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SMASH PRODUCTS, GROUP ACTIONS AND GROUP GRADED RINGS

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Introduction.

For a graded ring A, graded by a finite group G, one can define the smash product $A \# G^*$ [5]. The purpose of this paper is to continue the study of smash products from [5] and use the results obtained to get new results on group graded rings and on fixed point rings for groups acting on rings as well as to get new and simpler proofs for known results concerning skew group rings and fixed points rings.

We begin by proving that an $A \# G^*$-module M is flat (projective or injective) if and only if M_A is. This means that if A has a certain "homological" property so has $A \# G^*$. In general properties from $A \# G^*$ are not inherited by A, but for "separably" graded rings A and $A \# G^*$ are alike.

Finally we show that a ring is perfect if and only if A_1, the rings of constants, is.

We start by fixing some notation. k denotes a commutative ring with an identity element, we say that the k-algebra A is graded by the group G if

$$(*) \qquad A = \sum_{g \in G} \oplus A_g$$

where the A_g's are k-modules and $A_g A_h \subset A_{gh}$ for all $g, h \in G$.

1. General results on smash products.

Let A be a k-algebra graded by a finite group G. The smash product, denoted by $A \# G^*$, is the free left A-module with the set $\{p_g\}_{g \in G}$, as a free base and multiplication given by

$$(1) \qquad (ap_g)(bp_h) = ab_{gh^{-1}}p_h,$$

where for an element x in A, x_g denotes the g'th component in the decomposition of x given by $(*)$.

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The basic properties of the smash product is stated in the following proposition from [5, Proposition 1.4] and [5, Corollary 1.5].

PROPOSITION 1.1. *Let A be a group graded ring graded by the finite group G. $A \# G^*$ is a free right module with base $\{p_g \mid g \in G\}$. The set $\{p_g\}_{g \in G}$, is a set of orthogonal idempotent with sum 1. Moreover the following multiplication rules hold:*

(i) $p_h a = \sum_g a_{hg^{-1}} p_g$ *for all* $a \in A$ *and all* $h \in G$.
(ii) $p_h a_g = a_g p_{g^{-1}h}$ *for all* $a_g \in A_g$.
(iii) p_h *centralizes* A_1 *for all* $h \in G$.
(iv) $p_h(I \# G^*)p_g = I_{hg^{-1}} p_g = p_h I p_g$ *for all* $g, h \in G$ *and for all graded ideals* I.
(v) $p_1(I \# G^*)p_1 = I_1 p_1$, *which is ring isomorphic to* I_1, *for all graded ideals* I.

Furthermore G acts on $A \# G^*$ by $(ap_h)^g = ap_{hg}$ for all $g, h \in G$. The fixed-point ring under this action is $A \# 1$. This simple observation together with other results from [5] in fact give a very short proof of the following result of Fisher and Montgomery [7].

Let A be a semiprime ring and G a finite group of automorphisms of A. If A has no $|G|$-torsion, then $A * G$ is semiprime.

A short proof of this results is given in [9], that proof depends on the Bergman-Isaacs theorem [4] stating: Let A be a semiprime ring and G a finite group of automorphisms of A, such that A has no $|G|$-torsion. Then A^G, the fixed point ring, is semiprime.

We now show that the Fisher-Montgomery theorem is an immediate consequence of the results in [5] and the Bergman-Isaacs theorem.

By [5, Theorem 3.2] $(A * G) \# G^*$ is semiprime and G acts on this ring with fixed point ring $A * G$, which thus must be semiprime.

2. Homological results for smash products.

In [5, Theorem 2.3] a Maschke type theorem was proved, namely:

THEOREM 2.1. *Let V be a right $A \# G^*$-module and W an $A \# G^*$-submodule of V, which is an A-direct summand of V. Then W is an $A \# G^*$-direct summand.*

Similar to this is the following:

THEOREM 2.2. *Let V and W be two $A \# G^*$-modules and u an A-homomorphism from V to W. Then*

$$\tilde{u}(v) = \sum_{g \in G} u(vp_g)p_g$$

is an $A \# G^*$-homomorphism from V to W.

