MATH. SCAND. 70 (1992), 293–301

BOUNDARY CONVERGENCE IN NON-NONTANGENTIAL AND NONADMISSIBLE APPROACH REGIONS

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§0. Introduction.

Let $u(x,y) = Pf(x,y)$ be the Poisson integral in the upper halfspace $\mathsf{R}^{n+1}_+ = \{(x,y);\ x \in \mathsf{R}^n, y > 0\}$ of a function $f \in L^p(\mathsf{R}^n)$, $p \geqq 1$. The question whether $u(x,y)$ has boundary limits has been extensively studied. To state this problem more precisely we let Ω be an open set in R^{n+1}_+ having $(0,0)$ as a limit point and put $\Omega^x = x + \Omega$ for $x = (x,0) \in \mathsf{R}^n$. Then the question is for which Ω,

$$(1) \qquad \lim_{\Omega^{x_0}\ (x,y)\to(x_0,0)} u(x,y) = f(x_0), \text{ a.e. } x_0 \in \mathsf{R}^n,\ f \in L^p(\mathsf{R}^n).$$

The classical result of Fatou states that (1) holds if Ω is the cone $C_\alpha = \{(x,y) \in \mathsf{R}^{n+1}_+;\ |x| < \alpha y\}$; i.e. u has nontangential boundary values almost everywhere. It was proved by Littlewood [L] that (1) cannot hold if Ω contains a tangential curve ending at the origin. However, Nagel and Stein showed in [NS] that there are many approach regions Ω not contained in any C_α for which (1) holds, i.e. we may have non-nontangential convergence. The conditions on Ω are

$$\text{a)} \qquad \Omega + C_\alpha \subset \Omega$$

and

$$\text{b)} \qquad |\{x;\ (x,y) \in \Omega\}| \leqq Cy^n.$$

In §1 we provide an alternative proof of Nagel-Stein's result; in fact we show that if Ω satisfies a) and b), then the distribution functions of the maximal functions corresponding to Ω and C_α, respectively, are equivalent, so everything is reduced to the nontangential case. Thus we also get e.g. a generalization to real H^p-spaces, $p < 1$.

Received April 25, 1991.

In §2 we discuss the same problem in bounded domains in R^n with C^2-boundary. It turns out that if $\Omega = \Omega^{x_0}$ satisfies conditions as a) and b) for some $x_0 \in \partial D$ and if Ω^x is some smooth move of Ω^{x_0} for x near x_0, then an analogous result holds for $\{\Omega^x\}$.

It follows from the theory of harmonic functions in R^{2n} that holomorphic functions in $H^p(D)$ for smooth domains D in C^n and $p \geqq 1$, have nontangential boundary limits a.e. However, convergence holds for all $p > 0$ and in larger regions, the so-called admissible approach regions, that allow parabolic tangential approach in the complex tangential directions. It is a natural problem to find an analogue of Nagel-Stein's theorem in the complex case. This problem has been considered by Sueiro [Su2] in the case of the generalized halfspace and the ball.

In §3 we treat the complex case and find an analogous generalization of admissible convergence, "nonadmissible" convergence, in strictly pseudoconvex smooth domains in C^n.

Finally in §4 we consider a specific example, the generalized halfspace in C^n, and deduce a local fact about strictly pseudoconvex boundaries which we use in §3.

§1. Non-nontangential boundary values in the upper halfspace.

As usual the existence of boundary limits follows from estimates for the corresponding maximal function. If Ω is a region in E^{n+1}_+ having 0 as a limit point and $\Omega^x = x + \Omega$ for $x \in \mathsf{R}^n$, we define the maximal function by $M_\Omega u(x) = \sup_{\Omega^x} |u|$, for any measurable function u in R^{n+1}_+. In particular, $M_{C_\alpha} u$ is the nontangential maximal function of u. Let $\Omega^x(t) = \{x' \in \mathsf{R}^n; (x', t) \in \Omega^x\}$, $\Omega(t) = \Omega^0(t)$ and let Pf denote the Poisson integral of f. In [NS] Nagel and Stein proved

THEOREM 1. *Suppose that for some α, Ω satisfies*

a) $$\Omega + C_\alpha \subset \Omega$$

and

b) $$|\Omega(t)| \leqq Ct^n.$$

Then the map $f \mapsto M_\Omega Pf$ is of weak type $(1,1)$ *and strong type* (p,p) *if* $1 < p \leqq +\infty$.

