mathbf{C} \times \{0\}) \times \mathbf{C}^2$ is a geometric partial J*-triple corresponding to the 2-dimensional Reinhardt domain $\{(\zeta, \xi) \colon |\zeta|^2 + |\xi|^{2/p} < 1\}$ (cf. [11], [1, p. 162]). Here we have $\Omega_{(1,0)} = \{0, 1\}$ and $\mathrm{Sp}((1,0) \,\square\, (1,0)^*) = \{1, p\}$.

2. Joint eigenvectors of box opertors.

Throughout this section let E be a geometric partial J*-triple with triple product $\{*\}$ on $E \times E_0 \times E$ and assume that D is a bounded balanced domain in E in which the vectors fields $(b - \{zb^*z\})\partial/\partial z$ are complete for all $b \in E_0$. Let us also fix $a \in E_0$ arbitrarily. We denote by T the Gelfand representation [8], [6, Th. 10.38] of $\mathscr{C}_0(a)$, i.e. $T\, \mathscr{C}_0(\Omega_a) \xrightarrow{\ \simeq\ } \mathscr{C}_0(a)$ is a topological isomorphism such that

$$T(\varphi\bar{\chi}\psi) = \{T(\varphi)T(\chi)^*T(\psi)\} \quad (\varphi, \chi, \psi \in \mathscr{C}_0(\Omega_a)), \quad T(\xi) = a$$

where $\xi(\omega) := \sqrt{\omega}$ $(\omega \in \Omega_a)$ and $\mathscr{C}_0(\Omega_a) := \{\varphi \in \mathscr{C}(\Omega_a): \varphi(0) = 0\}$. Recall [8] that $\Omega_a \geqq 0$ and that $\{b \square b^*: b \in \mathscr{C}_0(a)\}$ is a commutative family of bounded E-hermitian operators. Define $\mathscr{L}(a) := \operatorname{Span}\{b \square b^*: b \in \mathscr{C}_0(a)\}$.

LEMMA 2.1. *$\mathscr{L}(a) = \mathscr{C}_0(a) \square \mathscr{C}_0(a)^*$ and there exists a linear homeomorphism $L: \mathscr{C}_0(\Omega_a) \xrightarrow{\sim} \mathscr{L}(a)$ such that*

$$L(\varphi\bar{\psi}) = T(\varphi) \square T(\psi)^* \quad (\varphi, \psi \in \mathscr{C}_0(\Omega_a)). \tag{2.3}$$

PROOF. Let $\mathscr{D} := \{\phi \in \mathscr{C}_0(\Omega_a): \phi$ vanishes in a neighbourhood of $0 \in \Omega_a\}$. We may define $L_0(\phi) := T(\phi/\xi) \square T(\xi)^*$ $(\phi \in \mathscr{D})$. It is well-known [4] that

$$T(p) \square T(q)^* = T(p\bar{q}/\xi) \square T(\xi)^* \quad \text{for} \quad p, q \in \mathscr{P} := \{\text{odd polynomials of } \xi).$$

Given $\varphi, \psi \in \mathscr{D}$, we can find sequences $(p_n), (q_n)$ in $\mathscr{P}$ tending uniformly to φ/ξ^2 and ψ/ξ^2, respectively. Then $L_0(\varphi\bar{\psi}) = T(\varphi\bar{\psi}/\xi) \square T(\xi)^* = \lim_n T(\xi^3 p_n q_n) \square T(\xi)^* = \lim_n T(\xi^2 p_n) \square T(\xi^2 q_n)^* = T(\varphi) \square T(\psi)^*$. Hence $\|L_0(\varphi)\| = \|T(\phi^{1/2}) \square T(\phi^{1/2})^*\| \leqq M\|\phi\|$ $(\phi \in \mathscr{D}_+)$ where $M := \sup\{\|T(\varphi) \square T(\psi)^*\|: \|\varphi\| = \|\psi\| = 1\} < \infty$. Decomposing the functions of $\mathscr{D}$ into linear combinations from $\mathscr{D}_+$, it follows $\|L_0\| \leqq 4M$. By the density of $\mathscr{D}$ in $\mathscr{C}_0(\Omega_a)$ there is a unique continuous linear extension $L: \mathscr{C}_0(\Omega_a) \to \mathscr{L}(a)$ of L_0 satisfying (2.3). On the other hand every $\phi \in \mathscr{C}_0(\Omega_a)$ can be written in the form $\phi = \varphi\bar{\psi}$ for some $\varphi, \psi \in \mathscr{C}_0(\Omega_a)$. Hence with $d := \max\{\|T\|, \|T^{-1}\|\}$ we get