PROOF. It suffices to show that $\tilde{u}(vap_g) = \tilde{u}(v)ap_g$ for all $v \in V$, $a \in A$ and $g \in G$.

Using Proposition 1.1 we get

$$\begin{aligned}\tilde{u}(v)ap_g &= \sum_{h\in G} u(vp_h)p_h ap_g = \sum_{h\in G} u(vp_h)a_{hg^{-1}}p_g \\ &= \sum_{h\in G} u(vp_h a_{hg^{-1}})p_g = \sum_{h\in G} u(va_{hg^{-1}}p_g)p_g \\ &= u\left(v\cdot \sum_{h\in G} a_{hg^{-1}}\cdot p_g\right)p_g = u(vap_g)p_g = \tilde{u}(vap_g)\end{aligned}$$

It is easy to show

PROPOSITION 2.1. *Let M be a right $A \# G^*$-module and N a right A-module. Then there is a natural isomorphism between*

$$\mathrm{Hom}_A(M,N) \text{ and } \mathrm{Hom}_{A\#G^*}(M, N\otimes_A(A \# G^*)).$$

PROOF. For $f\in \mathrm{Hom}_A(M,N)$ define $\tilde{f}$ by $\tilde{f}(m) = \sum_{g\in G} f(m)p_g \otimes p_g$ and for $g\in \mathrm{Hom}_{A\#G^*}(M, N\otimes_A(A \# G^*))$ define $\hat{g}(m) = \sum_g n_g$, where $g(m) = \sum_{g\in G} n_g \otimes p_g$.

PROPOSITION 2.1′. *Let M be a right A-module and N a right $A \# G^*$-module. Then there is a natural isomorphism between*

$$\mathrm{Hom}_A(M,N) \text{ and } \mathrm{Hom}_{A\#G^*}(M\otimes_A(A \# G^*), N).$$

SKETCH OF PROOF. For f in $\mathrm{Hom}_A(M,N)$ define $\tilde{f}$ by $\tilde{f}(m\otimes p_h) = f(m)p_h$ and for g in $\mathrm{Hom}_{A\#G}(M\otimes_A(A \# G^*), N)$ define $\hat{g}(m) = m\otimes 1$.

COROLLARY 1. *Let E be an injective right A-module. Then the right $A \# G^*$-module $E\otimes_A(A \# G^*)$ is injective.*

The following are also easy consequences

THEOREM 2.3. *Let V be a right $A \# G^*$-module. Then*
(i) *V is projective if and only if V_A is.*
(ii) *V is injective if and only if V_A is.*

PROOF. We have an exact sequence of $A \# G^*$-modules

$$0\to K\to F\to V\to 0,$$

where F is a free $A \# G^*$-module. F is A-free by Proposition 1.1 and hence (i) follows from Maschke's Theorem.

Now let V_A be injective and let W be an $A \# G^*$-module containing V. V_A is a direct summand in W and by Maschke's Theorem V is injective.

Finally assume V is injective and M, N A-modules, $M\subset N$ and if $f\colon M\to V_A$ a homomorphism, then $\tilde{f}\colon M\otimes_A(A \# G^*)\to V$ defined by $\tilde{f}(m\otimes p_g) = mp_g$ is an $A \# G^*$-homomorphism (Theorem 2.2 and Proposition 2.1′), $\tilde{f}$ can be extended to an $A \# G^*$-homomorphism g: $N \otimes_A (A \# G^*) \to V$. Now $\hat{g}(y) = g(y \otimes 1)$ extends f.

In [13, 3.4 lemma 1] it is shown that under an extra assumption on the grading (separability) an $A \# G^*$-module is flat if and only if it is A-flat, here we show that the extra assumption is superfluous. Let us first recall a well-known result of Villamayor.

