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LEWY'S THEOREM FAILS IN HIGHER DIMENSIONS

JOHN C. WOOD*

In [1] H. Lewy showed that, if $n = 2$, a one-to-one harmonic map $\mathsf{R}^n \to \mathsf{R}^n$ has non-vanishing Jacobian. In 1974, I included the following example in my thesis [3] to show that Lewy's theorem fails for $n \geqq 3$. At that time, I assumed that such examples would be known but I am now told by P. Duren that this is not the case (cf. [2]) so I offer the example here:

EXAMPLE. The harmonic map $\mathsf{R}^3 \to \mathsf{R}^3$ defined by $(x, y, z) \mapsto (x^3 - 3xz^2 + yz, y - 3xz, z)$ is a homeomorphism of R^3 with Jacobian which vanishes on the plane $x = 0$.

Indeed the Jacobian of this map is $3x^2$ and its inverse is

$$(x, y, z) \mapsto (\sqrt[3]{x - yz}, y + 3z\sqrt[3]{x - yz}, z).$$

The example can be trivially modified to give harmonic homeomorphisms of R^n with Jacobian vanishing on a hyperplane.

COROLLARY. *Lewy's Theorem is false for $n \geqq 3$.*

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SCHOOL OF MATHEMATICS,
UNIVERSITY OF LEEDS
LEEDS LS2 9JT
G.B.

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