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TRANSPOSITIONS OF UHF ALGEBRAS

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Introduction.

This paper deals with the conjugacy of period two *-antiautomorphisms. Two such maps, α and β, usually called transpositions, are conjugate if there is an automorphism θ, such that $\alpha = \theta \circ \beta \circ \theta^{-1}$. The conjugacy classes of transpositions of certain algebras have been classified. For example, $B(H)$, the bounded linear operators on the Hilbert space H, has exactly one conjugacy class of transpositions if the dimension of the Hilbert space is odd. If the dimension of H is even or infinite, then there is exactly two such classes [5]. One class is determined by the usual matrix transpose, τ_n, $n = \dim(H)$. When the dimension is even, the other class comes from $\sigma_{2n} = \mathrm{Ad}\, V_{2n} \circ \tau_{2n} = \tau_{2n} \circ \mathrm{Ad}\, V_{2n}$, where

$$V_{2n} = \begin{bmatrix} 0 & I_n \\ -I_n & 0 \end{bmatrix}.$$

When $n = \infty$, the obvious generalizations produce the desired maps.

Since the matrix algebra case is well-understood, it is reasonable to study *-algebras which are the direct limit of matrix algebras. Suppose $\{A_n\}_{n=1}^{\infty}$ is an increasing sequence of full matrix algebras, i.e., each A_n is isomorphic to $M(k(n), \mathbf{C})$ where $\lim_{n\to\infty} k(n) = \infty$ and $k(n)$ divides $k(n+1)$ for each n. Let $A = \bigcup_{n=1}^{\infty} A_n$. Such an algebra will be called a matricial algebra. Stacey [11] has shown that if $a(p) = \sup_n \{a\colon p^a \text{ divides some } k(n)\}$, then A has exactly one conjugacy class of transpositions when $a(2) = 0$ or ∞. Otherwise, A has exactly two conjugacy classes.

With $\{A_n\}_{n=1}^{\infty}$ as above, if A is the von Neumann algebra generated by $\bigcup_{n=1}^{\infty} A_n$, via the standard trace, Størmer [12] and Giordano [3] have independently shown that A has exactly one conjugacy class of transpositions. In this paper, we investigate the case where A is the C^*-algebra generated by $\{A_n\}_{n=1}^{\infty}$. This type of

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C^*-algebra is usually called a uniformly hyperfinite (UHF) C^*-algebra and was first studied by Glimm [4].

Using standard density arguments, one can extend Stacey's result to prove the corresponding statement for transpositions of UHF C^*-algebras as long as the transposition leaves $\bigcup_{n=1}^{\infty} A_n$ invariant. Gåsemyr [2] has independently shown this using the Jordan algebra techniques developed by Størmer. Consequently, this paper deals with transpositions which do not necessarily fix $\bigcup_{n=1}^{\infty} A_n$. We also limit ourselves to 2^∞ UHF algebras, i.e. where each A_n is isomorphic to $M(2^n, \mathbf{C})$. In this setting, τ_n and σ_n will induce transpositions τ_0 and σ_0. It can be shown that τ_0 and σ_0 are conjugate [1]. Refer to [7] for basic properties of C^*- and UHF algebras.

A major tool in our study of transpositions of UHF algebras is the central sequence algebra. This algebra, introduced by McDuff [9], was used by Herman and Jones [6] to study automorphisms of UHF algebras. Many of their techniques are used here.

Let A be a C^*-algebra. Consider $l^\infty(A) = \{\{a_n\}_{n=1}^{\infty}: a_n \in A \text{ and } \sup\{\|a_n\|\} < \infty\}$. Here $c_0(A) = \{\{a_n\}_{n=1}^{\infty}: a_n \in A \text{ and } \lim_{n\to\infty} \|a_n\| = 0\}$ is an ideal of $l^\infty(A)$. Let A^∞ denote the C^*-algebra given by $l^*(A)/c_0(A)$. A sequence $\{a_n\}_{n=1}^{\infty}$ of elements in A is called a central sequence if $\lim_{n\to\infty} \|[a_n, a]\| = 0$ for each a in A. The central sequence algebra A_∞ is the collection of all central sequences of A modulo $c_0(A)$ and is a C^*-subalgebra of A^∞. Note that A naturally imbeds in A^∞ as constant sequences and under this identification $A_\infty = A^\infty \cap A'$, where A' is the commutant of A. Mappings in A naturally induce mappings in A^∞ via $\alpha(\{a_n\}_{n=1}^{\infty}) = \{\alpha(a_n)\}_{n=1}^{\infty}$. It should be noted that the use of the central sequence algebra in this paper can be circumvented; however, it adds clarity and ease of calculation.

1. Conjugacy of Transpositions.

CONJECTURE. There is only one conjugacy class of transpositions of 2^∞ UHF algebras.

Unfortunately, this conjecture is still open. At this time, it is still necessary to have some information about the structure of a transposition in order to determine its conjugacy class. The following property will help us meet this need.

DEFINITION. An antiautomorphism α of a C^*-algebra A is said to have the Rokhlin property of there is a projection E in A_∞ with $\alpha(E) = I - E$.

The following theorem can be proved with spectral theory and polar decomposition techniques. (See [1] for the proof).

THEOREM 1.1. *Let A be a unital C^*-algebra with central sequence algebra A_∞.*

(a) *If E is a projection in A, then there is a sequence $\{e_n\}_{n=1}^{\infty}$ of projections in A which represents E.*

(b) *If A is a unital C^*-algebra and $\{E_{i,j}: 1 \leqq i,j \leqq m\}$ is an $m \times m$ system of matrix units in A_∞, then there are sequences $\{e_{i,j}^{(n)}\}_{n=1}^{\infty}$ representing $E_{i,j}$ for each i, j such that for each n, $\{e_{i,j}^{(n)}: 1 \leqq i,j \leqq m\}$ is an $m \times m$ system of matrix units in A.*

Let A be a 2^∞ UHF algebra. Denote by τ_0 the transposition of A generated by the transpose map. The following proposition is necessary for there to be any hope of using the Rohklin property to solve the conjecture.

PROPOSITION 1.2. (i) *If α is a transposition with the Rokhlin property and β is conjugate to α, then β has the Rokhlin property.*

(ii) *τ_0 has the Rokhlin property on* UHF *algebras with* $a(2) = \infty$.

