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# COMPLEX BORDISM AND FREE UNITARY ACTIONS OF FINITE SOLVABLE GROUPS WITH PERIODIC COHOMOLOGY ON SPHERES

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## 0. Introduction.

The purpose of this paper is to calculate the order of $[S^{2k-1}, G]$ in $U_{2k-1}(BG)$, the action of $G$ being induced by a $k$-dimensional unitary fixed point free representation $\sigma$ of $G$ which is supposed to be finite solvable with periodic cohomology. Sometimes we shall use the notation $[S^{2k-1}, \sigma]$ instead of $[S^{2k-1}, G]$. We have the following classification of all finite solvable groups with periodic cohomology (see [6] page 179).

I $G = \langle A, B \rangle$, $A^m = B^n = 1$, $BAB^{-1} = A^r$, $m \geqq 1$, $n \geqq 1$, $(n(r-1), m) = 1$, $r^n \equiv 1(m)$, $\operatorname{ord} G = mn$.

II $G = \langle A, B, R \rangle$ with $\langle A, B \rangle$ as in I, $R^2 = B^{n/2}$, $RAR^{-1} = A^s$, $RBR^{-1} = B^k$, $n = 2^u v$, $u \geqq 2$, $v$ odd, $s^2 \equiv r^{k-1} \equiv 1(m)$, $k \equiv -1(2^u)$, $k^2 \equiv 1(n)$, $\operatorname{ord} G = 2mn$.

III $G = \langle A, B, P, Q \rangle$ with $\langle A, B \rangle$ as in I, $P^4 = 1$, $P^2 = Q^2 = (PQ)^2$, $AP = PA$, $AQ = QA$, $BPB^{-1} = Q$, $BQB^{-1} = PQ$, $n \equiv 1\ (2)$, $n \equiv 0(3)$.

IV $G = \langle A, B, P, Q, R \rangle$ with $\langle A, B, P, Q \rangle$ as in II and $R^2 = P^2$, $RPR^{-1} = QP$, $PQR^{-1} = Q^{-1}$, $RAR^{-1} = A^s$, $RBA^{-1} = B^k$, $k^2 \equiv 1(n)$, $k \equiv -1(3)$, $r^{k-1} \equiv s^2 \equiv 1(m)$, $\operatorname{ord} G = 16mn$.

If $k > 0$, $q > 0$, $q = p_1^{\alpha_1} \dots p_r^{\alpha_r}$ the decompositions of $q$ into primes then we define $N(k,q) = p_1^{\alpha_1 + k_1} \dots p_r^{\alpha_r + k_r}$ with $k_i = [k/p_i - 1]$ the greatest integer $\alpha$, $\alpha < k/p_i - 1$.

Let $G$ be a finite solvable group with periodic cohomology and $\sigma$ a fixed point free unitary representation of $G$ with $\dim \sigma = k$ over $\mathbf{C}$.

The main result of this paper is as follows.

THEOREM. *If $G$ is of type* I, II, III, IV, *then ord*$[S^{2k-1}, \sigma]$ *is respectively*,

$$N(k,m)N(k,n), N(k,m)N(k,v)2^{k+u-1}, N(k,m)N(k,n)2^{k+1}, N(k,m)N(k,v)2^{k+2} \cdot 3^{u+k'-1}$$

$$(n = 3^u v, (v,6) = 1, k = 2k).$$

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These results are proved in section II, theorems 2.1 and 2.2.

Let $U^*(\mathrm{pt})[[X]]$ be the $U^*(\mathrm{pt})$-algebra of formal power series in $X$ with coefficients in $U^*(\mathrm{pt})$ graded by taking $\dim X = 2$. If $F$ denotes the formal group law then $[q](X)$ is defined inductively by $[q](X) = F([q-1](X), X)$, $[1](X) = X$, $q \geqq 1$. We shall denote $s_{2k-1} = [S^{2k-1}, k\sigma] \in U_{2k-1}(BZ_q)$, $\sigma$ being the unitary representation of $Z_q$ of dimension 1 defined by the $q$th primitive root of unity $\exp(2i\pi/q)$. It is well known that $\operatorname{ord} s_{2k-1} = N(k,q)$ (see [5]).

