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ON THE CONVERGENCE OF SEQUENCES OF OPERATORS AND THE CONVERGENCE OF THE SEQUENCE OF THEIR SPECTRA

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Abstract

Let (T_n) be a sequence of bounded linear invertible operators on the Banach space E over C, and assume that the double sequence $(T_n^k)_{n\in\mathsf{N},k\in\mathsf{Z}}$ satisfies a certain growth condition. We prove the following theorem:

If the sequence $(\sigma(T_n))_{n\in\mathsf{N}}$ of the spectra $\sigma(T_n)$ of T_n converges to the singleton $\{1\}$ with respect to the Hausdorff metric on bounded subsets of C, then (T_n) converges uniformly to the identity operator. We also establish a generalization of this result.

1. Introduction

In the following let E be a fixed Banach space over C. Let $(T_n)_{n\in\mathsf{N}}$ be a sequence of bounded linear operators on E which converges uniformly (i.e. with respect to the operator norm) to the identity I on E. Then the sequence $(\sigma(T_n))_{n\in\mathsf{N}}$ of the spectra $\sigma(T_n)$ of T_n converges to $\{1\}$ with respect to the Hausdorff metric which is defined for closed bounded sets A, B in two steps:

Set $\mathrm{dist(A,B)} := \sup_{\mathrm{a}\in\mathrm{A}}(\inf_{\mathrm{b}\in\mathrm{B}} |\mathrm{a}-\mathrm{b}|)$, then

$$d(A,B) := \max(\mathrm{dist(A,B)}, \mathrm{dist(B,A)}).$$

It is the main aim of our paper to prove a certain converse to this result. To this end let us first of all consider the "stationary" case, i.e. let T be a bounded linear operator with $\sigma(T) = \{1\}$. Then in general $T \neq I$. But a very old and famous theorem of Gelfand [4] says that if in addition $(T^k)_{k\in\mathsf{Z}}$ is norm bounded then $T = I$.

This theorem was generalized by many authors; we cite some of them: E. Hille [7] proved the result under the condition that $(\|T^k\|) = o(k)$. If one applies a result of Shilov [5, §41] together with the representation theory of

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Domar and Lindahl [3] then one gets the result under the conditions $(\|T^k\|) = 0(|k|^\alpha)$ for some $\alpha \geq 0$ together with $\liminf_{k\to\pm\infty} \frac{\|T^k\|}{k} = 0$. Atzmon [1] showed that the conditions (1) $\sup\{\|T^k\| : k \geq 0\} < \infty$ and (2) $\lim_{k\to\infty} \frac{\log\|T^{-k}\|}{\sqrt{k}} = 0$ together imply $T = I$. Other generalizations are to be found in [14] and [10]. The up to now most general case is treated in [9, §3]. We shall show that if the sequence (T_n) satisfies uniformly one of the conditions mentioned above and if $(\sigma(T_n))$ converges to $\{1\}$ then (T_n) converges uniformly to I.

Our paper is organized in the following manner: In the second section we recall some important facts on the Beurling algebras $l^1_w(\mathsf{Z})$ which we need in the sequel, in the third section we state and prove our main theorem where as the fourth section is devoted to a generalization of it.

2. Preliminaries

We start by a recapitulation of what we need about the Beurling algebra $l^1_w(\mathsf{Z})$ for a given weight w on Z (for notions not explained here we refer to [11]).

Definition 2.1. (a) A function $w : \mathsf{Z} \to [1, \infty[= \{x \in \mathsf{R} : x \geq 1\}$ is called a *weight* if $w(k + l) \leq w(k)w(l)$ holds for all k, l.

(b) Such a weight is called *nonquasianalytic* (*nqa* for short) if

$$\sum_{k\in\mathsf{Z}} \frac{\log w(k)}{1 + k^2} < \infty$$

For w being a weight the space $\{f \in \mathsf{C}^{\mathsf{Z}} : \sum_{k\in\mathsf{Z}} |f(k)|w(k) < \infty\} =: l^1_w(\mathsf{Z})$ is a subalgebra of $l^1(\mathsf{Z})$ with respect to convolution, which is a Banach algebra when equipped with the norm

$$\|f\| = \sum_{k\in\mathsf{Z}} |f(k)|w(k).$$

If w is nonquasianalytic then it follows from the theory of Domar [2] on general Beurling algebras that $l^1_w(\mathsf{Z})$ is a Wiener algebra with an approximate unit (see [11] for these notions).