PROOF. (i) If $\beta = \theta \circ \alpha \circ \theta^{-1}$, then $\theta(E)$ is a projection in A_∞ and $\beta(\theta(E)) = I - \theta(E)$.

(ii) Since $a(2) = \infty$, A can be written as $A = \bigotimes_{n=1}^{\infty} M(2k(n), \mathrm{C})$. Define $f_{k(n)}$ in $M(2k(n), \mathrm{C})$ as

$$f_{k(n)} = \begin{bmatrix} \frac{1}{2} I_{k(n)} & \frac{i}{2} I_{k(n)} \\ -\frac{i}{2} I_{k(n)} & \frac{1}{2} I_{k(n)} \end{bmatrix},$$

where $I_{k(n)}$ is the identity matrix in $M((k(n), \mathrm{C})$. Let $e_n = I_{k(1)} \otimes \ldots \otimes I_{k(n-1)} \otimes f_{k(n)}$. Easy calculations show that $\{e_n\}_{n=1}^{\infty}$ is the desired element of A_∞.

The next series of results give the structure of a transposition on a UHF algebra which has the Rohklin property.

LEMMA 1.3. *If A is a unital C^*-algebra and α is a transposition of A with $\alpha(E) = I - E$ for some projection E in A_∞, then there is a sequence $\{e_n\}_{n=1}^{\infty}$ in A which represents E with $\alpha(e_n) = I - e_n$ for all n.*

PROOF. By Theorem 1.1 (a), there is a sequence of projections $\{f_n\}_{n=1}^{\infty}$ representing E. Since $\alpha(E) = I - E$, $\lim_{n\to\infty} \|\alpha(f_n) - (I - f_n)\| = 0$. Setting $\varepsilon_n = \|\alpha(f_n) - (I - f_n)\|$, there is an N such that $n \geqq N$ implies that $\varepsilon_n < 1$. For $n < N$, let $e_n = e_N$, where e_N will be determined later. Assume $n \geqq N$. Set $u_n = 2f_n - I$. Then u_n is a self-adjoint unitary element of A and

$$\|\alpha(u_n) + u_n\| = 2\,\|\alpha(f_n) - (I - f_n)\| = 2\varepsilon_n.$$

Define $v_n = (u_n - \alpha(u_n))/2$. Note that $v_n = v_n^*$, $\alpha(v_n) = -v_n$ and

$$\|u_n - v_n\| = \tfrac{1}{2}\|\alpha(u_n) + u_n\| = \varepsilon_n.$$

Since u_n is unitary and $\|u_n - v_n\| < 1$, v_n is invertible. Since v_n is invertible, its polar decomposition can be formed. Let $v_n = w_n|v_n|$ where w_n is unitary. As v_n is self-adjoint, $v_n = w_n|v_n| = v_n^* = w_n^*(w_n|v_n|\,w_n^*)$ and by the uniqueness of the polar decomposition, it follows that $w_n = w_n^*$. Also, $\alpha(v_n) = -v_n$, so $\alpha(w_n|v_n|) = \alpha(|v_n|)\alpha(w_n) = -v_n = -w_n|v_n|$ and $-w_n|v_n| = \alpha(w_n)(\alpha(w_n^*)\alpha(|v_n|)\alpha(w_n))$, so using the uniqueness of the polar decomposition again yields $\alpha(w_n) = -w_n$. Now, let $e_n = (w_n + I)/2$. Easy calculations show that e_n is a projection and $\alpha(e_n) = I - e_n$. To conclude the proof, it has to be shown that $\{e_n\}_{n=1}^\infty$ represents E. To that end,

$$\|e_n - f_n\| = \tfrac{1}{2}\|w_n - u_n\| \leqq \tfrac{1}{2}(\|w_n - v_n\| + \|v_n - u_n\|) \leqq 2\varepsilon_n.$$

Since ε_n approaches 0 as n approaches ∞, $\{e_n\}_{n=1}^\infty$ represents E.

THEOREM 1.4. *Suppose A is a* UHF *algebra and α is a transposition of A with the Rokhlin property. If there exists an $m \times m$ system of matrix units $\{F_{i,j}: 1 \leqq i,j \leqq m\}$ in A_∞ with $\alpha(F_{i,j}) = F_{j,i}$ for each i,j, then there is an $m \times m$ system of matrix units $\{E_{i,j}: 1 \leqq i,j \leqq m\}$ in A_∞ with representing sequences of matrix units $\{e_{i,j}^{(n)}\}_{n=1}^\infty$ such that $\alpha(e_{i,j}^{(n)}) = e_{j,i}^{(n)}$ for all n, $1 \leqq i,j \leqq m$. (Note that this implies that $\alpha(E_{i,j}) = E_{j,i}$, $1 \leqq i,j \leqq m$).*

PROOF. By Theorem 1.1(b), there are sequences of matrix units $\{f_{i,j}^{(n)}: 1 \leqq i,j \leqq m\}_{n=1}^\infty$ which represent each $F_{i,j}$. Since $\{\alpha(f_{j,i}^{(n)}): 1 \leqq i,j \leqq m\}$ is an $m \times m$ system of matrix units for any n, given any ε, Lemma 1.8 in Glimm [4] yields a $\delta(\varepsilon)$ and a partial isometry w_n in A such that $w_n^* w_n = f_{1,1}^{(n)}$, $w_n w_n^* = \alpha(f_{1,1}^{(n)})$ and if $\|f_{1,1}^{(n)} - \alpha(f_{1,1}^{(n)})\| < \delta(\varepsilon)$, then $\|w_n - f_{1,1}^{(n)}\| < \varepsilon$. Here, $\delta(\varepsilon)$ does not depend on the matrix units, but only on ε. Let $n(k)$ be such that $\|\alpha(f_{i,j}^{(n(k))}) - f_{j,i}^{(n(k))}\| < \min(\delta(1/48mk), 1/48mk)$. Now, by Glimm's lemma, $\|w_{n(k)} - f_{1,1}^{(n(k))}\| < 1/48mk$. Let $v_k = \sum_{j=1}^m \alpha(f_{1,j}^{(n(k))})\, w_{n(k)} f_{1,j}^{(n(k))}$. Straightforward calculations show that $v_k f_{i,j}^{(n(k))} v_k^* = \alpha(f_{j,i}^{(n(k))})$, v_k is unitary and that $\|v_k - I\| < 1/24k$. The sequences $\{f_{i,j}^{(n(k))}\}_{k=1}^\infty$ are central and represent a (possibly) new $m \times m$ system of matrix units $\{E_{i,j}: 1 \leqq i, j \leqq m\}$ in A_∞.

