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ON COMPACTIFICATIONS OF INFINITE-DIMENSIONAL SPACES

VITALIJ A. CHATYRKO

Abstract

For every separable metrizable space X with $\operatorname{trind} X \neq \infty$ there exists a countable ordinal number $\beta(X) \geq \operatorname{trind} X$ such that for every countable ordinal number $\gamma \geq \beta(X)$ there exists a compactification $c_\gamma X$ of the space X with $\operatorname{trind} c_\gamma X = \gamma$ ($\beta(X) = \operatorname{trind} X$, if $\operatorname{trInd} X \neq \infty$).

0. Introduction

Throughout this note we shall consider only separable metrizable spaces. The necessary information about notions and notations we use can be found in [AP], [E].

It is well known the following Hurewicz's result (see for example [AP]):
(*) *for every space X with* $\operatorname{ind} X = n$ *there exists a compactification cX of the space X with* $\operatorname{ind} cX = n, n = 0, 1, 2, ...$

It is known (see [E]) that
(**) *for every space X with* $\operatorname{trind} X \neq \infty$ *there exists a compactification cX with* $\operatorname{trind} cX \neq \infty$ (trind *is the transfinite extension of the dimension function* ind).

However the exact extension of proposition (*) to the transfinite case is impossible. In [Lu1] Luxemburg has proved that for any limit ordinal number $\alpha : \omega \leq \alpha < \omega_1$ there exists a complete strongly countable-dimensional space X_α with $\operatorname{trind} X_\alpha = \alpha$ such that for every compactification cX_α of the space X_α we have $\operatorname{trind} cX_\alpha > \operatorname{trind} X_\alpha$ (by definition we assume $\infty > \alpha$ for every ordinal number α).

Recall that $\operatorname{trind} Z \leq \operatorname{trind} Y$, if $Z \subseteq Y$.

In [E] Engelking has remarked the following open

Problem. *Evaluate the increase of* trind *in the process of compactifying a separable metrizable space.*

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One of the results of this paper is

THEOREM 1. *For every space X with* $\operatorname{trind} X \neq \infty$ *there exists a countable ordinal number* $\beta(X) \geq \operatorname{trind} X$ *such that for every countable ordinal number* $\gamma \geq \beta(X)$ *there exists a compactification* $c_\gamma X$ *of the space X with* $\operatorname{trind} c_\gamma X = \gamma$. *Moreover, if* $\operatorname{trInd} X \neq \infty$, *then* $\beta(X) = \operatorname{trind} X$ (trInd *is the transfinite extension of the dimension function* Ind).

Note that for every space X with $\operatorname{trind} X \neq \infty$ we have $\operatorname{trind} X < \omega_1$ [AP].

1. The case of the locally compact noncompact spaces

Let X, Y be topological spaces. The notation $X \simeq Y$ will mean that the spaces X and Y are homeomorphic and the notation $X \hookrightarrow Y$ will mean that the space X is homeomorphic to a subset of the space Y. Let $X \subset Y$. The notation $[X]_Y$ will mean the closure of the space X in the space Y.

We shall need the following Theorem 2 which is a corollary from a fact established by Aarts and van Emde Boas [AE]. For the sake of completeness, let us outline its proof.

Let X be a locally compact noncompact space and $bX = X \cup \{p\}$ be the one-point compactification of the space X, where p is the compactification point.

It is evident that there exists a continuous function $f : bX \longrightarrow I = [0, 1]$ such that $f^{-1}\{0\} = p$. Put

$$X_f = \{(x, f(x)) : x \in X\} \subset X \times I, bX_f = \{(x, f(x)) : x \in bX\} \subset bX \times I.$$

Note that $X \simeq X_f$ and $bX \simeq bX_f$. Let $\mathrm{pr}_I : bX \times I \longrightarrow I$ be the projection of the compact space $bX \times I$ onto the closed interval I. It is easy to see that there exists a sequence $\{c_n\}_{n=1}^\infty$ of points from X_f with $\lim_{n\to\infty}(c_n) = \{p\} \times \{0\}$ such that $x_{n+1} < x_n$ for any $n \in \mathsf{N}$ and $\lim_{n\to\infty}(x_n) = 0$, where $x_n = \mathrm{pr}_I(c_n)$.