Let Γ be the Gelfand space of $l^1_w(\mathsf{Z})$. If w is nonquasianalytic then Γ can be identified with $\mathbf{T} := \{\xi \in \mathsf{C} : |\xi| = 1\}$ by setting $\varphi_\xi(f) = \sum f(k)\xi^k =: \widehat{f}(\xi)$.

If J is an ideal of $\mathscr{A} = l^1_w(\mathsf{Z})$ then $h(J) = \{\xi \in \Gamma : \widehat{f}(\xi) = 0 \text{ for all } f \in J\}$ is called the hull of J. It is always a closed subset of Γ. If conversely $\Delta \subset \Gamma$ is a closed set then we set $k(\Delta) = \{f \in \mathscr{A} : \widehat{f}(\Delta) = \{0\}\}$ and $m(\Delta) =$

$\overline{\{f \in \mathscr{A} : f \text{ vanishes on an open set containing } \Delta\}}$. $m(\Delta)$ is the smallest closed ideal J satisfying $h(J) = \Delta$, and $k(\Delta)$ is the largest one.

Definition 2.2 (cf. [9, Definition 3.3]). The *nqa*–weight w is called a *weight of uniqueness* (U–weight for short) if $m(\{\xi\}) = k(\{\xi\})$ for every $\xi \in \Gamma$.

Remark 2.3. A closed ideal J is called *primary* if the hull $h(J)$ is a singleton. w is an U–weight if every primary ideal is maximal.

Concerning concrete examples we mention the following ones:

Theorem 2.4. (a) *Let* $(w(k)) = 0(|k|^{\alpha})$ *for some* $\alpha \geq 0$ *and in addition let* $\liminf_{k\to\pm\infty} \frac{w(k)}{k} = 0$. *Then* w *is an* U*–weight.*

(b) *Assume that*

$$\lim_{n\to\infty}\left(\frac{w(n)}{n^{\alpha}} + \frac{\log(n)}{\sqrt{n}}\right) = 0 \text{ for some } \alpha \geq 0.$$

If $0 \leq \alpha < 1$ *or* $\liminf_{n\to\pm\infty} w(n) < \infty$ *then* w *is an* U*–weight.*

Remark 2.5. Part (a) is Shilov's theorem mentioned in the introduction. The corresponding assertion for $\alpha < 1$ in part (b) was announced in [6, Theorem 8.1 (i)], and a complete proof of it was given in [1]. The other assertion of part (b) is due to the first author [9, Prop. 3.12].

We shall make heavy use of results of Domar and Lindal [3] specialized to our situation of representations of $l^1_w(\mathsf{Z})$.

Let T be a bounded linear invertible operator on the Banach space E. Then $w(k) := \max(1, \|T^k\|)$ defines a weight. Let $Uf := \sum f(k)T^k$ for $f \in \mathscr{A} := l^1_w(\mathsf{Z})$. Then U is a contractive representation of $\mathscr{A}$ in $L(E)$. We denote by $\ker(U)$ its kernel $U^{-1}(\{0\})$.

Theorem 2.6. (cf. [3, Theorem 6.7], [13, Prop. 3.6], [8, Prop. 1.3.8]) *Let* w *be nonquasianalytic. Then* $h(\ker(U)) = \sigma(T)$ *where* $\sigma(T)$ *denotes the spectrum of* T.

This theorem gives us back our results in the first section concerning the "$T = I$" problem.

Corollary 2.7 (cf. [9, Theorem 3.10]). *Let* w *be an* U*–weight. Then* $\sigma(T) = \{1\}$ *implies* $T = I$.

Proof. Since w is an U–weight and $\sigma(T) = \{1\}$, the kernel of U has codimension 1. It follows that $\mathscr{A}/\ker(U) = \mathsf{C}1$ and the assertion follows from [13], Remark 3.7 (2) to Proposition 3.6.

3. The main result

THEOREM 3.1. *Let $(T_n)_{n\in\mathsf{N}}$ be a sequence of bounded linear invertible operators on the Banach space E. Assume that there exists an U–weight w on Z such that $\|T_n^k\| \le w(k)$ for all $n \in \mathsf{N}$ and $k \in \mathsf{Z}$. If the sequence $(\sigma(T_n))$ of the spectra $\sigma(T_n)$ of T_n converges to $\{1\}$ (with respect to the Hausdorff metric) then (T_n) converges uniformly to the identity.*

In order to prove this theorem and related results we use the theory of ultraproducts. (Compare the following paragraph with the introduction of section 2 of [12])

Let $\mathscr{U} \subset \mathscr{P}(\mathsf{N})$ be a free ultrafilter and denote by $m_{\mathscr{U}}$ the finitely additive $\{0,1\}$–valued measure on N, given by $m_{\mathscr{U}}(A) = 1$ iff $A \in \mathscr{U}$. Note that for such a measure the intersection of finitely many sets of measure 1 has also measure 1.