The goal is to construct new sequences of matrix units $\{e_{i,j}^{(n)}\}_{n=1}^\infty$ representing $E_{i,j}$ such that $\alpha(e_{i,j}^{(n)}) = e_{j,i}^{(n)}$. This will be accomplished if, for each k, $m \times m$ matrix units $\{e_{i,j}^{(k)}: 1 \leqq i,j \leqq m\}$ are constructed with $\alpha(e_{i,j}^{(k)}) = e_{j,i}^{(k)}$ and $\|e_{i,j}^{(k)} - f_{i,j}^{(n(k))}\| < 1/k$. For clarity, fix k and omit it from subscripts and superscripts. So that an $m \times m$ system of matrix units $\{f_{i,j}: 1 \leqq i,j \leqq m\}$ and a unitary v are given with $\|f_{i,j} - \alpha(f_{j,i})\| < 1/48mk$, $vf_{i,j}v^* = \alpha(f_{j,i})$ and $\|v - I\| < 1/24k$. What is needed is a system of matrix units $\{e_{i,j}: 1 \leqq i, j \leqq m\}$ with $\alpha(e_{i,j}) = e_{j,i}$ and $\|f_{i,j} - e_{i,j}\| < 1/k$.

Let $z = (v + \alpha(v))/2$, then $\|z - I\| < 1/12k$ and $\alpha(z) = z$. Also,

$$zf_{j,i} = \tfrac{1}{2}(vf_{j,i} + \alpha(v)f_{j,i}) = \tfrac{1}{2}(\alpha(f_{i,j})v + \alpha(\alpha(f_{j,i})v))$$
$$= \tfrac{1}{2}(\alpha(f_{i,j})v + \alpha(vf_{i,j})) = \tfrac{1}{2}(\alpha(f_{i,j})v + \alpha(f_{i,j})\alpha(v)) = \alpha(f_{i,j})z.$$

Using the fact that $\alpha(z) = z$,

$$z^*zf_{j,i} = z^*\alpha(f_{i,j})z = \alpha(f_{i,j}z^*)z = \alpha((zf_{j,i})^*)z$$
$$= \alpha((\alpha(f_{i,j})z)^*)z = \alpha(z^*\alpha(f_{j,i}))z = f_{j,i}z^*z.$$

Thus, z^*z is in $A \cap B'$ where B is the algebra generated by $\{f_{i,j}\colon 1 \leqq i, j \leqq m\}$. Thus, $|z|$ is in $A \cap B'$. Since $\|z - I\| < 1/24k < 1$, z is invertible and has a polar decomposition $z = w|z|$ with w unitary. Now,

$$w|z| = z = \alpha(z) = \alpha(|z|)\alpha(w) = \alpha(w)(\alpha(w^*)\alpha(|z|)\alpha(w)),$$

and the uniqueness of the polar decomposition gives $\alpha(w) = w$. Since

$$\alpha(f_{i,j})w|z| = \alpha(f_{i,j})z = zf_{j,i} = w|z|f_{j,i} = wf_{j,i}|z|,$$

multiplication by $|z|^{-1}$ yields $\alpha(f_{i,j})w = wf_{j,i}$. Note that $\|I - |z|\| \leqq \|I - z^*z\| \leqq \|I - z\|(1 + \|z\|) \leqq 2\|I - z\|$. Thus, $\|w - I\| \leqq \|I - |z|\| + \|z - I\| < 1/4k$.

Thus, $\sqrt{w}$ can be formed in the C^*-algebra generated by w. Also, $\alpha(\sqrt{w}) = \sqrt{w}$ since $\alpha(w) = w$. Let $y = \sqrt{w}$. Then y is a unitary with $\alpha(y)y = w$. Finally, define $e_{i,j} = yf_{i,j}y^*$. Then,

$$\alpha(e_{i,j}) = \alpha(y^*)\alpha(f_{i,j})\alpha(y) = yy^*\alpha(y^*)\alpha(f_{i,j})\alpha(y)yy^*$$
$$= yw^*\alpha(f_{i,j})wy^* = yf_{j,i}y^* = e_{j,i}.$$

Since y is unitary and $\{f_{i,j}\colon 1 \leqq i, j \leqq m\}$ is an $m \times m$ system of matrix units, $\{e_{i,j}\colon 1 \leqq i, j \leqq m\}$ will be one as well.

All that remains to be checked is that $\|f_{i,j} - e_{i,j}\| < 1/k$. Indeed,

$$\|f_{i,j} - e_{i,j}\| = \|yf_{i,j}y^* - f_{i,j}\| = \|yf_{i,j} - f_{i,j}y\|$$
$$\leqq \|yf_{i,j} - y^2f_{i,j}\| + \|y^2f_{i,j} - f_{i,j}y^2\| + \|f_{i,j}y^2 - f_{i,j}y\|$$
$$\leqq 2\|I - y\| + \|wf_{i,j} - f_{i,j}w\|$$
$$< 2\|I - w\| + \|\alpha(f_{j,i})w - f_{i,j}w\| < \frac{1}{2k} + \frac{1}{48mk} < \frac{1}{k}.$$

LEMMA 1.5. *Suppose A is a UHF algebra and $\{E'_{i,j}\colon i,j = 1,2\}$ and $\{F'_{i,j}\colon i,j = 1,2\}$ are two 2×2 systems of matrix units in A_∞ with representing sequences $\{e^{(n)}_{i,j}\}_{n=1}^\infty$ and $\{f^{(n)}_{i,j}\}_{n=1}^\infty$, respectively. In this case, there exists 2×2 systems of matrix units $\{E_{i,j}\colon i,j = 1,2\}$ and $\{F_{i,j}\colon i,j = 1,2\}$ with representing sequences which are subsequences of $\{e^{(n)}_{i,j}\}_{n=1}^\infty$ and $\{f^{(n)}_{i,j}\}_{n=1}^\infty$ and a unitary V in A_∞ with $VE_{i,j}V^* = F_{i,j}$.*

PROOF. Let A be the norm closure of $\bigcup_{n=1}^{\infty} A_n$. For each n, let $\delta'(n) = \min(\delta(1/n, 1/n)$, where $\delta(\varepsilon, k)$ is given in Lemma 1.10 of Glimm [4]. Let $\delta(n) = \delta(\delta'(n), 2)$. By induction, central subsequences $\{e_{i,j}^{k(n)}\}_{n=1}^{\infty}$ and $\{f_{i,j}^{k(n)}\}_{n=1}^{\infty}$ of $\{e_{i,j}^{(n)}\}_{n=1}^{\infty}$ and $\{f_{i,j}^{(n)}\}_{n=1}^{\infty}$ can be picked such that the set $\{e_{i,j}^{k(n)}, f_{i,j}^{k(n)}: i,j = 1,2\}$ is within $\delta(n)$ of $A'_n \cap A$ since the sequences are central.