EXAMPLE 1. Let γ be any tangential curve ending at 0. If $p_j = (x_j, y_j)$ is a sequence of points on γ and $x_j \to 0$ rapidly enough, then $\Omega = \bigcup_j p_j + C_\alpha$ satisfies a) and b) but clearly Ω is not contained in any cone. Thus we have non-nontangential convergence.

REMARK 1. Nagel and Stein also proved a converse of Theorem 1; namely if Ω is any set such that $M_\Omega P$ is of weak type (1, 1) and $\tilde{\Omega} = \Omega + C_\alpha$ then $|\tilde{\Omega}(t)| \leqq Ct^n$.

Conditions a) and b) clearly implies

i) $t \to \Omega(t)$ is increasing,

ii) if $B_p(t) \cap \Omega(t) \neq \emptyset$ then $p \in \Omega(Ct)$, $\left(\text{with } C = 1 + \dfrac{1}{\alpha}\right)$,

and

iii) $|\Omega(t)| \leqq Ct^n$.

Here $B_p(t)$ is the Euclidean ball with center p and radius t. The conditions are in fact essentially equivalent since if Ω satisfies i)–iii), then $\tilde{\Omega} = \Omega + C_\alpha$ satisfies a) and b). Hence Theorem 1 is a consequence of the following proposition that states that the distribution functions of M_Ω and M_{C_α} are equivalent.

PROPOSITION 1. *Suppose that Ω satisfies* i)–iii). *Then*

$$|\{x \in \mathbf{R}^n; M_\Omega u(x) > \lambda\}| \leqq C\,|\{x \in \mathbf{R}^n; M_{C_\alpha} u(x) > \lambda\}| \tag{1}$$

and hence

$$\|M_\Omega u\|_{L^p(\mathbf{R}^n)} \leqq C\,\|M_{C_\alpha} u\|_{L^p(\mathbf{R}^n)},\ p > 0, \tag{2}$$

for any measurable function u in $\mathbf{R}^{n+1}_+$

REMARK 2. The estimate (1) also follows from the proof of Theorem 1 given by Sueiro in [Su].

By Proposition 1 we may deduce estimates for M_Ω from known estimates for M_{C_α}. For instance if we let $u = Pf$, Theorem 1 follows from the classical estimates of the nontangential maximal function of Poisson integrals. As another example we may assume that $u \in H^p(\mathbf{R}^n)$ and deduce that $\|M_\Omega u\|_{L^p(\mathbf{R}^n)} \leqq C\,\|u\|_{H^p(\mathbf{R}^n)}$, p > 0 since the corresponding estimates holds for $M_{C_\alpha} u$, see [FS, Theorem 9].

To prove (1) we will use the idea, suggested by Carleson in [C, p. 67] that estimates of maximal functions can be deduced from results about Carleson measures. In fact, if we let μ_Ω be the outer measure in $\mathbf{R}^{n+1}_+$ defined by $\mu_\Omega(E) = |\{x \in \mathbf{R}^n;\ \Omega^x \cap E \neq \emptyset\}|$, then $|\{x \in \mathbf{R}^n;\ M_\Omega u(x) > \lambda\}| = \mu_\Omega(\{z \in \mathbf{R}^{n+1}_+;\ u(z) > \lambda\})$. Thus Proposition 1 is a consequence of the following two results.