Denote by E_∞ the Banach space of all bounded sequences $\tilde{x} = (x_n)_{n\in\mathsf{N}}$ with $x_n \in E$ for all n, equipped with the norm $\|\tilde{x}\| = \sup_n \|x_n\|$. Then $E_0 := \{\tilde{x} : \lim_{\mathscr{U}} \|x_n\| = 0\}$ is a closed subspace of E_∞. The quotient $\widehat{E} := E_\infty/E_0$ is called the *ultrapower* of E with respect to $\mathscr{U}$. The norm on $\widehat{E}$ is given by $\|\widehat{y}\| = \|\tilde{y} + E_0\| = \lim_{\mathscr{U}} \|y_n\|$. E is isometrically embedded into $\widehat{E}$ by means of $x \to (x, x, x, \ldots) + E_0$.

Every bounded sequence (S_n) of operators S_n on E defines an operator $\tilde{S}$ on E_∞ by $\tilde{S}\tilde{x} = (S_n x_n)_{n\in\mathsf{N}}$, with norm given by $\|\tilde{S}\| = \sup_n \|S_n\|$. So E_0 is invariant for $\tilde{S}$, and we obtain a uniquely defined operator $\widehat{S}$ on $\widehat{E}$ given by $\widehat{S}\widehat{y} = \tilde{S}\tilde{y} + E_0$. Moreover, $\|\widehat{S}\| = \lim_{\mathscr{U}} \|S_n\|$. If (R_n) is another bounded sequence of operators R_n on E_n such that $R_n = S_n$ $m_{\mathscr{U}}$-a.e., then $\widehat{R} = \widehat{S}$. Thus each subfamily $(S_n)_{n\in M}$ with $\mu_{\mathscr{U}}(M) = 1$ defines in a canonical way an operator on $\widehat{E}$ which coincides with $\widehat{S}$. It should cause no confusion if we denote the operator on $\widehat{E}$ induced by $(S_n)_{n\in M}$ also by $\widehat{S} = \widehat{(S_n)}$. If T is a bounded operator on E then the constant sequence $(T, T, \ldots)$ defines the extension $\widehat{T}$ of T on $\widehat{E}$. The mapping $T \to \widehat{T}$ is an isometric embedding of the algebra $\mathscr{L}(E)$ into $\mathscr{L}(\widehat{E})$.

It will become clear from the context whether $\widehat{S}$ denotes an operator coming from a constant sequence or from an arbitrary sequence. But one should keep in mind the following two different notations: For $(S_n)_{n\in\mathsf{N}}$ being a bounded sequence of operators we consider not only $\widehat{S} = \widehat{(S_n)}$ but also each individual operator $\widehat{S}_n$, defined through the constant sequence $(S_n, S_n, S_n, \ldots)$.

PROPOSITION 3.2. *Let w be a nonquasianalytic weight on Z. Let (T_n) be a sequence of bounded linear invertible operators on E satisfying $\|T_n^k\| \le w(k)$*

for all $n \in \mathsf{N}$ *and* $k \in \mathsf{Z}$. *Let finally* $A \subset \mathsf{T}$ *be closed. Then* $\lim_{n\to\infty} d(\sigma(T_n), A) = 0$ *implies* $\sigma(\widehat{T}) = A$.

PROOF. (I) The assumption $\|T_n^k\| \le w(k)$ for all k and n implies $\|\widehat{T}^k\| \le w(k)$. Consider the representation U of $\mathscr{A} := l_w^1(\mathsf{Z})$ in $\mathscr{L}(\widehat{E})$ given by $Uf := \sum f(k)\widehat{T}^k$. Then Theorem 2.6 implies that $\sigma(\widehat{T}) = h(\ker(U))$.

(II) For fixed $n \in \mathsf{N}$ we also consider $U_n : f \in \mathscr{A} \to U_n f := \sum f(k)\widehat{T}_n^k$. Then Theorem 2.6 combined with [13, Theorem 3.4] yields that $\sigma(T_n) = \sigma(\widehat{T}_n) = h(\ker(U_n))$.