VILLAMAYORS LEMMA. *Let*

$$0 \to K \to F \xrightarrow{\varphi} M \to 0$$

be an exact sequence of A-modules such that F is a free A-module and K the kernel of φ. The following conditions are equivalent:

(1) *M is flat.*
(2) *For all k in K there exists a $u \in \mathrm{Hom}_A(F, K)$ such that $u(k) = k$.*
(3) *For all $k_1, \ldots, k_t$ in K there exists a $u \in \mathrm{Hom}_A(F, K)$ such that $u(k_i) = k_i$ for all $i = 1, \ldots, t$.*

It is now easy to get

THEOREM 2.4. *Let M be an $A \# G^*$-module. M is flat if and only if M_A is flat.*

PROOF. We have an exact sequence of $A \# G^*$-modules

$$0 \to \mathrm{K} \to \mathrm{F} \to \mathrm{M} \to 0,$$

where F is a free $A \# G$-module. The lemma shows that if M is flat so is M_A.

Conversely let M_A be flat, $k \in K$ and let u: $F \to K$ be an A-homomorphism such that $u(kp_g) = kp_g$ for all $g \in G$.

$\tilde{u}$ constructed in Theorem 2.2 is an $A \# G^*$-homomorphism from F to K and

$$\tilde{u}(k) = \sum_{g \in G} u(kp_g)p_g = \sum_{g \in G} kp_g = k,$$

and M is $A \# G^*$-flat by Villamayors lemma.

We can now list a number of corollaries

COROLLARY 1. *Let N be an $A \# G^*$-module. Then*

$$\begin{aligned} \mathrm{rhd}\, N &= \mathrm{rhd}_A N_A \\ \mathrm{injdim}\, N &= \mathrm{injdim}_A N_A \\ \mathrm{whd}\, N &= \mathrm{whd}_A N_A. \end{aligned}$$

where rhd, (injdim *and* whd), *denotes the projective* (*injective and flat*) *dimension of a module.*

COROLLARY 2. *Let A be a group graded ring. Then*

$$r\,\mathrm{gldim}\, A \# G^* \leqq r\,\mathrm{gldim}\, A$$
$$w\,\mathrm{gldim}\, A \# G^* \leqq w\,\mathrm{gldim}\, A$$
$$r\,\mathrm{FDP}\, A \# G^* \leqq r\,\mathrm{FPD}\, A.$$

In Corollary 2 equality does not hold in general. Let A be the groupring $k[G]$, where k is a feld, then $A \# G^*$ is Morita-equivalent to k [5, Theorem 2.12]. But in case char $k \mid |G|$, A is not semisimple.

Before we state and prove the last corollary let us recall that a ring A is said to be of finite representation type (FRT) if A has only a finite number of finitely generated indecomposable left A-modules, and A moreover is left artinian. This concept is left/right symmetric, and furthermore every left A-module is a direct sum of finitely generated modules if A *is* FRT.

COROLLARY 3. *Let the ring A be graded by the finite group G. If A has finite representation type, then $A \# G^*$ is also a ring of finite representation type.*

PROOF. The argument showing [8, Theorem 2] together with Theorem 2.1 can easily be applied here.

REMARK. If k is a field of characteristic p and G is a finite group, then $k[G]$ is of finite representation type precisely when the p'th Sylow subgroups are cyclic. $k[G] \# G^*$ is Morita-equivalent to k, this shows that in general the converse to Corollary 3 does not hold. We will return to this question later on.

Finally we note that the assumption (separably graded) in [13, 3.7 Proposition and 3.10 Corollary] is not necessary because by Corollary 1 and Theorem 2.3 we get

COROLLARY 4. *Let A be a group graded ring, A is a* QF *(selfinjective) if and only if $A \# G^*$ is.*

By [5, Theorem 3.2] we get

COROLLARY 5. (cf. [6, Theorem 24.29]) *Let A be a ring and G a finite group of automorphisms of A. Then A is* QF *if and only if $A * G$ is* QF. *A is right self-injective if and only if $A * G$ is self-injective.*

Since for a strongly graded ring A $A \# G^*$ is Morita equivalent to A_1 most of the results in [12, Chapter 2] can easily be derived.