If $P(X) = \alpha_p X^p + \alpha_{p+1} X^{p+1} + \dots$ is a homogeneous formal power series, $\alpha_p \neq 0$, we denote $v(P) = 2p$. We have the following result whose proof follows the lines of that of [3], theorem 2.4 b) and therefore will be omitted. We recall that $\{s_{2n+1}\}$, $n \geqq 0$, is a system of generators for the $U_*(\mathrm{pt})$-module $\tilde{U}(BZ_q)$.

THEOREM 0.1. *If* $[q](X) = qX + a_2X^2 + \dots + a_nX^n + \dots$ *then we have:*

a) $qs_{2n+1} + a_2s_{2n-1} + \dots + a_{n+1}s_1 = 0$ *for every* $n \geqq 1$

b) *In* $\tilde{U}_{2p+1}(BZ_q)$: $\alpha_0 s_1 + \dots + \alpha_n s_{2n+1} = 0$ *iff there are a homogeneous polynomial* $H(X)$ *and a homogeneous formal power series* $E(X)$ *such that:*
$\alpha_n X + \alpha_{n-1}X^2 + \dots + \alpha_0 X^{n+1} = H(X)\cdot[q](X) + E(X),\ v(E) > 2(n+1)$.

In this paper we will use the notation and results contained in [3].

## I. Preliminaries.

Consider the following exact sequence of finite groups:

$$1 \to H \xrightarrow{i} G \xrightarrow{\pi} S \to 1$$

$H$ being a normal subgroup of $G$, $i$: $H \subset G$, $S \subset G$, $(\operatorname{ord} H, \operatorname{ord} S) = 1$.
There is a homomorphism $f$: $S \to G$ such that $\pi \circ f = 1$. If $[M, H] \in \tilde{U}_*(BH)$, $g \in G$, there is a new free action of $H$ on $M$ induced by the inner automorphism $i_g$: $H \to H$, $i_g(h) = g^{-1}hg$ and we obtain the element $[M, H]_g \in \tilde{U}_*(BH)$. We can identify the groups $S$ and $G/H$ by means of the unique isomorphism $\lambda$: $S \to G/H$ such that $\lambda \circ \pi = \psi$, $\psi$ being the quotient map: $G \to G/H$. If $t_H$ denotes the transfer map: $\tilde{U}_*(BG) \to \tilde{U}_*(BH)$ then we get:

$$t_H \circ i_*([M, H]) = \sum_{j=1}^{j=s} [M, H]_{g_j} \text{ with } s = \operatorname{ord} S,\ S = G/H = \bigcup_{j=1}^{j=s} g_j H$$

(see [1] or [2]). The action of $S$ on $\tilde{U}_*(BH)$ which derives from its action on $H$ by inner automorphisms is as follows: $\gamma[M, H] = [M, H]_g$, $\gamma = gH \in G/H = S$. We have the following result by $M$. Kamata and H. Minami (see [2]): the map $\varphi$: $\tilde{U}_k(BH)^s \oplus \tilde{U}_k(BS) \to \tilde{U}_k(BG)$ defined by $\varphi(x, y) = i_*(x) + f_*(y)$ is injective if $H$ is abelian. In the next result we do not suppose $H$ abelian.

THEOREM 1.1. *Suppose $G$ finite with periodic cohomology and* $1 \to H \xrightarrow{i} G \xrightarrow{\pi} S \to 1$ *an exact sequence such that* $(\operatorname{ord} H, \operatorname{ord} S) = 1$.