(III) Now let $\lambda \in \mathsf{T} \setminus A$ be given and set $\varepsilon := \mathrm{dist}(\lambda, \mathrm{A})/2$. Then there exists n_0 such that for $n \ge n_0$ the spectrum $\sigma(T_n)$ is contained in $C := \{\xi : \mathrm{dist}(\xi, \mathrm{A}) \le \varepsilon\}$

Since $m(C)$ is the smallest closed ideal J in $\mathscr{A}$ with $h(J) = C$, and since $\sigma(T_n) \subset C$ we obtain $m(C) \subset \ker(U)_n =: J_n$ by (II). Set

$$f = \lambda\delta_0 - \delta_1 \text{ where } \delta_k(r) = \begin{cases} 1, & r = k, \\ 0, & r \in \mathsf{Z} \setminus \{k\} \end{cases}$$

Then $\widehat{f}(\xi) = 0$ implies $\lambda = \xi$, hence $\widehat{f}$ does not vanish on C. Since $\mathscr{A}$ is a Wiener algebra, in particular regular, there exists $g \in \mathscr{A}$ such that $\widehat{f}\widehat{g} = \mathbf{1}$ on C. This in turn implies that $\tilde{f} := f + m(C)$ is invertible in the quotient algebra $\mathscr{A}/m(C)$. Let $\alpha := \|g + m(C)\|$ be the quotient norm of $\tilde{g} := g + m(C)$.

(IV) Since $m(C) \subset J_n$, $g + J_n$ is the inverse of $f + J_n$ in $\mathscr{A}/J_n$ and $\|g + J_n\| \le \alpha$. Since U_n is obviously contractive, $U_n g$ is the inverse of $(\lambda - T_n)$ and $\|U_n g\| \le \alpha$, or in other words $\lambda \in \rho(T_n)$ $(= \mathsf{C}\backslash\sigma(T_n))$ and $\|(\lambda - T_n)^{-1}\| \le \alpha$. This holds for all $n \ge n_0$ (see (III) above). But then $\lambda \in \rho(\widehat{T})$ by [12], Lemma 2.1.

(V) Let conversely $\lambda \in A$ be arbitrary. Then by assumption to every n there exists $\lambda_n \in \sigma(T_n)$ such that $\lim_{n\to\infty} \lambda_n = \lambda$ holds. Assume now that $\lambda \in \rho(\widehat{T})$. Then by Lemma 2.1 of [12] there exists $\delta > 0$ such that $\lambda \in \rho(T_n)$ and $\|(\lambda - T_n)^{-1}\| < \delta$ holds $m_{\mathscr{U}}$–a.e. To this δ there exists $n_0 \in \mathsf{N}$ such that $|\lambda_n - \lambda| < 1/\delta$ for all $n \ge n_0$. But this implies $|\lambda_n - \lambda| < \frac{1}{\|(\lambda - T_n)^{-1}\|}$ $m_{\mathscr{U}}$–a.e. From the equation

$$(\lambda_n - T_n) = (\lambda - T_n)(I - (\lambda - \lambda_n)(\lambda - T_n)^{-1})$$

we obtain that $\lambda_n \in \rho(T_n)$ holds m_U–a.e, a contradiction.

Proof of 3.1: If the assertion fails there exists a free ultrafilter $\mathscr{U}$ such that $\lim_{\mathscr{U}} \|T_n - I\| > 0$, or in other words, such that $\widehat{T} \ne I$ holds. But $\lim_{n\to\infty} d(\sigma(T_n), \{1\}) = 0$ implies $\sigma(\widehat{T}) = \{1\}$ by Proposition 3.3. Now from $\|T_n^k\| \le w(k)$ for all $n \in \mathsf{N}$ and $k \in \mathsf{Z}$ we obtain $\|\widehat{T}^k\| \le w(k)$ for all $k \in \mathsf{Z}$ (see part (I) of the proof of Prop. 3.3). Now w is an U–weight by hypothesis, so $\widehat{T} = I$, a contradiction.

4. A generalization

In this section we consider the convergence of $(\sigma(T_n))$ to a finite set $\{\lambda_1, \dots, \lambda_r\} = A$. For the formulation of our result we need the following definition:

DEFINITION 4.1. A *decomposition of the identity operator I* on the Banach space E is a finite set $\{P_1, \dots, P_r\}$ of pairwise disjoint nonvanishing projections which add up to I, i.e. $\sum_{k=1}^{r} P_k = I$.