By Glimm's lemma, there are 2×2 systems of matrix units $\{e_{i,j}^{(n)\prime}: i,j = 1,2\}$ and $\{f_{i,j}^{(n)\prime}: i,j = 1,2\}_{n=1}^{\infty}$ in $A'_n \cap A$ with $\|e_{i,j}^{(n)} - e_{i,j}^{k(n)}\| < \delta'(n)$ and $\|f_{i,j}^{(n)\prime} - f_{i,j}^{k(n)}\| < \delta'(n)$. Since A_n is finite dimensional, $A'_n \cap A$ is a UHF algebra and can be written as the norm closure of $\bigcup_{n=1}^{\infty} M_n$. Thus, there is a $M_{m(n)}$ with $\{e_{i,j}^{(n)\prime}: i,j = 1,2\} \cup \{f_{i,j}^{(n)\prime}: i,j = 1,2\}$ within $\delta(1/n, 2)$ of $M_{m(n)}$. Applying Glimm's lemma again, there are 2×2 systems of matrix units $\{e_{i,j}^{(n)\prime\prime}: i,j = 1,2\}$ and $\{f_{i,j}^{(n)\prime\prime}: i,j = 1,2\}$ with $\|e_{i,j}^{(n)\prime\prime} - e_{i,j}^{(n)\prime}\| < 1/n$ and $\|f_{i,j}^{(n)\prime\prime} - f_{i,j}^{(n)\prime}\| < 1/n$. Since $\{e_{i,j}^{(n)\prime\prime}: i,j = 1,2\}$ and $\{f_{i,j}^{(n)\prime\prime}: i,j = 1,2\}$ are in $M_{m(n)}$, matrix algebra techniques yield a unitary v_n in $M_{m(n)}$ with $v_n e_{i,j}^{(n)\prime\prime} v_n^* = f_{i,j}^{(n)\prime\prime}$. Let V be represented by $\{v_n\}_{n=1}^{\infty}$ in A^{∞}. Since v_n is in $A'_n \cap A$ for each n, V is in A_{∞}. Also, if $\{E_{i,j}: i,j = 1,2\}$ and $\{F_{i,j}: i,j = 1,2\}$ are the elements of A_{∞} represented by $\{e_{i,j}^{k(n)}\}_{n=1}^{\infty}$ and $\{f_{i,j}^{k(n)}\}_{n=1}^{\infty}$, respectively, then $VE_{i,j}V^* = F_{i,j}$. Indeed,

$$\|v_n e_{i,j}^{k(n)} v_n^* - f_{i,j}^{k(n)}\| \leqq \|v_n e_{i,j}^{k(n)} v_n^* - v_n e_{i,j}^{(n)\prime\prime} v_n^*\| + \|f_{i,j}^{(n)\prime\prime} = f_{i,j}^{k(n)}\|$$
$$< 2\left(\frac{1}{n} + \delta'(n)\right) \leqq \frac{4}{n}.$$

This completes the proof.

PROPOSITION 1.6. *If α is a transposition with the Rokhlin property on a UHF algebra with $a(2) = \infty$, then there exists a 2×2 system of matrix units $\{E_{i,j}: i,j = 1,2\}$ in A_{∞} such that $\alpha(E_{i,j}) = E_{j,i}$.*

PROOF. Since A has $a(2) = \infty$, there is a 2×2 system of matrix units $\{F'_{j,i}: i,j = 1, 2\}$ in A_{∞}. Let F be in A_{∞} with $\alpha(F) = I - F$. Since $\{\alpha(F'_{j,i}): i,j = 1,2\}$ forms another 2×2 system of matrix units in A_{∞}, Lemma 1.5 can be used to find systems of matrix units $\{E_{i,j}: i,j = 1,2\}$ and $\{F_{i,j}: i,j = 1,2\}$ and a unitary V in A_{∞} such that $VE_{i,j}V^* = F_{i,j}$. Since the representing sequences for $E_{i,j}$ and $F_{i,j}$ are subsequences of those for $\alpha(F'_{j,i})$ and $F'_{i,j}$, $E_{i,j}$ will be equal to $\alpha(F_{j,i})$. Therefore, it can be assumed that $V\alpha(F_{j,i})V^* = F_{i,j}$ for $i,j = 1, 2$. Let $\{f_n\}_{n=1}^{\infty}$ and $\{v_n\}_{n=1}^{\infty}$ be sequences representing F, $F_{i,j}$ and V, respectively. A subsequence $\{f_{n(k)}\}_{k=1}^{\infty}$ of $\{f_n\}_{n=1}^{\infty}$ can be chosen so that $\|f_{n(k)}f_{i,j}^{(k)} - f_{i,j}^{(k)}f_{n(k)}\| < 1/k$ and $\|f_{n(k)}v_k - v_k f_{n(k)}\| < 1/k$. Replacing $\{f_n\}_{n=1}^{\infty}$ with $\{f_{n(k)}\}_{k=1}^{\infty}$, it may be assumed that $[F_{i,j}, F] = 0 = [V, F]$, as well as $\alpha(F) = I - F$.