*Then* $\tilde{U}_*(BG)$ *is isomorphic to* $\tilde{U}_*(BH)^S \oplus \tilde{U}_*(BS)$ *as* $U_*(\mathrm{pt})$*-modules.*

PROOF. Let $\varphi$: $\tilde{U}_*(BH)^s \oplus \tilde{U}_*(BS) \to \tilde{U}_*(BG)$ be the $U_*(\mathrm{pt})$-homomorphism defined by $\varphi(x, y) = i_*(x) + f_*(y)$, where $f: S \to G$ is a homomorphism such that $\pi \circ f = 1$. Suppose $\varphi(x, y) = 0$; then $i_*(x) = -f_*(y)$ and $0 = \pi_* \circ i_*(X) = -\pi_* \circ f_*(y) = -y$. So: $y = 0$ and $i_*(x) = 0$. As $x \in \tilde{U}_*(BH)^s$ we have $t_H \circ i_*(x) = sx = 0$, $s = \operatorname{ord} S$. There is $\alpha > 0$ such that $h_x^\alpha = 0$, $h = \operatorname{ord} H$; since $(h^\alpha, s) = 1$ we get $x = 0$ and $\varphi$ is injective. Now let $\{S_1, S_2, \ldots, S_q\}$ be a complete set of Sylow-subgroups of $G$ ($S_i$, $S_j$ are not conjugate if $i \neq j$). The map $\chi: \bigcup_{i=1}^{i=q} \tilde{U}_*(BS_i) \to \tilde{U}_*(BG)$ induced by the inclusions $S_i \subset G$ is an epimorphism and as a consequence the map $i_* + f_*: \tilde{U}_*(BH) \oplus \tilde{U}_*(BS) \to \tilde{U}_*(BG)$ is epimorphic. It remains to prove that $i_*(\tilde{U}_*(BH)) = i_*(\tilde{U}_*(BH)^s)$. Let $x \in \tilde{U}_*(BH)$, $S = \{\gamma_1, \gamma_2, \ldots, \gamma_s\}$, $\gamma_i = g_i H$. Then we have: $i_*(\gamma_j x) = g_j \circ i_*(x) = i_*(x)$ because the action of $g_j$ on $\tilde{U}_*(BG)$ induced by the conjugation by $g_j$ on $G$ is trivial. Take $x_1 = \sum_{j=1}^{s} \gamma_j x \in \tilde{U}_*(BH)^S$. Obviously $i_*(x_1) = s i_*(x)$ and $h^\beta x = 0$ for some $\beta > 0$, $h = \operatorname{ord} H$. As $(h^\beta, s) = 1$ there are $a \in \mathrm{Z}$, $b \in \mathrm{Z}$ satisfying $ah^\beta + bs = 1$. Hence $i_*(bx_1) = i_*(x)$.

Let $t_H: \tilde{U}_*(BG) \to \tilde{U}_*(BH)$, $t_S: \tilde{U}_*(BG) \to \tilde{U}_*(BS)$ be the transfer homomorphisms. We keep the same hypotheses as in theorem 1.1.

PROPOSITION 1.2. *If* $x \in \tilde{U}_q(BG)$ *then* $\operatorname{ord} x = \operatorname{ord} t_H(x) \operatorname{ord} t_s(x)$.