LEMMA 1. *If* i)–iii) *holds then μ_Ω is a Carleson measure.*

LEMMA 2. *If μ is a Carleson measure, then*

$$\mu(\{z \in \mathbf{R}^{n+1}; |u(z)| > \lambda\}) \leqq C\,|\{x \in \mathbf{R}^n; M_{C_\alpha} u(x) > \lambda\}|.$$

We recall that μ is a Carleson measure if $\mu(B_p(t) \times (0, t)) \leqq Ct^n$. We use, somewhat incorrectly, the terminology "Carleson measure" also when μ only is an outer measure. Lemma 2 is well-known and is proved by a standard covering argument, for details see for instance [G, Ch.1, §5]. It also holds for outer measures since only subadditivity is needed in the proof. So it only remains to prove Lemma 1:

$$\mu(B_p(t) \times (0,t)) = |\{x; \Omega^x \cap B_p(t) \times (0,t) \neq \emptyset\}| = \text{by i)}$$
$$= |\{x; \Omega^x(t) \cap B_p(t) \neq \emptyset\}| \leqq \text{by ii)} \leqq |\{x; p \in \Omega^x(Ct)\}|$$
$$= |\{x; -x \in \Omega^{-p}(Ct)\}| \leqq \text{by iii)} \leqq Ct^n.$$

In the next two paragraphs we generalize this argument to domains in $\mathbf{R}^n$ and $\mathbf{C}^n$. As the boundaries are homogeneous spaces, Lemma 2 immediately generalizes. The problem therefore reduces to give natural conditions on the approach regions Ω^x that guarantee that μ_Ω is a Carleson measure

§2. Non-nontangential limits for bounded domains in $\mathbf{R}^n$.

Let D be a bounded domain in $\mathbf{R}^n$ with C^2 boundary. We fix a neighborhood U of ∂D in $\bar{D}$ so small that U can be identified with $\partial D \times [0, \varepsilon)$. For instance, if $n(x)$ is the inward normal vector field, $x \in \partial D$, then by the inverse function theorem the map $\partial D \times [0, \varepsilon) \in (x, t) \mapsto x + tn(x) \in \bar{D}$ is a C^2-diffeomorphism onto some neighborhood U of ∂D, and hence realizes such an identification. For $x \in \partial D$ we let $B_x(t)$ be the intersection of ∂D with the ball in $\mathbf{R}^n$ of radius t and center x. The cone C_α^x at $x \in \partial D$ is $C_\alpha^x = \{(x', y) \in U; x' \in B_x(\alpha y)\}$. If Pf denotes the Poisson integral of a function $f \in L^p(\partial D)$, then $M_{C_\alpha} P$ is of weak type $(1,1)$ and bounded on L^p, $1 < p \leqq +\infty$, see [S, §5]. Here of course $M_{C_\alpha} u(x) = \sup_{C_\alpha^x} |u|$.

As in §1, we want to extend this result to approach regions Ω^x, $x \in \partial D$, with $x \in \overline{\Omega^x}$ that are not contained in any cone C_α^x. In view of §1, it is natural to assume that Ω^x satisfies

i) $t \mapsto \Omega^x(t)$ is increasing,

ii) if $B_p(t) \cap \Omega^x(t) \neq \emptyset$, then $p \in \Omega^x(Ct)$,

and

iii) $|\Omega^x(t)| \leqq Ct^n$.

Here $\Omega^x(t) = \{x' \in \partial D; (x', t) \in \Omega^x\}$. Note that the cones C_α^x satisfie i)–iii). However, in order to obtain an analogue of Theorem 1, we also need some continuity in x. Suppose that to $x, x' \in \bar{\omega} \subset \partial D$ there are local diffeomorphisms $T_{xx'}$ mapping a neighborhood of x in ∂D to a neighborhood of x'. We also assume that $(x, x', x'') \mapsto T_{xx'}(x'')$ is smooth, $T_{xx'}(x) = x'$ and that T_{xx} is the identity. We extend $T_{xx'}$ to U by $T_{xx'}(x'', t) = (T_{xx'}(x''), t)$ and assume that $\Omega^{x'} = T_{xx'}(\Omega^x)$.