Let A be a nondegenerated AR-compactum and $S = \{u_n\}_{n=1}^\infty$ be a countable everywhere dense subset of A. Define the mapping $h : \{x_n\}_{n=1}^\infty \longrightarrow A$ as follows: $h(x_n) = u_n$ for any $n \in \mathsf{N}$. Let $g : (0, 1] \longrightarrow A$ be a continuous extension of the mapping h. Put

$$W = bX \times I \times A, Z = \{(x, f(x), g(f(x))) : x \in X\} \subset X \times (0, 1] \times A \subset W$$

It is evident that $X \simeq Z$, $[Z]_W = Z \cup (\{p\} \times \{0\} \times A)$ and $[Z]_W \setminus Z \simeq A$. Denote $[Z]_W = K[X, A]$.

We have proved

THEOREM 2. *Let X be a locally compact noncompact space and A be a non-*

degenerated AR-*compactum. Then there exists a compactification cX of the space X such that $cX \setminus X \simeq A$.*

We will say here that a dimension function F is monotone , if for every space X and any its closed subset Y we have $FY \leq FX$.

We will say that a dimension function F is ω_1-bounded, if for any space X we have $FX < \omega_1$ or $FX = \infty$.

Let X be a compact space and Y be a closed subset in X with $FY \neq \infty (FY = \beta)$, where F is a dimension function. Moreover, let for every closed subset $Q \subset X$ such that $Q \cap Y = \emptyset$, we have $FQ \neq \infty$ ($FQ \leq \alpha$ and $\beta \geq \alpha$). If $FX \neq \infty$ ($FX \leq \alpha + \beta$), then we will say that the dimension function F has the (strong) Dowker property.

Corollary 1. *Let F be a monotone ω_1-bounded dimension function which has the Dowker property. Moreover, let $\sup\{FP^\alpha : \alpha < \omega_1\} = \omega_1$, where $P^\alpha, \alpha < \omega_1$, are* AR-*compacta. Then for every locally compact noncompact space X such that $FQ \neq \infty$ for any compactum $Q \subset X$ we have $\sup\{FcX : cX$ is a compactification of space X with $FcX \neq \infty\} = \omega_1$.*

Recall [KM] that any ordinal number α can be uniquely represented as $\alpha = \omega^{\eta_1} \cdot n_1 + ... + \omega^{\eta_k} \cdot n_k$, where $n_i \in \mathsf{N}$ and $\eta_1 > ... > \eta_k \geq 0$ are ordinal numbers. Note that for every ordinal number $\beta \geq \omega^{\eta_1+1}$, we have $\alpha + \beta = \beta$.

Corollary 2. *Let F be a monotone dimension function which has the strong Dowker property. Moreover, let for every countable ordinal number γ there exists an* AR-*compactum A^γ with $FA^\gamma = \gamma$. Then for every locally compact noncompact space X such that $FQ \leq \alpha$ for every compactum $Q \subset X$, and for any ordinal number $\gamma : \alpha \leq \gamma < \omega_1$ we have $\gamma \leq F(K[X, A^\gamma]) \leq \alpha + \gamma$. In particular, if $\alpha = \omega^{\eta_1} \cdot n_1 + ... + \omega^{\eta_k} \cdot n_k$, where $n_i \in \mathsf{N}$ and $\eta_1 > ... > \eta_k \geq 0$ are ordinal numbers, then for every countable ordinal number $\beta \geq \omega^{\eta_1+1}$, we have $F(K[X, A^\beta]) = \beta$.*

Recall [E] the definitions of dimension functions trind, trInd, D, trdim which are different transfinite extensions of the finite dimension *dim* in the class of separable metrizable spaces.