Now, let $W = VF + (I - F)\alpha(V)$. Simple calculations show that $[W, F] = 0$,

W is unitary and $\alpha(W) = W$. Observe, also, that

$$\begin{aligned} F_{i,j}W &= F_{i,j}(VF + (I - F)\alpha(V)) \\ &= FVV^*F_{i,j}V + (I - F)\alpha(V)\alpha(V\alpha(F_{i,j})V^*) \\ &= FV\alpha(F_{j,i}) + (I - F)\alpha(F_{j,i}) = W\alpha(F_{j,i}), \end{aligned}$$

so $W\alpha(F_{j,i})W^* = F_{i,j}$. Let $U = WF + I - F$. Again, easy calculations show that $U\alpha(U) = W$ and U is unitary. Define $E_{i,j} = U^*F_{i,j}U$. Then $\{E_{i,j}\colon i,j = 1, 2\}$ is a 2×2 system of matrix units in A_∞ with

$$\begin{aligned} \alpha(E_{i,j}) &= \alpha(U^*F_{i,j}U) = \alpha(U)\alpha(F_{i,j})\alpha(U^*) \\ &= U^*W\alpha(F_{i,j})W^*U = U^*F_{j,i}U = E_{j,i}. \end{aligned}$$

COROLLARY 1.7. *If A is a* UHF *algebra with $a(2) = \infty$ and α is a transposition of A which satisfies the Rokhlin property, then there is a 2×2 system of matrix units in A_∞, $\{E_{i,j}\colon i,j = 1,2\}$, with representing sequences $\{e_{i,j}^{(n)}\colon i,j = 1,2\}_{n=1}^{\infty}$ of 2×2 systems of matrix units such that $\alpha(E_{i,j}) = E_{j,i}$ and $\alpha(e_{i,j}^{(n)}) = e_{j,i}^{(n)}$ for all n and i, $j = 1,2$.*

PROOF. This follows from Proposition 1.6 and Theorem 1.4.

The following theorem gives a description of the behavior of α on a 2^∞ UHF algebra. It can be generalized to other types of UHF algebras since much of the preliminary work is done for arbitrarily sized matrix units. However, we will use the 2^∞ case and will state the theorem in that form. The proof is nearly identical to one used in Herman and Jones [6], where they deal with automorphisms.

THEOREM 1.8. *If A is a* UHF *algebra with $a(2) = \infty$ and α is a transposition of A which satisfies the Rokhlin property, then there is a 2^∞* UHF *subalgebra F of A such that A is isomorphic to $F \otimes (F' \cap A)$ and α is conjugate to $\tau_0 \otimes \alpha|_{F' \cap A}$ with respect to this decomposition.*

PROOF. Let $\{a_i\}_{i=1}^{\infty}$ be a dense sequence in A. A sequence of mutually commuting 2×2 matrix units $\{e_{i,j}^{(k)}\colon i,j = 1, 2\}_{k=1}^{\infty}$ will be constructed by induction on k such that:

(i) $\|[e_{i,j}^{(k)}, a_s]\| \leqq \dfrac{1}{2^k}$ for $1 \leqq s \leqq k$ and $i,j = 1,2$

(ii) $\alpha(e_{i,j}^{(k)}) = e_{j,i}^{(k)}$ for $i,j = 1, 2$.

By Corollary 1.7, there is a 2×2 system of matrix units $\{E_{i,j}\colon i,j = 1, 2\}$ in A_∞ such that $\alpha(E_{i,j}) = E_{j,i}$ and here are representing sequences for the $E'_{i,j}s$, namely $\{f_{i,j}^{(n)}\}_{n=1}^{\infty}$, which form systems of matrix units and which satisfy $\alpha(f_{i,j}^{(n)}) = f_{j,i}^{(n)}$. Since the sequences are central, an integer $N(1)$ can be chosen such that

$\|[f_{i,j}^{(N(1))}, a_1]\| < 1/2$ for $i,j = 1, 2$. To start the induction, set $e_{i,j}^{(1)} = f_{i,j}^{(N(1))}$. This clearly satisfies the required conditions.

Now assume that $\{e_{i,j}^{(m)}: i,j = 1, 2\}_{m=1}^{k-1}$ has been found satisfying the conditions above. Let B be the C^*-algebra generated by $\{e_{i,j}^{(m)}: i, j = 1, 2\}_{m=1}^{k-1}$. Since B is a finite dimensional subalgebra of A and invariant under α, α restricted to $A \cap B'$ will have the Rokhlin property. Since A had $a(2) = \infty$, so will $A \cap B'$. Therefore, Corollary 1.7 yields central sequences of matrix units $\{f_{i,j}^{(n)\prime}: i,j = 1, 2\}_{n=1}^{\infty}$ in $A \cap B'$ with $\alpha(f_{j,i}^{(n)\prime}) = f_{j,i}^{(n)}$.

Choose matrix units $\{f_{i,j}: 1 \leqq i, j \leqq m\}$ for B and use Lemma 1–6 in Powers [10] to write each a_s for $1 \leqq s \leqq k$ as

$$a_s = \sum_{i,j=1}^{m} a_{i,j}^{(s)} f_{i,j}$$

where $a_{i,j}^{(s)}$ is in $A \cap B'$. Since each sequence $\{f_{i,j}^{(n)\prime}: i,j = 1, 2\}_{n=1}^{\infty}$ is central in $A \cap B'$, there is an integer $N(k)$ with $\|[f_{i,j}^{(N(k))\prime}, a_{t,u}^{(s)}]\| < 1/2^k m^2$ for all $1 \leqq s \leqq k$, $1 \leqq t, u \leqq m$. Let $e_{i,j}^{(k)} = f_{i,j}^{(N(k))\prime}$. These elements commute with B and hence with $e_{i,j}^{(n)}$, $1 \leqq n \leqq k - 1$. Also, $\alpha(e_{i,j}^{(k)}) = e_{j,i}^{(k)}$ for $i, j = 1, 2$, thus condition (ii) is satisfied. To check condition (i), let $1 \leqq s \leqq k$ and $i, j = 1$ or 2, then since $f_{t,u}$ is in B

$$\begin{aligned}
\|[e_{i,j}^{(k)}, a_s]\| &= \left\|e_{i,j}^{(k)}\left(\sum_{t,u=1}^{m} a_{t,u}^{(s)} f_{t,u}\right) - \left(\sum_{t,u=1}^{m} a_{t,u}^{(s)} f_{t,u}\right) e_{i,j}^{(k)}\right\| \\
&= \left\| \sum_{t,u=1}^{m} f_{t,u}(e_{i,j}^{(k)} a_{t,u}^{(s)} - a_{t,u}^{(s)} e_{i,j}^{(k)})\right\| \\
&\leqq \sum_{t,u=1}^{m} \|[e_{i,j}^{(k)}, a_{t,u}^{(s)}]\| < m^2\left(\frac{1}{2^k m^2}\right) = \frac{1}{2^k}
\end{aligned}$$

by the choice of $e_{i,j}^{(k)}$. Thus condition (i) is satisfied.