Remark 1. Such a family diffeomorphisms $T_{xx'}$ always exists locally in $x, x' \in \partial D$. One can for instance use the translation structure induced from some local coordinate system on ∂D, so that in these coordinates $T_{xx'}(x'') = x'' + x' - x$. However global existence of $T_{xx'}$ is impossible unless the tangent bundle $T(\partial D)$ is trivial. Furthermore, to obtain Ω^x and $T_{xx'}$ as above it is enough to prescribe Ω^{x_0} for some x_0, satisfying i)–iii), and local diffeomorphisms F_x on ∂D with $F(x_0) = x$; if we extend F_x to U by $F(x', t) = (F_x(x'), t)$ and put $\Omega^x = F_x(\Omega^{x_0})$, then Ω^x also satisfies i)–iii) (perhaps with a larger C) and Ω^x are connected via $T_{xx'} = F_{x'} \circ F_x^{-1}$.

Under these assumptions we can prove

Proposition 1. *If $\Omega^x, x \in \bar{\omega}$, are connected via local diffeomorphisms as above and Ω^x satisfies* i)–iii), *then*

$$|\{x \in \bar{\omega}; M_\Omega u(x) > \lambda\}| \leqq C\,|\{x \in \partial D; \mathrm{M}_{C_\alpha} u(x) > \lambda\}|$$

and

$$\|M_\Omega u\|_{L^p(\bar{\omega})} \leqq C\, \|M_{C_\alpha} u\|_{L^p(\partial D)}.$$

As Lemma 2 in § 1 applies in this setting. Proposition 1 follows from

Lemma 1. *If Ω^x is as in Proposition* 1, *then μ_Ω is a Carleson measure.*

Here $\mu_\Omega(E) = |\{x \in \bar{\omega}; \Omega \cap E \neq \emptyset\}|$. As in the proof of Lemma 2 in § 1, we obtain

$$\mu_\Omega(B_p(t) \times (0, t)) \leqq |\{x; p \in \Omega^x(Ct)\}|.$$

Now we cannot deduce that this equals $|\{x; -x \in \Omega^{-p}(Ct)\}|$. However, if we let $F_x = T_{x_0 x}$ and $G(x, y) = F_x^{-1}(y)$, then for fixed y, $G(x, y)$ is defined for x near y. Since $G(y, y) = x_0$, $\partial G/\partial x|_{(p,p)} + \partial G/\partial y|_{(p,p)} = 0$ or

$$\left.\frac{\partial G(x, y)}{\partial x}\right|_{(p,p)} = -\left[\left.\frac{\partial F_x(y)}{\partial y}\right|_{(px_0)}\right]^{-1}.$$

Hence $\partial G/\partial x|_{(p,p)}$ is non-singular and $g(x) = g_p(x) = G(x, p)$ is a diffeomorphism. Thus

$$\begin{aligned}
&\mu_\Omega(B_p(t) \times (0, t)) \leqq |\{x; p \in \Omega^x(Ct)\}| \\
&= |\{x; p \in T_{x_0 x}(\Omega^{x_0}(Ct))\}| = |\{x; g(x) \in \Omega^{x_0}(Ct)\}| \\
&= |g^{-1}(\Omega^{x_0}(Ct))| \leqq C\,|\Omega^{x_0}(Ct)| \leqq Ct^n.
\end{aligned}$$

§ 3. Boundary values of H^p-functions.