Let X be a space. Define

(i) $\operatorname{trind} X = -1 \Leftrightarrow X = \emptyset$;

(ii) $\operatorname{trind} X \leq \alpha$, where α is an ordinal number, if for every point $x \in X$ and each neighborhood V of the point x there exists an open set $U \subset X$ such that $x \in U \subset V$ and $\operatorname{trind} \operatorname{Fr} U < \alpha$;

(iii) $\operatorname{trind} X = \alpha$ if $\operatorname{trind} X \leq \alpha$ and the inequality $\operatorname{trind} X \leq \beta$ holds for no $\beta < \alpha$;

(iv) $\operatorname{trind} X = \infty$ if $\operatorname{trind} X \leq \alpha$ holds for no ordinal number α.

The definition of trInd one can get through the substitution of the point x in (ii) from the definition above with a closed subset of the space X.

Observe that for each ordinal number α there exist a uniquely determined limit number $\lambda(\alpha) \geq 0$ and an integer $n(\alpha) \geq 0$ such that $\alpha = \lambda(\alpha) + n(\alpha)$.

We let $\mathrm{D}(\emptyset)$ = -1, and for every non-empty space X we define $\mathrm{D}(X)$ as the smallest ordinal number α such that there exists a closed cover $\{A_\beta\}_{\beta\leq\lambda(\alpha)}$ of the space X satisfying the following conditions:

(D1) The union $\cup\{A_\beta : \delta \leq \beta \leq \lambda(\alpha)\}$ is closed for every $\delta \leq \lambda(\alpha)$;

(D2) For every $x \in X$ the set $\{\beta \leq \lambda(\alpha) : x \in A_\beta\}$ has a largest element;

(D3) $\dim A_\beta < \infty$ for every $\beta < \lambda(\alpha)$, and $\dim A_{\lambda(\alpha)} \leq n(\alpha)$;

if no such ordinal number exists, we let $\mathrm{D}(X) = \infty$.

It is clear that $\mathrm{D}Z \leq \mathrm{D}Y$, if $Z \subseteq Y$.

Let L be an arbitrarary set. By $\mathrm{Fin}\, L$ we shall denote the collection of all finite, non-empty subsets of L. Let M be a subset of $\mathrm{Fin}\, L$. For $\sigma \in \{\emptyset\} \cup \mathrm{Fin}\, L$ we put $M^\sigma = \{\tau \in \mathrm{Fin}\, L| \quad \sigma \cup \tau \in M$ and $\sigma \cap \tau = \emptyset\}$. Let $M^a = M^{\{a\}}$.

Define the ordinal number $\mathrm{Ord}\, M$ inductively as follows

(i) $\mathrm{Ord}\, M = 0$ iff $M = \emptyset$,

(ii) $\mathrm{Ord}\, M \leq \alpha$ iff for every $a \in L$ $\mathrm{Ord}\, M^a < \alpha$,

(iii) $\mathrm{Ord}\, M = \alpha$ iff $\mathrm{Ord}\, M \leq \alpha$ and $\mathrm{Ord}\, M < \alpha$ is not true, and

(iv) $\mathrm{Ord}\, M = \infty$ iff $\mathrm{Ord}\, M > \alpha$ for every ordinal number α.

Let X be a non-empty space. A finite sequence $\{(A_i, B_i)_{i=1}^m\}$ of pairs of disjoint closed sets in the space X is called inessential if we can find open sets $O_i, i = 1, ..., m$ such that $A_i \subset O_i \subset [O_i]_X \subset X \setminus B_i$ and $\cap_{i=1}^m \mathrm{Fr}\, O_i = \emptyset$. Otherwise it is called essential.

Put $L(X) = \{(A, B)| \quad A, B \subset X,$ closed, disjoint$\}$ and $M_{L(X)} = \{\sigma \in \mathrm{Fin}\, L(X)| \quad \sigma$ is essential in X $\}$.

We let $\mathrm{trdim}\,(\emptyset) = -1$, and for every non-empty space X we define $\mathrm{trdim}\, X = \mathrm{Ord}\, M_{L(X)}$.