The conditions show that the hypotheses of Theorem A.1 in Herman and Jones [6] can be satisfied. Consequently, A is isomorphic to $F \otimes (F' \cap A)$ where F is the UHF algebra generated by the sequence $\{M_k\}_{k=1}^{\infty}$. Since each M_k is 2×2, F has generalized integer 2^∞. Condition (ii) now says that α acts as the transpose on each M_k, thus α is τ_0 on F.

We want to use Theorem 1.8 to show that all transpositions which are conjugate via a unitary to τ_0 are conjugate as transpositions. We first fix some terminology.

DEFINITIONS. If A is a C^*-algebra and α and β are antiautomorphisms of A, then

(i) α is inner conjugate to β if there is a unitary u in A with $\alpha \circ \beta = \operatorname{Ad} u$,

(ii) α is positive (resp. negative) inner conjugate to β if there is a unitary u in A with $\alpha \circ \beta = \operatorname{Ad} u$ and $\alpha(u) = \beta(u) = u$ (resp. $-u$).

If A is any C^*-algebra with trivial center and α and β are inner conjugate transpositions, then $\mathrm{id} = \mathrm{Ad}\, u\beta(u^*)$. Thus, there is a complex number λ with $u\beta(u^*) = \lambda I$. Now, $u = \lambda\beta(u) = \lambda\beta(\lambda\beta(u)) = \lambda^2 u$, so $\lambda^2 = 1$ and $\beta(u) = \pm u$. Also, $\alpha(u) = \mathrm{Ad}\, u \circ \beta(u) = \pm u$, with the same polarity as $\beta(u)$. Thus, $\alpha(u) = \beta(u) = \pm u$. Since UHF algebras have trivial centers, any inner conjugate transpositions on UHF algebras are either positive or negative inner conjugate. Finally, if u has a square root v in the C^*-algebra generated by u and $\alpha(u) = u$, then $\alpha(v) = v$ and α is conjugate to β via $\mathrm{Ad}\, v$.

The goal of the following is to show that any transposition of a 2^∞ UHF algebra which is inner conjugate to τ_0 is, in fact, conjugate to τ_0. In light of Proposition 1.2, it is necessary that such transpositions have the Rokhlin property. Lemma 1.9 gives the assurance that the goal will not be denied on this account.

LEMMA 1.9. *If α is an antiautomorphism of a C^*-algebra A and α has the Rokhlin property, then so does any antiautomorphism inner conjugate to α.*

PROOF. Since α has the Rokhlin property, there is a projection E in A_∞ such that $\alpha(E) = I - E$. Observe that $I - E = \alpha^{-1}(\alpha(I - E)) = \alpha^{-1}(E)$. Now, suppose β is inner conjugate to α, so $\beta = \mathrm{Ad}\, v \circ \alpha^{-1}$ for some unitary v in A. Letting V be the element in A^∞ generated by v, $\beta = \mathrm{Ad}\, V \circ \alpha^{-1}$ in A^∞. Since E is in A_∞ and V is a constant sequence, $EV = VE$. Now, $\beta(E) = \mathrm{Ad}\, V \circ \alpha^{-1}(E) = V\alpha^{-1}(E)V^* = I - E$. Thus, β has the Rokhlin property.

We saw that a sufficient condition for the conjugacy of two positive inner conjugate transpositions is the existence of the square root of the unitary which implements the inner conjugacy. Of course, this is not always assured in a C^*-algebra. Theorem 1.10 shows that this difficulty can be eliminated when the transpositions have the Rokhlin property.

THEOREM 1.10. *Let A be a unital C^*-algebra and α and β transpositions of A with α possessing the Rokhlin property. If α is positive inner conjugate to β, then α is conjugate to β.*

PROOF. Since α has the Rokhlin property, Lemma 1.9 guarantees that β does also. Let u be a unitary with $\alpha \circ \beta = \mathrm{Ad}\, u$ and $\alpha(u) = u = \beta(u)$. Applying Lemma 1.3, there is a projection e in A with $\beta(e) = I - e$ and $\|[u, e]\| < 1/8$. Let $v = ue + (I - e)$. Then

$$\|v^*v - I\| = \|eu^*(I - e) + (I - e)ue\| \leqq 2\,\|[u, e]\| < \tfrac{1}{4}$$

and $\|vv^* - I\| < 1/8$. So, both vv^* and v^*v are invertible and v is invertible. Thus, v has a polar decomposition, namely, $v = t|v|$ with t unitary. Also note that

$$\|v\beta(v) - u\| = \|ue + (I - e)u - u\| = \|[u, e]\| < \tfrac{1}{8}.$$

Now, $sp(v^*v) \subseteq B(1/4, 1)$, so $sp(|v|) \subseteq B(1/4, 1)$ and $\|I - |v|\|_{sp} < 1/4$. Also,

$$\|t - v\| = \|t - t|v|\| = \|I - |v|\| = \|I - |v|\|_{sp} < \tfrac{1}{4}.$$

Additionally,

$$\|\beta(v)\| = \|v\| = \|v^*v\|^{1/2} = (\|v^*v\|_{sp})^{1/2} < (\tfrac{5}{4})^{1/2}.$$

Thus,

$$\begin{aligned}\|I - u\beta(t^*)t^*\| &= \|t\beta(t) - u\| \\ &\leqq \|t\beta(t) - t\beta(v)\| + \|t\beta(v) - v\beta(v)\| + \|v\beta(v) - u\| \\ &< \tfrac{1}{4} + \tfrac{1}{4}(\tfrac{5}{4})^{1/2} + \tfrac{1}{8} < 1.\end{aligned}$$

Let $\gamma = \mathrm{Ad}\, t\beta(t) \circ \beta$. Then $\alpha \circ \gamma = \mathrm{Ad}\, u \circ \beta \circ \gamma = \mathrm{Ad}\, u\beta(t^*)t^*$. Thus, α is inner conjugate to γ and easy calculations show that, in fact, α is positive inner conjugate to γ and γ is a transposition. Since $\|I - u\beta(t^*)t^*\| < 1$, $u\beta(t^*)t^*$ has a square root in A and γ is conjugate to α. But $\gamma = \mathrm{Ad}\, t\beta(t) \circ \beta = \mathrm{Ad}\, t \circ \beta \circ \mathrm{Ad}\, t^*$, so γ is conjugate to β and α is conjugate to β.