PROOF. Since $\operatorname{ord} t_H(x)$, $\operatorname{ord} t_s(x)$ are coprime we have $\operatorname{ord} t_H(x) \operatorname{ord} t_s(x) \mid \operatorname{ord} x$. By theorem 1.1 $x = i_*(x_1) + f_*(y)$, $x_1 \in \tilde{U}_*(BH)^S$, $y \in \tilde{U}_*(BS)$. So: $t_H(x) = sx_1 + t_H \circ f_*(y) = sx_1$ and $\operatorname{ord} t_H(x) \cdot x_1 = 0$ because $(s, h) = 1$, $s = \operatorname{ord} S$, $h = \operatorname{ord} H$. Similarly $t_S(x) = t_S(f(y))$ and $t_S f(\operatorname{ord} t_S(x) y) = 0$. Take $y_1 = f(\operatorname{ord} t_S(x) \cdot y)$. We have $t_S y_1 = 0$. Suppose $y_1 \neq 0$, $y_1 \in \tilde{U}_{2k-1}(BG)$. There is $q \geqq 1$ such that $y_1 = J_{2q-1,2k-2q}$, $y_1 \notin J_{2q-2,2k-2q+1}$ where $J_{*,*}$ denotes the filtration of $\tilde{U}_{2k-1}(BG)$ associated to the Atiyah-Hirzebruch spectral sequence for $BG$. Let $p$ be the quotient map: $J_{2q-1,2k-2q} \to J_{2q-1,2k-2q}/J_{2q-2,2k-2q+1} = H_{2q-1}(BG) \otimes U_{2(k-q)}(pt)$. The map $(f \circ t_s) \otimes 1$ defined on $H_{2q-1}(BG) \otimes U_{2(k-q)}(\mathrm{pt})$ is the product by $h = \operatorname{ord} H$ and as $s^\alpha y = 0$ for some $\alpha > 0$, $s = \operatorname{ord} S$, we get $s^\alpha y_1 = 0$. Consequently since $p(y_1) = 0$ we obtain $((f \circ t_s) \otimes 1)(p(y_1)) = 0$. Hence $f_* \circ t_S(y_1) = 0$ which is impossible. So $y_1 = f_*(\operatorname{ord} t_s(x) y) = 0$ and $\operatorname{ord} t_S(x) y = 0$ since $f$ is injective. As $\operatorname{ord} t_H(x) x_1 = 0$ we have $\operatorname{ord} t_H(x) \operatorname{ord} t_S(x) x = 0$. We have seen that $\operatorname{ord} t_H(x) \operatorname{ord} t_S(x) \mid \operatorname{ord} x$. Hence: $t_S(x) = \operatorname{ord} t_H(x) \operatorname{ord} t_S(x)$.

Let $\sigma$ be a unitary fixed point free representation of $Z_q$, $\dim \sigma = k$.

PROPOSITION 1.3. *We have* $\operatorname{ord}[S^{2k-1},\sigma] = N(k,q)$.

PROOF. If $\mu:\tilde{U}_{2k-1}(BZ_q) \to \tilde{H}_{2k-1}(BZ_q)$ denotes the Thom-homomorphism then $\mu([S^{2k-1},\sigma])$ is a generator of $\tilde{H}_{2k-1}(BZ_q)$. So:

1) $[S^{2k-1},\sigma]) = ms_{2k-1} + \sum_{i\geqq 1} \lambda_i s_{2(k-i)-1}$, $\lambda_i \in U_*(\text{pt})$, $m \in Z$, $(m,q) = 1$.

As $\operatorname{ord} s_{2k-1} \geqq \operatorname{ord} s_{2(k-i)-1}$, $i \geqq 1$ we have:

2) $\operatorname{ord}[S^{2k-1},\sigma] \leqq \operatorname{ord} s_{2k-1} = N(k,q)$.

There are $a \in Z$, $b \in Z$ such that $am + bq = 1$ and then : $s_{2k-1} = ams_{2k-1} + bqs_{2k-1}$. By using theorem 0.1 a) we obtain:

$$a[S^{2k-1},\sigma] = s_{2k-1} + \sum_{i\geqq 1} \lambda_i' s_{2(k-i)-1} \in U_*(\text{pt}).$$

Consequently if $c = \operatorname{ord}[S^{2k-1},\sigma]$ we get : $cs_{2k-1} + \sum_{1\leqq i \leqq k-1} c\lambda_i' s_{2(k-i)-1} = 0$.