Note that the dimension functions trind, trInd, D, trdim are monotone, ω_1-bounded and they have the strong Dowker property (for the dimensions trind, trInd about the strong Dowker property see for example [B1], for D – [He1], for trdim – [Ha]).

In [He2] Henderson have constructed AR-compacta H^α, $\alpha < \omega_1$, and have proved that $\mathrm{trInd}\, H^\alpha = \alpha$,$\alpha < \omega_1$. Observe that $FH^\alpha = \alpha, \alpha < \omega_1$, for $F = \mathrm{D}$ (see [Ch]), trdim (see [B1]). Moreover, from Levshenko's inequality [Le] $\mathrm{trInd}\, X \leq \omega \cdot \mathrm{trind}\, X$, which is true for any compact space X, we have $\sup\{\mathrm{trind}\, H^\alpha : \alpha < \omega_1\} = \omega_1$.

Recall the construction of Henderson's AR-compacta $H^\alpha, \alpha < \omega_1$ [He2]. Let $H^1 = I = [0, 1], p_1 = \{0\} \in I$. Assume that for every $\beta < \alpha$ the compacta

H^β and the points $p_\beta \in H^\beta$ have already been defined. If $\alpha = \beta + 1$, then we set $H^{\beta+1} = H^\beta \times I$ and $p_\alpha = (p_\beta, 0)$. If α is a limit ordinal number, then K_β is the union of the H^β and a half-open arc A_β such that $A_\beta \cap H^\beta = \{p_\beta\} = \{$ endpoint of the arc $A_\beta\}, \beta < \alpha$. Let us define H^α as the one-point compactification of the free sum $\bigoplus_{\beta<\alpha} K_\beta$ and let p_α be the compactification point.

Recall also [T] that $\operatorname{trind} X \times I \leq \operatorname{trind} X + 1$, for any space X.

Now it is easy to note that one can choose from the collection $\{H^\alpha : \alpha < \omega_1\}$ a new collection $\{P^\alpha : \alpha < \omega_1\}$ such that for every ordinal number $\alpha < \omega_1$ we have $\operatorname{trind} P^\alpha = \alpha$.

Remark 1. *The dimension functions* trind, trInd, D, trdim *satisfy the conditions of Corollary 1, 2.*

2. The general case

Theorem 3. *Let X be a noncompact space and c_1X be a compactification of the space X. Then for every nondegenerate* AR*-compact space A and any point $p \in c_1X \setminus X$ there exists a compactification cX of the space X such that $cX \hookleftarrow c_1X \setminus \{p\}$ and $cX \setminus X \hookleftarrow A$.*

Proof. Denote $X_1 = c_1X \setminus \{p\} \hookleftarrow X$. Then $cX = K[X_1, A]$.

Corollary 3. *Let F be a monotone ω_1-bounded dimension function which has the Dowker property. Moreover, let $\sup\{FP^\alpha : \alpha < \omega_1\} = \omega_1$, where $P^\alpha, \alpha < \omega_1$, are* AR*-compacta. Then for every noncompact space X such that X has a compactification c_1X with $Fc_1X \neq \infty$ we have $\sup\{FcX : cX$ is a compactification of space X with $FcX \neq \infty\} = \omega_1$.*

Corollary 4. *Let F be a monotone dimension function which has the strong Dowker property. Moreover, let for every countable ordinal number γ there exists an* AR*-compactum A^γ with $FA^\gamma = \gamma$. Then for every noncompact space X, such that X has a compactification cX with $FcX = \alpha$, and for any ordinal number $\gamma : \alpha \leq \gamma < \omega_1$ there exists a compactification $c_\gamma X$ with $\gamma \leq Fc_\gamma X \leq \alpha + \gamma$. In particular, if $\alpha = \omega^{\eta_1} \cdot n_1 + ... + \omega^{\eta_k} \cdot n_k$, where $n_i \in \mathbb{N}$ and $\eta_1 > ... > \eta_k \geq 0$ are ordinal numbers, then for every countable ordinal number $\beta \geq \omega^{\eta_1+1}$ we have $Fc_\beta X = \beta$.*

Remark 2. *The dimension functions* trind, trInd, D, trdim *satisfy the conditions of Corollary* 4.