Let D be a bounded strictly pseudoconvex domain of class C^2 in $\mathbf{C}^n$. In this section we will study boundary values of H^p-functions in D. For a background to this problem we refer to [S]. The appropriate analogue of nontangential convergence in this situation is the so-called admissible convergence, i.e. convergence in

Korányi cones, which are larger than the standard cones. (We give the precise definition of the Korányi cones and the associated maximal function M_{K_α} in a moment.) The basic theorem is, se [S, §9] or [H],

THEOREM A. *If D is a bounded C^2-domain and $f \in H^p(D)$, then*

$$\|M_{K_\alpha} f\|_{L^p} \leqq C_p \|f\|_{H^p}, 0 < p \leqq +\infty,$$

and f has admissible boundary values for almost all $x \in \partial D$.

In analogy to the non-nontangential approach regions in §1 and §2, we will generalize this result to nonadmissible approach regions that are strictly larger than the Korányi cones. As before we will show that the corresponding maximal functions are comparable. As a consequence, we obtain a generalization of Theorem A to these nonadmissible approach regions. As another corollary, we obtain the result of Sueiro [Su2] about Poisson-Szegö integrals in the generalized halfspace and the ball.

We start by briefly discussing the nonisotropic structure on ∂D, the associated Korányi balls and the admissible approach regions. For each $p \in \partial D$ we have a preferred $(2n-2)$-dimensional real subspace M_p, the complex tangent space, of the real tangent space $T_p(\partial D)$. If J is the complex structure, then $M_p = T_p(\partial D) \cap JT_p(\partial D)$. If $D = \{\rho < 0\}$, then the real 1-form $\alpha = Jd\rho = d^c\rho = i(\bar\partial - \partial)\rho$ defines M_p, i.e. $M_p = \{v \in T_p(\partial D); \alpha . v = 0\}$. That D is strictly pseudiconvex means that the quadratic form (the Levi form) $d\alpha(v, w) = dd^c(v, w)$ is positive definite on $M_p(\partial D)$ for each $p \in \partial D$. We define a basis of neighborhoods $B_p(t)$ in ∂D, the Korányi balls at p, by requiring that $B_p(t)$ have length $\sim\sqrt{t}$ in the M_p-directions and length $\sim t$ in the last one. Then $|B_p(t)| \sim t^n$ and $B_p(t)$ is determined up to equivalence. $B_p(t)$ and $B'_p(t)$ are equivalent if $B_p(t/C) \subset B'_p(t) \subset B_p(Ct)$ for small t. From now on we fix a choice of $B_p(t)$ at each p continuously in p. (Any other such a choice will be equivalent to $B_p(t)$ uniformly for p in compact sets). The Korányi balls satisfy

$$\text{if } s \leqq t \text{ and } B_p(t) \cup B_q(s) \neq \emptyset, \text{ then } B_q(s) \subset B_p(Ct), \tag{1}$$

and

$$|B_p(2t)| \leqq C\,|B_p(t)|. \tag{2}$$

Thus ∂D equipped with these balls $B_p(t)$ and surface measure is a homogeneous space, and the standard tools from harmonic analysis such as covering lemmas, L^p-estimates of maximal functions, etc. can be applied.

As in §2 we identify a neighborhood U of ∂D with $\partial D \times [0, \varepsilon)$. The admissible approach regions, or Korányi cones, are $K_\alpha^x = \{(x', t); x' \in B_x(\alpha t)\}$. We consider approach regions Ω^x, $x \in \partial D$, with $x \in \overline{\Omega^x}$ that satisfies

i) $t \mapsto \Omega^x(t)$ is increasing,

ii) if $\Omega^x(t) \cap B_p(t) \neq \emptyset$, then $p \in \Omega^x(Ct)$

and

iii) $|\Omega^x(t)| \leqq Ct^n$.