THEOREM 4.2. *Let w be an U–weight on Z and let (T_n) be a sequence of bounded linear invertible operators such that $\|T_n^k\| \le w(k)$ for all $n \in \mathsf{N}$ and $k \in \mathsf{Z}$. Assume that $\lim_{n\to\infty} d(\sigma(T_n), A) = 0$ for some finite set $A = \{\lambda_1, \dots, \lambda_r\}$. Then there exists n_0 such that to each $n \ge n_0$ there exists a partition $\{P_{1n}, \dots, P_{rn}\}$ of I satisfying* $\lim_{n\to\infty} \|T_n - \sum_{k=1}^{r} \lambda_k P_{k,n}\| = 0$

REMARK 4.3. Note that the sequence $(\sum_{k=1}^{r} \lambda_k P_{k,n})$ itself needs not converge as easy examples show.

PROOF. (I) Let $\delta := \inf\{|\lambda_i - \lambda_j| : i \ne j\}/8$ and let $n_0 \in \mathsf{N}$ be chosen so that $d(\sigma(T_n), A) < \delta$ for all $n \ge n_0$. Then, we have for $D_{k,n} := \{\lambda \in \sigma(T_n) : |\lambda - \lambda_k| < \delta\}$ that

(i) $D_{k,n} \cap D_{l,n} = \emptyset$ for $k \ne l$; and

(ii) $\sigma(T_n) = \bigcup_{k=1}^{r} D_{k,n}$.

(II) Let $P_{k,n}$ be the spectral projection corresponding to $D_{k,n}$, i.e.,

$$P_{k,n} = \frac{1}{2\pi i} \int_{|\xi - \lambda_k| = 2\delta} (\xi - T_n)^{-1}\, d\xi.$$

The set $\{P_{k,n} : k = 1, \dots, r\}$ is a partition of I, and we want to prove that $(T_n - \sum_{k=1}^{r} \lambda_k P_{k,n})$ converges to 0.

(III) We use an ultrapower $\widehat{E}$ of E with respect to an arbitrary free ultrafilter. Proposition 3.3 yields $\sigma(\widehat{T}) = \{\lambda_1, \dots, \lambda_r\}$. Let Q_j be the spectral projection corresponding to λ_j; moreover set $F_j := Q_j(\widehat{E})$. Then $\bar{\lambda}_j \widehat{T}|_{F_j} = I$ by Theorem 2.6 since $\|(\bar{\lambda}_j \widehat{T})^k\| \le w(k)$ (cf. the proof of 3.1). Hence $\widehat{T} = \sum \lambda_j Q_j$.

(IV) Set $C_k := \{\xi : |\xi - \lambda_k| \le \delta\}$ and $C := \bigcup_{k=1}^{r} C_k$.

As in the proof of Proposition 3.3 we have $m(C) \subset \ker(U_n) =: J_n$ at least for $n \geq n_0$. The function $\xi \to g_\xi := \xi\delta_0 - \delta_1$ has the property that $\widehat{g}_\xi$ does not vanish on C, hence $\tilde{g}_\xi =: g_\xi + m(C)$ is invertible and uniformly continuous (as a function of ξ). Since $m(C) \subset J_n$ for $n \geq n_0$ and since $U_n : l^1_w(\mathsf{Z}) \to \mathscr{L}(E)$ $(U_n f := \sum_{-\infty}^{\infty} f(k) T_n^k)$ is a contraction we obtain for the induced representation $\overline{U_n} : l^1_w(\mathsf{Z})/J_n \to \mathscr{L}(E)$ that $(\xi - T_n)^{-1} = \overline{U_n}((g_\xi + J_n)^{-1})$ is uniformly equicontinuous. Thus for each $\varepsilon > 0$ there exists a partition $\{\xi_1, \ldots \xi_m\}$ of the circle C_k such that

$$\left\| P_{k,n} - \frac{1}{2\pi} \sum (\xi_j - T_n)^{-1} (\xi_{j+1} - \xi_j) \right\| < \varepsilon/2 \text{ uniformly in } n \geq n_0.$$

Passing to $\widehat{E}$ we obtain $\|Q_k - \frac{1}{2\pi}\sum(\xi_j - \widehat{T})^{-1}(\xi_{j+1} - \xi_j)\| \leq \varepsilon/2$ as well as $\|\widehat{P}_k - \frac{1}{2\pi}\sum(\xi_j - \widehat{T})^{-1}(\xi_{j+1} - \xi_j)\| \leq \varepsilon/2$. Since $\varepsilon > 0$ was arbitrary we get $Q_k = \widehat{P}_k$, hence $(\widehat{T} - \sum \lambda_j \widehat{P}_k) = 0$. Since the free ultrafilter $\mathscr{U}$ was chosen arbitrarily the assertion follows.

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