By theorem 0.1, b), there is a homogeneous polynomial $H(X)$ and a homogeneous formal power series $E(X)$ such that:

$$cX\left[1 + \sum_{1\leqq i\leqq k-1} \lambda_i' X^i\right] = H(X)\cdot[q](X) + E(X),\ v(E) > 2k.$$

It follows that $cX = H_1(X)\cdot[q](X) + E_1(X)$, $v(E_1) > 2k$. Let $D_1 \in \tilde{U}^2(BZ_q)$ be the Euler class of the universal complex vector bundle over $BZ_q$ corresponding to $\sigma$ (see the introduction). We have: $D_1 \cap s_{2k+1} = s_{2k-1}$ (see proposition 2.3 of [3] for a similar assertion). As $[q](D_1) = 0$ we get $cD_1 = E_1(D_1) = F(D_1) \cap D_1^{k+1}$, $F(X) \in U^*(\text{pt})\ [[X]]$. It follows that $cs_{2k-1} = cD_1 \cap s_{2k+1} = F(D_1) \cap (D_1^{k+1} \cap s_{2k+1}) = 0$ because $D_1^{k+1} \cap s_{2k+1} \in \tilde{U}_{-1}(BZ_q) = 0$. So: $c \geqq \operatorname{ord} s_{2k-1} = N(k,q)$. Finally: $\operatorname{ord}[S^{2k-1},\sigma] = N(k,q)$. ($\cap$ denotes the cap-product).

Let $\Gamma_m$ be the generalized quaternion group of order $2^m$; $\Gamma = \langle x, y\rangle$ subject to the relations $x^t = y^2$, $xyx = y$, $t = 2^{m-2}$. We refer the reader to [3] section III for the notation and results we shall use in the next proposition. Let $\sigma$ be a unitary fixed point free representation of $\Gamma_m$, $k = \dim\sigma$.

PROPOSITION 1.4. *We have* $\operatorname{ord}[S^{2k-1},\sigma] = 2^{k+m-2}$.

PROOF. We give a proof in the case $m \geqq 4$ (if $m = 3$ it is simpler).
The unitary representations of $\Gamma_m$ of dimension 1 are not fixed point free. Hence $k = \dim\sigma$ must be even: $k = 2k'$. If $\mu$ denotes the Thom homomorphism: $\tilde{U}(B\Gamma_m) \to \tilde{H}(B\Gamma_m)$ then $\mu([S^{2k-1},\sigma])$ is a generator of $\tilde{H}_{2k-1}(B\Gamma_m)$ and consequently there is $p \in Z$ such that:

$$[S^{2k-1},\sigma] = (2p+1)w'_{4k'-1} + \sum_{i\geqq 1} \lambda_i w'_{4(k'-i)-1} + \sum_{i\geqq 1} \lambda_i' u'_{4(k-i)+1} + \sum_{i\geqq 1} \lambda_i'' v'_{4(k-i)+1}$$

(see [3], section III). We have $\operatorname{ord} w'_{4k'-1} = 2^{k+m-2}$, $\operatorname{ord} w'_{4(k'-i)-1} = 2^{k-2i+m-2}$ $\operatorname{ord} u'_{4(k'-i)+1} = \operatorname{ord} v'_{4(k'-i)+1} = 2^{k'-i+1}$, $i \geqq 1$. Hence $2^{k+m-2}[S^{2k-1},\sigma] = 0$, $2^{k+m-3}[S^{2k-1},\sigma] \neq 0$ because $k+m-2 \geqq k-2i+m-2$, $k+m-3 \geqq k'-i+1$, $i \geqq 1$.

## II. Main results.

Let $G$ be a finite solvable group with periodic cohomology and $\sigma$ a fixed point free unitary representation of $G$ of dimension $k$ over $\mathbf{C}$. Let

$$\Theta = \operatorname{ord}[S^{2k-1},{}^{B}\sigma] \text{ in } \tilde{U}_{2k-1}(BG).$$

THEOREM 2.1. *Suppose $G$ of type I, II or III. Then we have respectively* $\Theta = N(k,m)N(k,n)$, $\Theta = N(k,m)N(k,v)2^{k+u-1}$, $\Theta = N(k,m)N(k,n)2^{k+1}$.