Here of course $\Omega^x(t) = \{x' \in \partial D; (x', t) \in \Omega^x\}$. (Note that the Korányi cones satisfy these conditions.) We further assume that Ω^x are connected via local diffeomorphisms $T_{xx'}$ as in §2. Hence $\Omega^{x'} = T_{xx'}(\Omega^x)$. However an arbitrary diffeomorphismmmm $T_{xx'}$ will destroy condition ii). Hence we further need to assume that $T_{xx'}$ preserves the nonisotropic structure, that is $T_{xx'}(B_x(t))$ is equivalent to $B_{x'}(t)$. This is equivalent to that $dT_{xx'}|_p$ maps M_p onto $M_{T_{xx'}}(p)$. Then if Ω^x satisfies i)–iii), so will $\Omega^{x'}$. We will show in §4 that such a family $T_{xx'}$ always exists locally if D is strictly pseudoconvex.

REMARK 1. The requirement is *not* that $T_{xx'}$ are CR-mappings. For that one also requires that $dT_{xx'}|_p$ commute with the complex structure J. This implies that the Levi form is preserved and this cannot hold in general.

Let $M_\Omega u(x) = \sup_{\Omega^x} |u|$, $x \in \bar{\omega}$. We have

THEOREM 1. *Suppose that D is a strictly pseudoconvex bounded domain with C^2 boundary. Further assume that we have a family of regions Ω^x, $x \in \bar{\omega} \subset \partial D$, connected via Korányi ball preserving diffeomorphisms as above. If Ω^x, $x \in \bar{\omega}$, satisfies* i)–iii), *then for $f \in H^p(D)$,*

$$\|M_\Omega f\|_{L^p(\bar{\omega})} \leqq C\,\|f\|_{H^p(D)},\ 0 < p \leqq +\infty,$$

and f has Ω-limits for almost all $x \in \bar{\omega}$.

EXAMPLE 1. As in Example 1 in §1, for any tangential curve γ we can choose points $p_j = (x_j, y_i)$ on γ such that $x_j \to x_0$ and such that $\Omega^{x_0} = \bigcup_j \{(x', y_j + y);\ x' \in B_{x_j}(y)\}$ satisfies i)–iii). Then if we put $\Omega^x = T_{x_0 x}(\Omega^{x_0})$, Ω^x satisfies the conditions in Theorem 1. In particular, we may choose γ so that Ω^x is not contained in any K_α^x.

Theorem 1 is a consequence of

PROPOSITION 1. *If Ω^x, $x \in \bar{\omega}$, satisfy the assumptions of Theorem* 1, *we have*

$$|\{x \in \bar{\omega}; M_\Omega u(x) > \lambda\}| \leqq C\,|\{x \in \partial D; M_{K_\alpha} u(x) > \lambda\}|$$

and

$$\|M_\Omega u\|_{L^p(\bar{\omega})} \leqq C\,\|M_{K_\alpha} u(x)\|_{L^p(\partial D)}, p > 0.$$

To prove Proposition 1, we need two lemmas about Carleson measures as in

§1. The Carleson measure is defined with respect to the Korányi balls, $\mu(B_p(t) \times (0,t)) \leqq C|B_p(t)|$. We also define $\mu_\Omega(E) = |\{x \in \bar{\omega}; \Omega^x \cap E \neq \emptyset\}|$.

LEMMA 1. *If Ω^x, $x \in \bar{\omega}$, satisfy the conditions of Theorem 1, then μ_Ω is a Carleson measure.*

This is proved exactly as Lemma 1 in §2. Finally we have

LEMMA 2. *If μ is a Carleson measure, then*

$$\mu(\{x \in D; |u(x)| > \lambda\}) \leqq C|\{x \in \partial D; M_{K_\alpha}u(x) > \lambda\}|.$$

This follows by the same covering argument as proved Lemma 2 in §1; the covering argument works because of (1) and (2).

§4. Korányi ball preserving mappings.

In this last paragraph we show that locally on a strictly pseudoconvex boundary, there always exists a family of diffeomorphisms as in §2 that also preserves the Korányi balls. As was noted in §3, the last requirement is equivalent to that the derivatives preserve the real complex tangent spaces M_x.