3. Separability.

A general theme for the results in Section 2 was that properties from the group graded ring were inherited by the smash product. Also examples showed, that in the general the converse results were not always true. We will describe the extra assumption which is needed to obtain the converse results. The reader might also consult the papers [11] and [14] for the strong group graded case and [2] and [13] for the general situation.

Let A be a strongly group graded ring and suppose we have fixed decompositions of the identity element

$$1 = \sum_i u_g^{(i)} v_{g-1}^{(i)} \quad u_g^{(i)} \in A_g \quad \text{and} \quad v_{g-1}^{(i)} \in A_{g-1} \quad \text{for all} \quad g \in G.$$

The Miyashita automorphism of the center of A_1 is defined by

$$g(a) = \sum_i u_g^{(i)} a v_{g-1}^{(i)}.$$

g is characterized by the property $g(a)x = ax$ for all $x \in A_g$.

For a in the center of A_1 we can thus define $\operatorname{tr}(a) = \sum_g g(a)$.

If $|G|$ is a unit of A, then clearly A_1 has an element with tr equal to 1. Some consequences of the existence of an element with trace 1 are given in [11] and [13]. Let us also recall from [14], that A has an element a in the center of A_1 with $\operatorname{tr}(a) = 1$ if and only if the natural epimorphism from $A \otimes_{A_1} A$ to A is split as a homomorphism of bimodules.

In [2] the notion of a separable functor is introduced as a generalization of the above concept to not necessarily strongly group graded rings.

Following [13, 3.6 Theorem] we make the following:

DEFINITION 3.1. Let the ring A be graded by the finite group G. We say A is separably graded if there is a family of elements $\{x^g\}_{g \in G}$ in the center of A_1 such that

(i) $\sum_g x^g = 1$

(ii) $rx^g = x^{hg} r$ for all $r \in A_h$ and all $g \in G$.

We have the following interpretation in terms of the smash product.

THEOREM 3.1. *Let A be a graded ring. A is separably graded if and only if the A-bimodule epimorphism from $A \# G^*$ to A defined by $p_g \to 1$ is a split epimorphism as a bimodule homomorphism.*

PROOF. Let r denote the right A-module homomorphism from $A \# G^*$ to A defined by $\tau(ap_g) = a$ for all $g \in G$. τ is a bimodule homomorphism by Proposition 1.1 (i). τ is split as a bimodule homomorphism precisely when there is an element $\delta(1)$ in $A \# G^*$ such that $a\delta(1) = \delta(1)a$, $a \in A$ and $\tau\delta(1) = 1$.

Suppose first A is separably graded and $\{x^g\}_{g \in G}$ is a family of elements satisfying i) and ii). Then define $\delta(1) = \sum_g x^g p_g$. By i) $\tau\delta(1) = 1$ and for $a \in A_h$ we have by ii)

$$a\sum_g x^g p_g = \sum_g x^{hg} a p_g = \sum_g x^{hg} p_{hg} a = \delta(1)a.$$

Conversely assume that there is an element $\delta(1)$ in A such that $a\delta(1) = \delta(1)a$ and $\tau\delta(1) = 1$. We can write $\delta(1) = \sum_g y^g p_g$, $y^g \in A$. Now $\sum_g y^g = 1$ since $\tau\delta(1) = 1$. Moreover, for $a \in A_h$ we have

$$a\sum_g y^g p_g = \left(\sum_g y^g p_g\right)\cdot a,$$

consequently we get $\sum_g a y^g p_g = \sum_g y^g a p_{h^{-1}g}$ thus $ay^g = y^{hg}a$. If one projects each element of the family $\{y^g\}$ onto A_1 it follows that A is separably graded.

COROLLARY 1. *Let A be separably graded and let M be a right A-module. M is an A-direct summand in the right A-module $M \otimes_A (A \# G^*)$.*

PROOF. This is immediate since A is a bimodule direct summand in $A \# G^*$.