For any space X we will denote by $P(X)$ a closed subset of the space X such that $X \setminus P(X)$ is the union of all finite-dimensional sets, open in X.

In [Lu1] Luxemburg has proved that

(***) *for any space* X *with* $\operatorname{trInd} X \neq \infty$ *the set of all homeomorphisms* $f : X \to I^\omega$ *of the space* X *to the Hilbert cube* I^ω *such that the equalities*

(a) $FX = F([fX]_{I^\omega})$,

(b) $PX = P([fX]_{I^\omega})$,

where F *is one of the dimension functions* trind, trInd, D, *are satisfied contains an everywhere dense set of type* G_δ *in the space* $C(X, I^\omega)$. *In particular, there exists a compactification* cX *with* $FcX = FX$ *and* $P(cX) = PX$, *where* $F = \mathrm{trind}$, trInd *or* D.

Kimura [Ki] has proved the same for trdim.

Let F be one of the dimension functions trind, trInd, trdim or D. Recall (see for example [E] and [B1] for trdim) that if a space X can be represented as the union of two closed subspaces B_1 and B_2 such that $FB_i \leq \alpha \geq \omega_0$ for $i = 1, 2$ and the subspace $B_1 \cap B_2$ is finite-dimensional, then $FX \leq \alpha$.

Theorem 4. *Let* X *be a noncompact space with* $\operatorname{trInd} X \neq \infty$ *and* F *be one of the dimension functions* trind, trInd, trdim *or* D. *Then for every countable ordinal number* $\alpha \geq FX$ *there exists a compactification* $c_{\alpha,F}X$ *such that* $Fc_{\alpha,F}X = \alpha$.

Proof. Let α be a countable ordinal number $\geq FX$ and let cX be a compactification of the space X such that $FcX = FX$ and $P(cX) = PX$ (see (***)). Consider a point $p \in cX \setminus X$. Observe that there exists an open finite-dimensional set $U \subset cX$ such that $p \in U$. Let A be an AR-compact with $FA = \alpha$. Set $c_{\alpha,F}X = K[cX \setminus \{p\}, A]$. Note that the compactification $c_{\alpha,F}X$ of the space X can be represented as the union of two closed subspaces B_1 and B_2 such that $FB_1 \leq FA = \alpha$, $FB_2 \leq FX \leq \alpha$ and the subspace $B_1 \cap B_2$ is finite-dimensional. Consequently $Fc_{\alpha,F}X = \alpha$.

Theorem 1 follows proposition (**), Corollary 4 and Theorem 4. The same statements hold for dimensions trInd, D, trdim (see part 4.).

3. Examples

Let L be the space of irrational numbers.

Observe that

(i) $L \times L \simeq L$;

(ii) $\bigoplus_{n=1}^{\infty} L_n \simeq L$, where $\bigoplus_{n=1}^{\infty} L_n$ is the free sum of the spaces $L_n \simeq L$, $n = 1, 2, \ldots$;

(iii) $\operatorname{ind} L = 0$.

Recall the construction of Smirnov's compacta $S^\alpha, \alpha < \omega_1$ [S]. Let S^0 be the one-point space. Assume that for every $\beta < \alpha$ the compacta S^β have already been defined. If $\alpha = \beta + 1$, then we set $S^{\beta+1} = S^\beta \times I$. If α is a limit ordinal number, then let us define S^α as the one-point compactification of the free sum $\bigoplus_{\beta<\alpha} S_\beta$, where p_α is the compactification point.