PROPOSITION 1. *Let $D \in \mathbf{C}^n$ be strictly pseudoconvex with C^∞-boundary. For any point $x_0 \in \partial D$ there is a compact neighborhood $\bar{\omega} \in \partial D$ of x_0 and local diffeomorphisms F_x, $x \in \bar{\omega}$, such that $F_x(x_0) = x$, F_{x_0} is the identity, $(x, x') \to F_x(x')$ is smooth and $dF_x|_{x'}$ maps $M_{x'}$ onto $M_{F_x(x')}$.*

Before the proof we study an important example.

EXAMPLE 1. Let $\Pi = \{(z_1, z') \in \mathbf{C}^n;\ |z'|^2 - \operatorname{Im} z_1 < 0\}$ be the generalized halfspace with its strictly pseudoconvex boundary $\partial\Pi = \{(z_1, z') \in \mathbf{C}^n;\ |z'|^2 = \operatorname{Im} z_1\}$. The Heisenberg group H, with underlying manifold $\mathbf{R} \times \mathbf{C}^n$ and noncommutative group structure $(t, a)\cdot(t', a') = (t + t' + 2\operatorname{Im} a\cdot\overline{a'}, a + a')$ acts on Π from the left by $(t, a)\cdot(z_1, z') = (z_1 + t + i|a|^2 + 2i\operatorname{Im} z'\cdot\bar{a}, z' + a)$. Note that for fixed $g \in \Pi$, $(z_1, z') \to g\cdot(z_1, z')$ is a biholomorphic mapping of Π (and of $\mathbf{C}^n$) and hence a CR-mapping from $\partial\Pi$ to $\partial\Pi$, so in particular its derivate maps M_p onto $M_{g.p}$. Since for each pair $p_0, p \in \partial\Pi$ there is a unique $g \in H$ such that $p = g\cdot p_0$, we can put $F_p(x) = g\cdot x$. Then F_p is defined globally on $\partial\Pi$ and we have proved Proposition 1 in this particular case.

By the linear fractional mapping ψ, where $w = \psi^{-1}(z)$ is given by

$$z_1 = i\frac{1 - w_1}{1 + w_1} \text{ and } z_i = \frac{w_i}{1 + w_1} \text{ if } 2 \leqq i \leqq n,$$

Π is mapped biholomorphically onto the unit ball B and $\psi|_{\partial\Pi}$ is a CR-mapping from $\partial\Pi$ onto the boundary of B minus one single point. Together with Example 1, this shows Proposition 1 for the ball.

Now we consider the general strictly pseudoconvex case. We claim that there is locally a diffeomorphism ψ from ∂D to $\partial\Pi$ such that $d\psi|_x$ maps $M_x(\partial D)$ onto $M_{\psi(x)}(\partial\Pi)$. Then the proposition is a consequence of Example 1 above.

Contrary to the case of the ball, it is not possible in general to have ψ as the restriction to ∂D of a biholomorphic mapping, since then it also would be a CR-mapping.

However, what is relevant for us is that we have a real $(2n-1)$-dimensional manifold X $(=\partial D)$ and a smooth distribution (subbundle) M of $T(X)$. Such a pair (X, M) is called a contact manifold if M is nondegenerate. For a discussion of contact manifolds see [A, appendix 4H]. The nondegeneracy conditions means that M is as far as possible from being integrable (in the sense of Frobenius theorem) and can be stated as follows: If α is a 1-form that defines M, i.e. $\alpha|_x \perp M_x$ for all x, then for any $0 \neq v \in M_x$ there is a $w \in M_x$ such that $d\alpha|_x(v, w) \neq 0$.

If ∂D is strictly pseudoconvex, then the Levi form $L(v, w)|_x = \pm \operatorname{id} \alpha|_x(v, w)$ is positive definite, compare p. 9, so in particular $d\alpha|_x$ is nondegenerate on M_x and hence (X, M) is a contact manifold. Now the claim follows from

LEMMA 1 (see [A, appendix 4H)]. *Any two contact manifolds are locally contact-diffeomorphic.*

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