Corollary 1 and the results in section 2 imply

COROLLARY 2. *Let A be separably graded and M an A-module. Then*
(1) $\mathrm{rhd}_A M_A = \mathrm{rhd}_{A \# G^*} M \otimes_A (A \# G^*)$
(1′) $\mathrm{injdim}_A M_A = \mathrm{injdim}_{A \# G^*} M \otimes_A (A \# G^*)$
(2) $r\,\mathrm{gldim}(A \# G^*) = r\,\mathrm{gldim}\, A$
(3) $r\,\mathrm{FDP}(A \# G^*) = r\,\mathrm{FDP}(A)$

If moreover A is strongly graded $A \# G^*$ is Morita equivalent to A_1 [5, Theorem 2.12] and

$$r\,\mathrm{gldim}\, A_1 = r\,\mathrm{gldim}\, A \# G^* = r\,\mathrm{gldim}\, A.$$

This last equation is known in case $r\,\mathrm{gldim}\, A_1 = 1$ [11, 2.3 Proposition].

THEOREM 3.2. *Let A be a separably graded ring such that $A \# G^*$ is of finite representation type. Then A has finite representation type.*

PROOF. First notice that A is left and right artinian for instance by Proposition 1.1 (v). We know that for a finitely generated $A \# G^*$-module M, M is a finite direct sum of indecomposable $A \# G^*$-modules, i.e. we have a finite number of finitely generated indecomposable $A \# G^*$-modules M_α, $\alpha = 1,\dots,k$ such that every finitely generated $A \# G^*$-module is a direct sum of copies of the M_α's. Thus let M be an indecomposable finitely generated A-module; then $M \otimes_A (A \# G^*) \simeq \oplus M_\alpha^{(n_\alpha)}$ as A-modules. Each of the M_α's is a direct sum of finitely generated indecomposable A-modules, $M_\alpha = \oplus M_{\alpha\beta}$, $\beta = 1,\dots,m_\alpha$. By the Krull-Schmidt Theorem [1, Theorem 12.9] M is isomorphic to one of the finitely many $M_{\alpha\beta}$'s. Finally note that the indecomposable A-modules come from decompositions as A-modules of indecomposable $A \# G^*$-modules.

COROLLARY. ([8]) *Let the ring A be of finite representation type and let G be a finite group of automorphisms of A such that $|G|^{-1} \in A$. $A * G$ has finite representation type.*

PROOF. $(A * G) \# G^*$ has finite representation type by [5, Theorem 3.2].

REMARK. Let M be a simple right A-module, where A is graded by G. Then $M \otimes_A A \# G^*$ is $A \# G^*$-semisimple. To see this consider M^n as a $M_n(A)$-module in the natural way. Noting that $M_n(A) \simeq (A \# G^*) * G$ and applying Clifford's Theorem [10, I.3.33 Theorem] we arrive at M^n being semisimple as an $A \# G^*$-module. The module action of $A \# G^*$ on M^n is (cf. [2, Remarks after Lemma 3.1])

$$(m_g)_{g \in G} a p_h = (x_l)_{l \in G}$$

where $x_l = 0$ for $l \neq h$ and $x_h = \sum_g m_g a_{gh^{-1}}$. One easily verifies that $M^n \simeq M \otimes_A A \# G^*$ as $A \# G^*$-modules. This establishes the claim.

We give two applications of this. For a separably graded ring A we have (cf. [13, 3.9 Proposition]) A is a V-ring if and only if $A \# G^*$ is a V-ring.

Suppose first, that A is a V-ring, and let S be a simple $A \# G^*$-module. Since S is graded simple it is semisimple of length at most $|G|$ as an A_1-module and hence it has finite length as an A-module. Since simple A-modules are injective, S is A-semisimple of finite length and hence A-injective. By Theorem 2.3 S is $A \# G^*$-injective.

Conversely, let S be a simple A-module. Then from the above $S \otimes_A A \# G^*$ is semisimple of finite length as an $A \# G^*$-module and hence $A \# G^*$-injective. Theorem 2.3 and Theorem 3.1 Corollary 1 establish the claim.