Theorem 1.10 gives the desired relationship between τ_0 and those transpositions which are positive inner conjugate to it. The first three corollaries to Theorem 1.10 gradually expand the conditions on the C^*-algebra A in order to deal with the case of negative inner conjugacy. This results in Corollary 1.13 which deals with this case in the desired way for 2^∞ UHF algebras. Independently, Størmer [13] proved a statement like Corollary 1.11 with a similar proof.

COROLLARY 1.11. *Under the conditions of Theorem* 10, *if α is negative inner conjugate to β, then $\alpha \otimes \tau_2$ is conjugate to $\beta \otimes \sigma_2$ on $A \otimes M(2,\mathrm{C})$.*

PROOF. Let $\alpha \circ \beta = \mathrm{Ad}\, u$ and $\alpha(u) = -u$. Define $\gamma = (\alpha \otimes \tau_2) \circ (\beta \otimes \sigma_2)$ on $A \otimes M(2, \mathrm{C})$. Observe that if $a \in A$ and $m \in M(2, \mathrm{C})$, then

$$\gamma(a \otimes m) = \alpha \circ \beta(a) \otimes \tau_2 \circ \sigma_2(m) = \mathrm{Ad}\, u(a) \otimes \mathrm{Ad}\, V_2(m) = \mathrm{Ad}\,(u \otimes V_2)(a \otimes m).$$

Since $A \otimes M(2, \mathrm{C})$ is the linear span of the elementary tensors, $\gamma = \mathrm{Ad}\,(u \otimes V_2)$ on $A \otimes M(2, \mathrm{C})$. Also,

$$\alpha \otimes \tau_2(u \otimes V_2) = \beta \otimes \sigma_2(u \otimes V_2) = (-u) \otimes (-V_2) = u \otimes V_2,$$

so $\alpha \otimes \tau_2$ is positive inner conjugate to $\beta \otimes \sigma_2$ on $A \otimes M(2, \mathrm{C})$. Also, if $\alpha(E) = I - E$ in A_∞, $\alpha \otimes \tau_2(E \otimes I) = (I - E) \otimes I = I \otimes I - E \otimes I$. Since $E \otimes I$ is clearly in $(A \otimes M(2, \mathrm{C}))_\infty$, $\alpha \otimes \tau_2$ has the Rokhlin property. Theorem 1.10 implies that $\alpha \otimes \tau_2$ is conjugate to $\beta \otimes \sigma_2$.

COROLLARY 1.12. *If A is a 2^∞* UHF *algebra and α is negative inner conjugate to τ_0, then $\alpha \otimes \tau_2$ is conjugate to τ_0.*

PROOF. Corollary 1.11 says that $\alpha \otimes \tau_2 \sim \tau_0 \otimes \sigma_2$, but basic facts about UHF algebras and Stacey's result show that $\tau_0 \otimes \sigma_2 \sim \tau_0$.

COROLLARY 1.13. *If A is a 2^∞ UHF algebra and α is a transposition inner conjugate to τ_0, then α is conjugate to τ_0.*

PROOF. We know that α is either positive or negative inner conjugate to τ_0. In the first case, Theorem 1.10 applies and α is conjugate to τ_0. In the second case, Corollary 1.12 says that $\alpha \otimes \tau_2$ is conjugate to τ_0. Now, Theorem 1.8 gives that $\alpha \sim \alpha|_{F' \cap A} \otimes \tau_0$ with respect to some tensor product decomposition of A as $(A \cap F') \otimes F$. Thus, $\alpha \otimes \tau_2 \sim \alpha|_{F' \cap A} \otimes \tau_0 \otimes \tau_2$. But, since $F \otimes M(2, \mathbf{C})$ is isomorphic to F, $\tau_0 \otimes \tau_2$ on $F \otimes M(2, \mathbf{C})$ is conjugate to τ_0 on F. Consequently, $\tau_0 \sim \alpha \otimes \tau_2 \sim \alpha|_{F' \cap A} \otimes \tau_0 \otimes \tau_2 \sim \alpha|_{F' \cap A} \otimes \tau_0 \sim \alpha$.

Despite Corollary 1.13, it is still impossible to tell whether a general transposition is conjugate to τ_0. In [13], Størmer shows that two transpositions of a von Neumann algebra which are "close" to one another are conjugate. The following corollary proves a similar result in the present setting. The proof uses a result of E. C. Lance [12] which states that if θ is an automorphism of a UHF algebra and if $\|\theta - \mathrm{id}\| < 2$, then θ must be inner.

COROLLARY 1.14. *If A is a 2^∞ UHF algebra and α is a transposition with $\|\tau_0 - \alpha\| < 2$, then α is conjugate to τ_0.*

PROOF. Since $\|\tau_0 - \alpha\| < 2$, $\|\tau_0 \circ \alpha - \mathrm{id}\| < 2$ and Lance's result shows that τ_0 and α are inner conjugate. An application of Corollary 1.13 completes the proof.

2. Antiautomorphisms Not of Period Two.

In most of this paper, the antiautomorphisms under consideration are of period 2. In this section, some information is derived about slightly more general antiautomorphisms. In particular, antiautomorphisms whose square is an inner automorphism are examined. Again the Rokhlin property plays a role in the study of the structure of such maps. The following lemma will aid in this study; it appears implicitly in the proof of Remarque 1.7 in [3].