Note that

a) if $\{\alpha_i\}_{i=1}^\infty$ is a sequence of ordinal numbers such that $\alpha_i < \alpha_{i+1}$ and $\sup_i \alpha_i = \alpha < \omega_1$, then $S^\alpha \hookrightarrow \{b\} \cup \bigoplus_{i=1}^\infty S^{\alpha_i} \hookrightarrow S^\alpha$, where $\{b\} \cup \bigoplus_{i=1}^\infty S^{\alpha_i}$ is the one-point compactification of the free sum $\bigoplus_{i=1}^\infty S^{\alpha_i}$ and b is the compactification point (see [Ch]).

b) if $[X_1]_X = X$ and $[Y_1]_Y = Y$, then $[X_1 \times Y_1]_{X\times Y} = X \times Y$.

Let $i : L \hookrightarrow C$ be an embedding of the space L to the Cantor set C. Denote $c_0 L = [iL]_C$. Let M be the irrational numbers of the interval $(0, 1)$. Observe that $M \simeq L$. Denote $c_1 L = I = [0, 1]$.

It is easy to note that $c_0 L$ is a zero-dimensional compactification of L and $c_1 L$ is a one-dimensional compactification of L. Let $c_\alpha L = c_\beta L \times I$ for $\alpha = \beta + 1$. If α is a limit ordinal number $< \omega_1$, then let $c_\alpha L$ be the one-point compactification of the free sum $\bigoplus_{1\le\beta<\alpha} c_\beta L$ and let p_α be the compactification point. It is clear that $c_\alpha L$ is a compactification of the space L for any $\alpha < \omega_1$.

By induction one can prove the following

Proposition 1. *For every countable ordinal number* $\alpha \geq 1$ *we have* $S^\alpha \hookrightarrow c_\alpha L \hookrightarrow S^\alpha$.

Corollary 5. *Let F be a monotone dimension function such that*

(i) *for every ordinal number $\alpha < \omega_1$ there exists an ordinal number $\beta < \omega_1$ such that $FS^\beta = \alpha$;*

(ii) $F(X \times Y) = FX$ *for any spaces X, Y with* $\operatorname{ind} Y = 0$.

Then for every ordinal number $\alpha < \omega_1$ there exists a space X_α such that

a) $FX_\alpha = \alpha$;

b) *for any ordinal number $\beta \geq \alpha$ there exists a compactification $c_\beta X_\alpha$ with $Fc_\beta X_\alpha = \beta$.*

Proof. The spaces X_α should be chosen from the collection $\{S^\gamma \times L : \gamma < \omega_1\}$ and the compactifications $c_\beta X_\alpha$ can be found in the collection $\{S^\gamma \times c_\beta L : \gamma, \beta < \omega_1\}$. Recall (see [Ch]) that for any countable ordinal numbers ν, μ we have $S^{\nu(+)\mu} \hookrightarrow S^\nu \times S^\mu \hookrightarrow S^{\nu(+)\mu}$, where $(+)$ is the natural sum of Hessenberg [KM].

Remark 3. *The dimensions* trind, D *satisfy the conditions of Corollary* 5, *in particular condition* (ii) *for* trind *see* [*T*], *for* D - [*He1*].

Note also that $\operatorname{trInd}(S^\gamma \times L) = \operatorname{trdim}(S^\gamma \times L) = \infty$, if $\gamma \geq \omega_0$.

4. Questions

Recall that for every space X with

a) $\mathrm{trInd}X \neq \infty$ there exists a compactification cX such that $\mathrm{trInd}\, cX = \mathrm{trInd}\, X$ [Lu1];

b) $\mathrm{D}X \neq \infty$ there exists a compactification cX such that $\mathrm{D}X \leq \mathrm{D}cX \leq \mathrm{D}X + 1$ [K] (moreover for every ordinal number α: $\omega_0 \leq \alpha < \omega_1$ there exists a space X_α such that $\mathrm{D}X_\alpha = \alpha$ and for any compactification cX_α of the space X_α we have $\mathrm{D}cX_\alpha > \alpha$ [Lu1]);

c) $\mathrm{trdim}\, X \neq \infty$ there exists a compactification cX such that $\mathrm{trdim}\, cX = \mathrm{trdim}\, X$ [Ki].