$$d \cdot \|L(\phi)\| \geqq \sup_{\|\chi\|=1} \|L(\phi)T(\chi)\| =$$
$$= \sup_{\|\chi\|=1} \|T(\varphi)T(\psi)^*T(\chi)\}\| \geqq \sup_{\|\chi\|=1} \frac{1}{d}\|\varphi\bar{\psi}\chi\| = \frac{1}{d}\|\phi\|.$$

Thus L is a linear homeomorphism. In particular the range of L is a closed subspace of $\mathscr{L}(a)$ and $\operatorname{ran}(L) = L\{\varphi\bar{\psi}: \varphi, \psi \in \mathscr{C}_0(\Omega_a)\} = T(\mathscr{C}_0(\Omega_a)) \square T(\mathscr{C}_0(\Omega_a))^* = \mathscr{C}_0(a) \square \mathscr{C}_0(a)^*$.

The following fact seems to be known. We sketch a proof because we do not know a reference.

LEMMA 2.2. *Let F be a Banach space and $\mathscr{A}$ a separable linear subspace of $\mathscr{L}(F)$ consisting of commuting operators and let $\alpha_0 \in \mathscr{A}$. Then to every approximate eigenvalue λ_0 of α_0 there exist a sequence (x_n) in F and a continuous linear functional Λ on $\mathscr{A}$ such that $\lambda_0 = \Lambda(\alpha_0)$ and*

$$\|x_n\| \to 1, \ \|\alpha x_n - \Lambda(\alpha)x_n\| \to 0 \quad (n \to \infty, \alpha \in \mathscr{A}).$$

PROOF. Every $\alpha \in \mathscr{A}$ acts on $\ell^\infty(\mathsf{N}, F)$ by $(x_n) \mapsto (\alpha x_n)$ and hence also on $\tilde{F} := \ell^\infty(\mathsf{N}, F)/M$ where $M := \{(x_n) \in \ell^\infty(\mathsf{N}, F): \lim_n x_n = 0\}$. Denote this operator by $\tilde{\alpha}$. Then $\tilde{\mathscr{A}} := \{\tilde{\alpha}: \alpha \in \mathscr{A}\}$ is a commutative subspace of $\mathscr{L}(\tilde{F})$. It suffices to show that the operators in $\tilde{\mathscr{A}}$ admit a joint eigenvector in the λ_0-eigenspace of $\tilde{\alpha}_0$.

It is clear that $\tilde{F}_0 := \{\tilde{x} \in \tilde{F} : \tilde{\alpha}_0 \tilde{x} = \lambda_0 \tilde{x}\} \neq 0$ and that $\tilde{F}_0$ is left invariant by all $\tilde{\alpha} \in \tilde{\mathscr{A}}$. Let (α_n) be a dense sequence in $\mathscr{A}$ and for each $n \in \mathsf{N}$ define an $\tilde{\mathscr{A}}$-invariant subspace $\tilde{F}_n$ and $\lambda_n \in \mathsf{C}$ recursively in the following way: Let λ_n be an approximate eigenvalue of the operator $\alpha_n | \tilde{F}_{n-1}$ and let $\tilde{F}_n := \{\tilde{x} \in \tilde{F}_{n-1} : \tilde{\alpha}_n \tilde{x} = \lambda_n \tilde{x}\}$. This is possible since the approximate point spectrum of every bounded linear operator on a Banach space is not empty [11, p. 310]. The only thing we have to verify is that

$$\bigcap_n \tilde{F}_m \neq 0.$$

First we show by induction that $\tilde{F}_n \neq 0$ (n $= 0, 1, \ldots$). Assume $\tilde{F}_{n-1} \neq 0$. By the definition of λ_n there is a sequence $(\tilde{x}^k)$ in $\tilde{F}_{n-1}$ with $\|\tilde{x}^k\| = 1$ $(k \in \mathsf{N})$ and $\tilde{\alpha}_n \tilde{x}^k \to 0\,(k \to \infty)$. Since $\tilde{F}_0 \supset \ldots \supset \tilde{F}_n$, we also have $\tilde{\alpha}_j \tilde{x}^k = \lambda_j \tilde{x}^k$ $(0 \leqq j < n)$ for all $k \in \mathsf{N}$. For any k chose a representing sequence $(y^k_m : m \in \mathsf{N})$ in F for $\tilde{x}^k$. It follows that for each $\ell \in \mathsf{N}$ we can find $k(\ell)$ such that, by setting $z_{n,\ell} := y^{k(\ell)}_{m(\ell)}$, we have