PROOF. Theorem 2.1 is a consequence of propositions 1.2, 1.3, 1.4.

a) Suppose $G$ of type I. We have an exact split sequence, $(m,n) = 1$:

$$1 \to \mathbf{Z}_m = \langle A \rangle \to G \to \mathbf{Z}_n = \langle B \rangle \to 1.$$

By propositions 1.2, 1.3, we get $\Theta = N(k,m)N(k,m)$.

b) Suppose $G$ of type II. We have exact split sequences $((\operatorname{ord}\langle R, B^v\rangle, m) = 1$, $(\operatorname{ord}(\langle R, B^v\rangle, v) = 1))$:

$$1 \to \mathbf{Z}_m = \langle A \rangle \to G \to \langle R,B \rangle \to 1$$

$$1 \to \mathbf{Z}_v = \langle B^{2^u} \rangle \to R,B \to \Gamma_{u+1} = \langle R, B^v \rangle \to 1.$$

Then $\Theta = N(k,m)N(k,v)2^{k+u-1}$

c) Suppose $G$ of type III. We use the following exact split sequence, $((\operatorname{ord}\Gamma_3, \operatorname{ord}\langle A,B\rangle) = 1)$:

$$1 \to \Gamma_3 = \langle P,Q \rangle \to G \to \langle A,B \rangle \to 1.$$

Then $\Theta = N(k,m)N(k,n)2^{k+1}$.

If $G$ is of type IV we have the split exact sequences with $n = \operatorname{ord} B = 3^u v$, $(v,6) = 1$:

1) $1 \to \mathbf{Z}_m = \langle A \rangle \to G \to \langle B,P,Q,R \rangle = G_1 \to 1$,

2) $1 \to \mathbf{Z}_v = \langle B^{3^u} \rangle \to G_1 \to \langle B^v,P,Q,R \rangle = G_2 \to 1$

We give some information about $G_2 = \langle B_1,P,Q,R \rangle$, $B_1 = B^v$. We have $\langle P,Q,R \rangle = \Gamma_4$ the generalized quaternion group of order $2^4$ (see [6]). If $x = RP$, $y = R$ we obtain $\langle P,Q,R \rangle = \{x^\alpha y^\beta, 0 \leqq \alpha \leqq 7, \beta = 0,1, x^4 = y^2, xyx = y\}$. Furthermore: $\langle B_1 \rangle = \mathbf{Z}_{3^u}$. We have the following relations: $\{B_1PB_1^{-1} = Q, B_1QB_1^{-1} = PQ, RB_1R^{-1} = B_1^{-1}\}$ or $\{B_1PB_1^{-1} = PQ, B_1QB_1^{-1} = P, RB_1R^{-1} = B_1^{-1}\}$ according as $v \equiv 1\,(3)$ or $v \equiv 2\,(3)$. We consider the first case only the second one being similar. As $H^2(BG_2) = \operatorname{Hom}(G_2, U(1))$ it follows easily that $H^2(BG_2) = Z_2 a, a = c_1(\tau)$ the first Chern-class of the unitary representation $\tau$ of $G_2 : x \to -1, y \to -1, B_1 \to 1$. Moreover well known results by R. Swan (see [4]) show that $H^4(BG_2) = \mathbf{Z}_q g, q = 3^u 2^4$ and $H^*(BG_2)$ is periodic of period 4. As a consequence we have $H_{4n+1}(BG_2) = \mathbf{Z}_2$, $H_{4n+3}(BG_2) = \mathbf{Z}_q$, $q = 3^u 2^4$ and if $i: \Gamma_4 \subset G_2$, $j: Z_3 \subset G_2$ are the natural inclusions then we have: $(i_* + j_*)(\tilde{H}_{4n-1}(B\Gamma_4) \oplus \tilde{H}_{4n-1}(B\mathbf{Z}_{3^u})) = \tilde{H}_{4n-1}(BG_2)$ and $i_*(H_{4n+1}(B\Gamma_4)) = H_{4n+1}(BG_2)$. If $\{w'_{4n+3}, u'_{4n+1}, v'_{4n+1}\}$, $n \geqq 0$ is the system of generators for the $U_*(\mathrm{pt})$-module $\tilde{U}(B\Gamma_4)$ considered in [3] section III, $\{s_{2n+1}\}, n \geqq 0$, the system of generators of the $U_*(\mathrm{pt})$-module $\tilde{U}(B\mathbf{Z}_{3^u})$ described above, $\mu: \tilde{U}_*(BG_2) \to \tilde{H}_*(BG_2)$ the Thom-homomorphism then: $\mu(a_{4n-1})$ with $a_{4n-1} = i_*(w'_{4n-1}) + j_*(s_{4n-1})$ is a generator of $\tilde{H}_{4n-1}(BG_2)$ and there is $b_{4n+1}$ of the set $\{i_*(u'_{4n+1}), i_*(v'_{4n+1})\}$ such that $\mu b_{4n+1})$ is a generator of $\tilde{H}_{4n+1}(BG_2)$ (In fact, we will show in a forthcoming paper that $b_{4n+1} = i_*(v'_{4n+1})$).