LEMMA 2.1. *If A is a C^*-algebra with trivial center and α is an antiautomorphism of A with $\alpha^2 = \mathrm{Ad}\, w$ for some unitary w in A, then there is a unitary u in A with $\alpha^2 = \mathrm{Ad}\, u$ and $\alpha(u) = u^*$.*

PROOF. Since $\mathrm{Ad}\, w \circ \alpha = \alpha^3 = \alpha \circ \mathrm{Ad}\, w = \mathrm{Ad}\, \alpha(w^*) \circ \alpha$, $\mathrm{Ad}\, \alpha(w) w = \mathrm{id}$. Hence, $\alpha(w)w$ is a central element, but the center is trivial, so $\alpha(w)w = \lambda I$ for some complex number λ. Since $\alpha(w)w$ is unitary, $|\lambda| = 1$. Let $u = \lambda^{-1/2}w$. Then,

$$\alpha^2 = \operatorname{Ad} w = \operatorname{Ad} \lambda^{1/2} u = |\lambda| \operatorname{Ad} u = \operatorname{Ad} u, \text{ and}$$

$$\alpha(u) = \alpha(\lambda^{-1/2} w) = \lambda^{-1/2} \alpha(w) = \lambda^{-1/2}(\lambda w^*) = \lambda^{1/2} w^* = u^*.$$

The following proposition states that antiautomorphisms with the Rokhlin property on a C^*-algebra with trivial center which have inner squares can be perturbed in such a way that they become transpositions. The second conclusion will not be needed later, but says that the size of the perturbation can be controlled.

PROPOSITION 2.2. *If α is an antiautomorphism with the Rokhlin property on a C^*-algebra A with trivial center and $\alpha^2 = \operatorname{Ad} u$, then there is a unitary u' in A with $(\operatorname{Ad} u' \circ \alpha)^2 = \mathrm{id}$. Furthermore, if $\|u - I\| < \varepsilon$, then u' can be chosen so that $\|u' - I\| < 2\varepsilon$.*

PROOF. By Lemma 2.1, it can be assumed that $\alpha(u) = u^*$. Since α has the Rokhlin property, Lemma 1.3 applies and a projection e can be chosen in A such that $\|[u, e]\| < \delta$ (where δ will be specified later) and $\alpha(e) = I - e$. Let $v = u^*e + I - e$. Then $\|v^*v - I\| \leqq 2\,\|[u, e]\| < 2\delta$. If $\delta < 1/2$, then v^*v is invertible. Similarly, vv^* will be invertible and v is invertible. Let $v = r|v|$ be the polar decomposition of v. Then r is unitary and $\|v - r\| = \||v| - I\| \leqq \|v^*v - I\| < 2\delta$. Let $\beta = \operatorname{Ad} r \circ \alpha$. Then $\beta^2 = \operatorname{Ad} w$ where $w = r\alpha(r^*)u$. Now,

$$\begin{aligned} \|w - I\| &= \|r\alpha(r^*) - u^*\| \\ &\leqq \|r\alpha(r^*) - r\alpha(v^*)\| + \|r\alpha(v^*) - v\alpha(v^*)\| + \|v\alpha(v^*) - u^*\| \\ &\leqq \|r - v\| + \|r - v\|\,\|v\| + \|v\alpha(v^*) - u^*\|. \end{aligned}$$

But,

$$\|v\| = \|u^*e + (I - e)\| \leqq \|u^*e\| \leqq \|I - e\| = 2$$

and

$$\|v\alpha(v^*) - u^*\| \leqq 2\,\|u^*, I - e]\| = 2\,\|[\alpha(u), \alpha(e)]\| \leqq 2\,\|[u, e]\| < 2\delta.$$

So, $\|w - I\| < 2\delta + 4\delta + 2\delta = 8\delta$.

Also, easy calculations show that $\beta(w) = w^*$. So, $\beta^2 = \operatorname{Ad} w$ with $\beta(w) = w^*$ and $\|w - I\| < 8\delta$. Let w be identified with the identity function on $\operatorname{sp}(w)$ and let $(\beta f)(x) = f(\bar{x})$ for each f in the C-algebra $C(\operatorname{sp}(w))$ and x in $\operatorname{sp}(w)$. If $\delta < 1/8$, -1 is not in $\operatorname{sp}(w)$ and so y can be defined by $y(e^{i\theta}) = e^{-i\theta/2}$. Now, $\beta(y) = y^*$ and $y^2 = w^*$. Then $(\operatorname{Ad} y \circ \beta)^2 = \operatorname{Ad} y\beta(y^*)w = \mathrm{id}$. Letting $u' = yr$ and $\delta < 1/8$ yields the first assertion.

For the second assertion, note that

$$\|u' - I\| \leqq \|y - I\| + \|r - I\|$$
$$\leqq \|w - I\| + \|r - v\| + \|v - I\| < 10\delta + \|u - I\| < 10\delta + \varepsilon.$$

Thus, the statement is proved if $\delta < \varepsilon/10$.

This section concludes with a theorem which ties together earlier results with the situation under discussion.

THEOREM 2.3. *Let* α *be an antiautomorphism with* $\alpha^2 = \mathrm{Ad}\, u$ *on a* $*$*-algebra* A.

(i) *If* A *is matricial with* $a(2) = \infty$ *or* 0, *then there is a unitary* v *in* A *with* $\mathrm{Ad}\, v \circ \alpha$ *conjugate to* τ_0.

(ii) *If* A *is a* UHF *algebra with* $a(2) = \infty$ *and* $\alpha \circ \tau_0 = \mathrm{Ad}\, w$ *for some unitary* w *in* A, *then there is a unitary* v *in* A *with* $\mathrm{Ad}\, v \circ \alpha$ *conjugate to* τ_0.

PROOF. (i) Lemma 2.1 says that, without loss of generality, it can be assumed that $\alpha(u) = u^*$. Let v be a square root of u^* picked as y is in the proof of Proposition 2.2. Then $\alpha(v^*) = v$ and $u^* = v\alpha(v^*)$. Now, $(\mathrm{Ad}\, v \circ \alpha)^2 = \mathrm{Ad}\, v\alpha(v^*) \circ \alpha^2 = \mathrm{Ad}\, u^* \circ \mathrm{Ad}\, u = \mathrm{id}$. Stacey's result completes the proof.

(ii) Since α is inner conjugate to τ_0, Lemma 1.9 shows that α has the Rokhlin property. Thus, Proposition 2.2 produces a unitary v in A with $\mathrm{Ad}\, v \circ \alpha$ a transportation. Then $(\mathrm{Ad}\, v \circ \alpha) \circ \tau_0 = \mathrm{Ad}\, v \circ \mathrm{Ad}\, w = \mathrm{Ad}\, vw$, so the result follows from Corollary 1.13.

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