$$|\,\|z_{n,\ell}\| - 1| < \ell^{-1} \quad \text{and} \quad \|\tilde{\alpha}_j z_{n,\ell} - \lambda_j z_{n,\ell}\| < \ell^{-1} \quad (0 \leqq j \leqq n).$$

Hence the relation $\tilde{F}_n \neq 0$ is immediate.

We complete the proof by observing that the vector $\tilde{z} \in \tilde{F}$ which is represented by the diagonal $(z_{n,n})$ of the double sequence $(z_{n,\ell})$ constructed above satisfies $\|\tilde{z}\| = 1$ and $\tilde{\alpha}_j \tilde{z} = \lambda_j \tilde{z}$ $\quad (j \in \mathsf{N})$.

Let $\mathscr{U}$ be a non-trivial ultrafilter on N and $E^{\mathscr{U}}$ the $\mathscr{U}$-ultrapower of E that is $\ell^\infty(\mathsf{N}, E)/N$ where $N := \{(x_n) \in \ell^\infty(\mathsf{N}, E) : \lim_{\mathscr{U}} x_n = 0\}$. The elements of $E^{\mathscr{U}}$ are the cosets $(x_n)_{\mathscr{U}} := (x_n) + N$ with the norm $\|(x_n)_{\mathscr{U}}\| := \lim_{\mathscr{U}} \|x_n\|$ $((x_n) \in \ell^\infty(\mathsf{N}, E))$. We regard E as a subspace of $E^{\mathscr{U}}$ by the imbedding $x \mapsto (x, x, \ldots)_{\mathscr{U}}$. Taking $E_0^{\mathscr{U}} := \{(a_n)_{\mathscr{U}} : (a_n) \in \ell^\infty(\mathsf{N}, E_0)\}$, the canonical extension

$$\{(x_n)_{\mathscr{U}} (a_n)^*_{\mathscr{U}} := (\{x_n a_n^* y_n\})_{\mathscr{U}} \quad ((x_n), (y_n) \in \ell^\infty(\mathsf{N}, E); \quad (a_n) \in \ell^\infty(\mathsf{N}, E_0))$$

of the triple product makes $(E^{\mathscr{U}}, E_0^{\mathscr{U}}, \{*\}_{\mathscr{U}})$ into a partial J*-triple. We denote it also by $E^{\mathscr{U}}$ and write simply $\{*\}$ instead of $\{*\}_{\mathscr{U}}$. Note that the vector fields $(\tilde{b} - \{\tilde{z}\tilde{b}^*\tilde{z}\})\partial/\partial\tilde{z}$ are complete in the closed set $\tilde{D} := \{(z_n)_{\mathscr{U}} : z_1, z_2, \ldots \in D\}$ (the arguments of [5, Th. 9] apply with straightforward modifications). Since these vecctor fields are locally bounded it follows that they are complete also in the interior of $\tilde{D}$.

Since the spectrum of a hermitian operator is real [2], by [11, p. 310] it coincides with the approximate point spectrum. Therefore we can summarize the previous results as follows:

PROPOSITION 2.3. *Let E be a geometric partial* J*-*triple and $\mathscr{U}$ a non-trivial ultrafilter on* N. *Then $E^{\mathscr{U}}$ is also a geometric partial* J*-*triple. Given $a \in E_0$ and*

$\lambda_0 \in \mathrm{Sp}(a \,\square\, a^*)$ there exists a complex Radon measure μ of bounded variation on Ω_a and $0 \neq \tilde{x} \in E^{\mathscr{U}}$ such that

$$\lambda_0 = \int \omega \, d\mu(\omega) \tag{2.4}$$

$$\{T(\varphi)T(\psi)^*\tilde{x}\} = \int \varphi\bar{\psi} \, d\mu \cdot \tilde{x} \quad (\varphi, \psi \in \mathscr{C}_0(\Omega_a)). \tag{2.5}$$

3. Proof of Theorem 1.2.

Assume D is a bounded balanced domain in E in which the vector fields $(b - \{zb^*z\})\partial/\partial z$ are complete for all $b \in E_0$. Let us fix $a \in E_0$ arbitrarily and denote by T the Gelfand representation of $\mathscr{C}_0(a)$ (see Section 2). Let $\mathscr{U}$ be a non-trivial ultrafilter on N and regard E as a subtriple of $E^{\mathscr{U}}$. Set $\lambda_0 := \mathrm{Sp}(a \,\square\, a^*)$.