THEOREM 2.2. *If G is of type IV then* $\mathrm{ord}\,[S^{2k-1}, \sigma] = N(k,m)N(k,v)2^{k+2}$ $3^{u+k'-1}$, $k = 2k'$ *(k is even)*, $k = \dim \sigma$ *over C*.

PROOF. By restriction to $\Gamma_4 \subset G_2$ we see that $k$ is even by the proof of proposition 1.4. From the split exact sequences 1), 2) we get: $\mathrm{ord}\,[S^{2k-1}, \sigma] = N(k,m)N(k,v)\,\mathrm{ord}\,[S^{2k-1}, \sigma_2]$ where $\sigma_2$ denotes the restriction of $\sigma$ to $G_2$. The Sylow subgroups of $G_2$ are conjugate either to $\Gamma_4 = \langle P, Q, R \rangle$ or to $\langle B \rangle = \mathbf{Z}_{3^u}$. It is easily seen by using the transfer homomorphisms and propositions 1.3, 1.4 that $\mathrm{ord}\,[S^{2k-1}, \sigma_2] \geqq N(k, 3^u)2^{k+2} = 3^{u+k'-1}2^{k+2}$, $k = 2k'$. The remarks made before the statement of theorem 2.2 show that:

$$[S^{2k-1}, \sigma_2] = S^{4k'-1}, \sigma_2] = \sum_{0 \leqq i \leqq k'-1} \lambda_i a_{4(k'-i)-1} + \sum_{0 \leqq i \leqq k'} \lambda'_i b_{(k'-i)+1}$$

As $\mathrm{ord}\, w'_{4n-1} = 2^{2n+1}$, $\mathrm{ord}\, u'_{4n+1} = \mathrm{ord}\, v'_{4n+1} = 2^{n+1}$ (see [3], section III), $\mathrm{ord}\, s_{4n-1} = N(4n, 3^u) = 3^{u+n-1}$ it follows that $\mathrm{ord}\,[S^{2k-1}, \sigma_2] \leqq 2^{k+2}3^{u+k'-1}$. Hence $\mathrm{ord}\,[S^{2k-1}, \sigma_2] = 2^{k+2}3^{u+k'-1}$.

REMARK. The results of this paper will be used in a forthcoming paper about the determination of the $U_*(\mathrm{pt})$-structure of $\tilde{U}_*(BG)$ where $G$ denotes a finite solvable group with periodic cohomology.

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