It is interesting to note that there exists a space Y with $\mathrm{trdim}\, Y = \omega_0 + 1$ which has a compactification cY with $\mathrm{trdim}\, cY = \omega_0$ [B2]. Recall that for dimension trInd, which has very similar properties to dimension trdim, the following statement holds:

if $X \subset Y$ and $\mathrm{trInd}X$, $\mathrm{trInd}\, Y \neq \infty$, then $\mathrm{trInd}X \leq \mathrm{trInd}\, Y$ [Lu2].

In connection with this paper one can pose

PROBLEM 1. *Let X be a noncompact space, cX be a compactification of the space X and $F(cX) \neq \infty$, where F is one of the functions* $\mathrm{trind}, \mathrm{trInd}, \mathrm{D}, \mathrm{trdim}$. *Is it true that for any countable ordinal number $\alpha \geq F(cX)$ there exists a compactification $c_\alpha X$ such that $F(c_\alpha X) = \alpha$?*

Let us recall [Lu1] here

LUXEMBURG'S CONJECTURE. *If X is a space and* $\mathrm{trind}\, X = \alpha + p$, *where α is a limit ordinal number and p = 0, 1, 2, ..., then there exists a compactification $cX \supset X$ such that* $\mathrm{trind}\, X \leq \alpha + 2p + 1$.

REFERENCES

[AE] J. M. Aarts and P. van Emde Boas, *Continua as remainders in compact extensions,* Nieuw Arch. Wisk. 15 (1967), 34–37.

[AP] P. S. Aleksandrov and B. A. Pasynkov, *Introduction to Dimension Theory* (Russian), Moscow 1973.

[B1] P. Borst, *Transfinite Classifications of Weakly Infinite-dimensional Spaces,* dissertation, Amsterdam 1986.

[B2] P. Borst, *On weakly infinite-dimensional subspaces,* Fund.Math. 140 (1992), 225–235.

[Ch] V. A. Chatyrko, *Ordinal products of topological spaces,* Fund. Math. 144 (1994), 95–117.

[E] R. Engelking, *Theory of Dimensions Finite and Infinite,* Sigma Ser. Pure Math. vol. 10, 1995.

[Ha] Y. Hattori, *Remarks on weak large transfinite dimension $w -$ Ind,* Q & A in General Topology 4 (1986), 59–66.

[He1] D. W. Henderson, D-dimension, I. *Anew transfinite dimension,* Pacific J. Math. 26 (1968), 91–107.

[He2] D. W. Henderson, *A lower bound for transfinite dimension,* Fund. Math. 64 (1968), 167–173.

[Ki] T. Kimura, *A note on compactification theorem for* trdim, Topology Proc. 20 (1995), 145–159.

[K] I. M. Kozlovski, *Two theorems on metric spaces,* Soviet Math. Dokl. 13 (1972), 743–747.

[KM] K. Kuratowski, A. Mostowski, *Set Theory,* PWN, 1976.

[Lu1] L. A. Luxemburg, *On compactifications of metric spaces with transfinite dimensions,* Pacific. J. Math. 101 (1982), 399–450.

[Lu2] L .A. Luxemburg, *On transfinite inductive dimensions,* Soviet Math. Dokl. 14 (1973), 388–393.

[Le] B. T. Levshenko, *Spaces of transfinite dimensionality,* Amer. Math. Soc. Transl. 73 (1968), 135–148.

[S] Ju. M. Smirnov, *On universal spaces for certain classes of infinite-dimensional spaces,* Amer. Math. Soc. Transl. 21 (1961), 35–50.

[T] G. H. Toulmin, *Shuffling ordinals and transfinite dimension,* Proc. London Math. Soc. 4 (1954), 177–195.

DEPARTMENT OF MATHEMATICS
LINKÖPING UNIVERSITY
581 83 LINKÖPING
SWEDEN
Email: vitja@mai.liu.se