Suppose that $\lambda_0 < 0$. According to Proposition 2.3 choose $0 \neq \tilde{x} \in E^{\mathscr{U}}$ and a Radon measure μ of bounded variation on Ω_a satisfying (2.4) and (2.5).

We shall establish that in this case necessarily

$$\{\tilde{x}\mathscr{C}_0(\Omega_a)^*\tilde{x}\} = 0. \tag{3.1}$$

Assuming (3.1) for the moment, we finish the proof of the theorem as follows: We may assume $\tilde{x} \in \tilde{D}$ (defined in Section 2). Then given any $\varphi \in \mathscr{C}_0(\Omega_a)$, the solution $\tilde{z}_\varphi \colon \mathsf{R} \to E^{\mathscr{U}}$ of the initial value problem

$$\frac{d}{dt}\tilde{z}_\varphi(t) = T(\varphi) - \{\tilde{z}_\varphi(t)T(\varphi)^*\tilde{z}_\varphi\}, \quad \tilde{z}_\varphi(0) = \tilde{x}$$

must stay in $\tilde{D}$ for all time. One verifies directly [cf. [4]) that for $\varphi \geqq 0$ we have

$$\tilde{z}_\varphi(t) = T(\tanh(t\varphi)) + \exp\left[-2\int \log\cosh(t\varphi)\, d\mu\right]\tilde{x}.$$

Since $\tilde{D}$ is bounded, this means that $\sup\{\exp[-2\int \log\cosh(\psi)\, d\mu] \colon \psi \in \mathscr{C}_0(\Omega_a)_+\} = \sup\{\exp[-\int \phi \, d\mu] \colon \phi \in \mathscr{C}_0(\Omega_a)_+\} < \infty$. Hence $\int \phi \, d\mu \geqq 0 \; (\phi \in \mathscr{C}_0(\Omega_a)_+)$ which contradicts (2.4).

Proof of (3.1): Choose $\delta > 0$ such that $\|T(\varphi) \,\square\, T(\varphi)^* - a \,\square\, a^*\| < -\lambda_0/3$ for all $\varphi \in \mathscr{C}_0(\Omega_a)$ with $\|\varphi - \xi\| \leqq \delta$ where $\xi := \sqrt{\mathrm{id}}$ on Ω_a. Since $\mathscr{C}_0(\Omega_a) = \mathrm{Span}\{\psi \in \mathscr{C}_0(\Omega_a) \colon \mathrm{diam}\,\mathrm{supp}\,\psi < \delta\}$, it suffices to see that $\{\tilde{x}T(\psi)^*\tilde{x}\} = 0$ whenever the support of $\psi \in \mathscr{C}_0(\Omega_a)$ has diameter $\leqq \delta$.

Let $I := (\lambda, \lambda + \delta^2) \subset \mathsf{R}_+$ be an interval of length δ^2 and $\psi \in \mathscr{C}_0(\Omega_a)$ such that $\mathrm{supp}\,\psi \subset I$. Let φ denote the function $\varphi(\omega) := \mathrm{length}([0, \omega] \backslash I)^{1/2}$ $(\omega \in \Omega_a)$ and define $b := T(\varphi)$, $e := T(\psi)$. We have $\varphi(I) = \sqrt{\lambda}$ and hence $(b \,\square\, b^*)e = T(\varphi^2\psi)$