The above remark also applies to yield a general result of Greszczuk (cf. [2, 3.15 Corollary]) requiring no assumptions on the grading: if M is A-semisimple of finite length then M is semisimple as an A_1-module and $\mathrm{long}_{A_1}(M) \leqq |G| \, \mathrm{long}_A(M)$. It suffices to prove the claim in case M is a simple A-module. In this case $M \otimes_A A \# G^*$ is semisimple as an $A \# G^*$-module of length at most $|G|$ and hence (as it is graded semisimple) A_1-semisimple of length at most $|G|^2$. Since as A_1-modules $M^n \simeq M \otimes_A A \# G^*$ the claim follows.

The last part of the paper is concerned with perfect rings. In [13, 3.11 Proposition] it is proved that for a separably graded ring A, A is left perfect if and only if $A \# G^*$ is.

We show again that the assumption on the grading is superfluous and, which may be more surprising A perfect if and only if A_1 is.

As a basic reference we use [1].

THEOREM 3.3 *Let the ring A be graded by the finite group G. The following conditions are equivalent:*

1) *A is left perfect*
2) *A_1 is left perfect*
3) *$A \# G^*$ is left perfect.*

PROOF. Let us recall [1, 28.11 Proposition]:

Let A be a ring and $e_1, \ldots, e_n$ a complete set of orthogonal idempotents. A is left perfect if and only if $e_i A e_i$ is left perfect for all i $1 \leqq i \leqq n$.

To show that 1) implies 2) it suffices to show that any descending chain of principal right ideals of A_1 terminates.

Suppose given

$$(a_1 A_1) \supset (a_1 a_2 A_1) \supset \ldots \supset (a_1 a_2 \ldots a_k A_1).$$

We can find an element $a \in A$ and a $k \in \mathrm{N}$ such that $a_1 a_2 \ldots a_k a_{k+1} a = a_1 \ldots a_k$. The desired result follows by projecting on A_1.

Next we show 2) implies 3). This is done by showing that $p_g(A \# G^*)p_g$ is left perfect for all $g \in G$.

$p_1(A \# G^*)p_1$ is left perfect by Proposition 1.1 (v). By (iv) in the same proposition we get $p_g(A \# G^*)p_g = p_g A p_g = A_1 p_g$. Thus we can write a descending chain of principal right ideals in $p_g(A \# G^*)p_g$ as follows:

$$(a_1 p_g)p_g A p_g \supset a_1 p_g a_2 p_g A p_g \supset \ldots$$

or

$$(a_1 p_g)p_g A p_g \supset a_1 a_2 p_g p_g A p_g \supset \ldots$$

but this chain terminates.

We finally have to show that 3) implies 1). Let

$$(a_1) \supset (a_1 a_2) \supset \ldots \supset (a_1 a_2 \ldots a_k) \supset \ldots$$

be a descending chain of principal ideals of A. In $A \# G^*$ we have the same descending chain of principal right ideals. Thus we can find $\sum_g b_g p_g$ such that $a_1 \ldots a_k = a_1 \ldots a_k a_{k+1} (\sum_g b_g p_g)$. Suppose b_g is non-zero. Then multiply the above equation by p_g on the right

$$a_1 \ldots a_k p_g = a_1 \ldots a_{k+1} b_g p_g,$$

thus $a_1 \ldots a_k = a_1 \ldots a_k a_{k+1} b_g$ and the result is proved.

THEOREM 3.3′. *Let the ring A be graded by the finite group G. The following conditions are equivalent:*

1) *A is semiprimary*
2) *A_1 is semiprimary*
3) *$A \# G^*$ is semiprimary.*

PROOF. By Theorem 3.3 we may assume that all three rings here are left and right perfect. 1) implies 2) by [5, Corollary 5.4].

To show that 2) implies 3) we only have to show that $p_g(A \# G^*)p_g$ is semiprimary [1, 28.10 lemma]. $p_g(A \# G^*)p_g = p_g A p_g$ is a perfect ring, so we only have to show that its primeradical is nilpotent. The argument in Theorem 3.3 showing that 2) implies 3) shows that an element $a_1 p_g$ is in the prime radical of $p_g A p_g$ exactly when a_1 is in the primeradical of A_1 and the result now follows easily.

3) implies 1) since a perfect subring of a semiprimary ring is semiprimary [4].

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