$= \lambda \cdot e$. On the other hand, $(b \square b^*)\tilde{x} = \eta\tilde{x}$ where $\eta := \int |\varphi|^2\, d\mu$ and $|\lambda_0 - \eta| = \|(a \square a^* - b \square b^*)\tilde{x}\|/\|\tilde{x}\| \leqq \|a \square a^* - b \square b^*\| < \lambda_0/3$ since $\|\varphi - \xi\| \leqq \delta$. In particular $\eta < 2\lambda_0/3$. Observe that, by (J2), the eigen-subspaces $S(\kappa) := \{\tilde{y} \in E^{\mathscr{U}}: (b \square b^*)\tilde{y} = \kappa\tilde{y}\}$, $S_0(\kappa) := \{\tilde{y} \in E_0^{\mathscr{U}}: (b \square b^*)\tilde{y} = \kappa\tilde{y}\}$ satisfy

$$\{S(\kappa_1)S_0(\kappa_2)^*S(\kappa_3)\} \subset S(\kappa_1 - \kappa_2 + \kappa_3) \quad (\kappa_1, \kappa_2, \kappa_3 \in \mathrm{R}).$$

In particular $\{\tilde{x}e^*\tilde{x}\} \in \{S(\eta)S_0(\lambda)^*S(\eta)\} \subset (2\eta - \lambda)$. According to Sinclair's Theorem $\|a \square a^* - \nu \cdot \mathrm{id}\| = \mathrm{rad}\,\mathrm{Sp}(a \square a^* - \nu \cdot \mathrm{id}) = \nu - \min \mathrm{Sp}(a \square a^*)$ and similarly $\|b \square b^* - \nu \cdot \mathrm{id}\| = \nu - \min \mathrm{Sp}(b \square b^*)$ whenever $\nu \geqq \|a \square a^*\|$, $\|b \square b^*\|$. By the triangle inequality it follows $|\min \mathrm{Sp}(a \square a^*) - \min \mathrm{Sp}(b \square b^*)| \leqq \|a \square a^* - b \square b^*\| < -\lambda_0/3$. Hence $2\eta - \lambda < 2\eta < 4\lambda_0/3 < \min \mathrm{Sp}(b \square b^*)$. Thus $S(2\eta - \lambda) = 0$ which completes the proof.

4. New proof of Kaup's spectral estimate (1.1) for geometric J*-triples

Let $E_0 = E$ be a geometric J*-triple and fix $a \in \mathrm{E}$, $\lambda_0 \in \mathrm{Sp}(a \square a^*)$ arbitrarily. Choosing any non-trivial ultrafilter $\mathscr{U}$ on N, from Proposition 2.3 we see that there exists a Radon measure of bounded variation on Ω_a and $0 \neq \tilde{x} \in E^{\mathscr{U}}$ satisfying (2.4) and (2.5) where T is the Gelfand representation of $\mathscr{C}_0(a)$.

Consider any $\varphi \in \mathscr{C}_0(\Omega_a)_+$ and set $e := T(\varphi)$. Since $E^{\mathscr{U}}$ equipped with the binary product $u \bullet v := \{ue^*v\}$ is a commutative Jordan algebra, by [3, p. 145. (3.3)] (or for an elementary proof see [6, Prop. 10.42])

$$\{\{\{ee^*e\}e^*e\}e^*\tilde{x}\} = 3\{\{ee^*e\}e^*\{ee^*\tilde{x}\}\} - 2(ee \square e^*)^3\tilde{x}$$

$$\{T(\varphi^5)T(\varphi)^*\tilde{x}\} = 3\{T(\varphi^3)T(\varphi)^*\{T(\varphi)T(\varphi)^*\tilde{x}\}\} - 2(T(\varphi) \square T(\varphi)^*)^3\tilde{x}$$

Hence from (2.5) we obtain

$$\int \varphi^6\, d\mu = 3\int \varphi^4\, d\mu \int \varphi^2\, d\mu - 2\left(\int \varphi^2\, d\mu\right)^3 \quad (\varphi \in \mathscr{C}_0(\Omega_a)_+).$$

Given a compact subset $S \subset \Omega_a$, we can find a bounded sequence $\varphi_1, \varphi_2, \ldots \in \mathscr{C}_0(\Omega_a)_+$ converging pointwise to 1_S. Therefore

$$\mu(S) = 3\mu(S)^2 - 2\mu(S)^3$$

$$\mu(S) \in \{0, \tfrac{1}{2}, 1\} \quad (S \text{ compact} \subset \Omega_a). \tag{4.1}$$

This is possible only if the support of μ consists of at most 2 points, and hence (4.1) and (2.4) entail $\lambda_0 \in \frac{1}{2}\Omega_a + \frac{1}{2}\Omega_a$.

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