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COMPLETE ORDER ISOMORPHISMS BETWEEN NON-COMMUTATIVE L^2-SPACES

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Abstract

In this article we shall study the completely positive maps between non-commutative L^2-spaces. Especially, we deal with a complete order isomorphism and a completely o.d. homomorphism between the Hilbert spaces associated with the matrix ordered standard forms of von Neumann algebras, and we investigate the relationship between these maps and the homomorphisms of von Neumann algebras.

1. Introduction

Many authors have studied the positive maps on an ordered Hilbert space defined by a selfdual cone The linear map preserving the order and the orthogonal decomposition in a selfdual cone (called o.d. homomorphism) is introduced as the non-commutative version of the lattice homomorphism in orthogonally decomposable spaces by Yamamuro [Y1], and we have interesting results (see for example [DY],[D], [Y2], [I]). We consider here such a map and a more general map between non-commutative L^2-spaces from the point of view of complete positivity in the category of matrix ordered standard forms of von Neumann algebras introduced by Schmitt and Wittstock [SW].

Let $(H, H_n^+, n \in \mathsf{N})$ and $(\hat{H}, \hat{H}_n^+, n \in \mathsf{N})$ be matrix ordered Hilbert spaces. A (bounded) linear map h of H into $\hat{H}$ is said to be n-positive when the multiplicity map $h_n = h \otimes 1_n$ maps H_n^+ into $\hat{H}_n^+$. If h is n-positive for every natural number n, then h is called a completely positive map (or a complete order homomorphism). A bijective linear map h of H onto $\hat{H}$ is called an order isomorphism if $hH^+ = \hat{H}^+$. We call h a complete order isomorphism if $h_n H_n^+ = \hat{H}_n^+$ for every $n \in \mathsf{N}$. We call h an o.d.(orthogonal decomposition) homomorphism if h is 1-positive and $(h\xi, h\eta) = 0$ whenever $\xi, \eta \in H^+$ and $(\xi, \eta) = 0$. If h is completely positive and h_n is an o.d. homomorphism for

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every $n \in \mathsf{N}$, we call h is a complete o.d. homomorphism. A bijective map h is called a complete o.d. isomorphism if both h and h^{-1} are complete o.d. homomorphisms.

We shall use the notation as introduced in [SW] with respect to matrix ordered standard forms and their construction.

From now on, let $(M, H, H_n^+, n \in \mathsf{N})$ and $(\hat{M}, \hat{H}, \hat{H}_n^+, n \in \mathsf{N})$ be matrix ordered standard forms of σ-finite von Neumann algebras. We use here the notation

$$\mathrm{Ad}(h) : x \in B(H) \mapsto hxh^{-1} \in B(\hat{H})$$

for the invertible map $h : H \to \hat{H}$.

2. Complete order isomorphism between matrix ordered Hilbert spaces

This section is devoted to the study of the complete order isomorphism between two matrix ordered Hilbert spaces. At first, we shall consider that an isomorphism between von Neumann algebras induces a complete order isomorphism between the related Hilbert spaces. We need a lemma.

LEMMA 2.1. *For* $(M, H, H_n^+, n \in \mathsf{N})$ *and* $(\hat{M}, \hat{H}, \hat{H}_n^+, n \in \mathsf{N})$, *if* h *is a completely positive map of* H *onto* $\hat{H}$ *such that* $hH^+ = \hat{H}^+$, *then* h *is a complete order isomorphism.*

PROOF. Consider the polar decomposition $h = u|h|$ of h. There exists by [C, Theorem 3.3] a positive invertible operator k such that $|h| = kJ_{H^+}kJ_{H^+}$. Since H_n^+ is generated by all elements $[x_i J_{H^+} x_j J_{H^+} \xi], x_i \in M, \xi \in H^+$ by [SW, Lemma 1.1], it follows that $|h|_n H_n^+ = H_n^+$, so $h_n H^+ = \hat{H}_n^+$.

PROPOSITION 2.2. *For* $(M, H, H_n^+, n \in \mathsf{N})$ *and* $(\hat{M}, \hat{H}, \hat{H}_n^+, n \in \mathsf{N})$, *if* ρ *is an (not necessarily* $*$*-preserving) isomorphism of* M *onto* $\hat{M}$, *then there exists a complete order isomorphism* h *of* H *onto* $\hat{H}$ *satisfying* $\rho = \mathrm{Ad}(h)|_M$.

PROOF. Suppose that ρ is an isomorphism of M onto $\hat{M}$. By [C, Theorem 3.1] there exists a bijection h of H^+ onto $\hat{H}^+$ satisfying $\rho(x) = hxh^{-1}$, $\forall x \in M$. If $x_1, \cdots, x_n \in M$ and $\xi \in H^+$, then we have

$$h_n[x_i J x_j J \xi] = [\rho(x_i) \hat{J} \rho(x_j) \hat{J} h \xi].$$

Note that $hJh^{-1} = \hat{J}$ because of $hJh^{-1}\xi = \xi$ for every $\xi \in \hat{H}^+$. It follows from Lemma 2.1 that $h_n H_n{}^+ = H_n^+$.

LEMMA 2.3. *For* $(M, H, H_n^+, n \in \mathsf{N})$, *if* u *is a* 1*-positive unitary on* H *with* $u \in M \cup M'$, *then* $u = 1$.

PROOF. By symmetry it suffices to prove in the case $u \in M'$. Take an arbitrary element $\xi \in H$. Then ξ is written as $\xi = \xi_1 - \xi_2 + i(\xi_3 - \xi_4)$ such that $\xi_1 \perp \xi_2$ and $\xi_3 \perp \xi_4$, $\xi_i \in H^+$. Since $u\xi = JuJ\xi$, we have $u = JuJ$. Hence $u \in M \cap M'$ and $u = u^*$. In addition, since $s(\xi_1) \perp s(\xi_2)$ and $s(\xi_3) \perp s(\xi_4)$, where $s(\xi)$ denotes the support projection of a vector functional ω_ξ on M, and $uH^+ = H^+$, we have

$$(u\xi, \xi) = \sum_{i=1}^{4} (u\xi_i, \xi_i) \geq 0.$$

Hence $u \geq 0$, and so $u = 1$.

PROPOSITION 2.4. *For* $(M, H, H_n^+, n \in \mathsf{N})$ *and* $(\hat{M}, \hat{H}, \hat{H}_n^+, n \in \mathsf{N})$, *if* ρ *is a* $*$*-isomorphism of* M *onto* $\hat{M}$, *then there exists uniquely a completely positive isometry* u *of* H *onto* $\hat{H}$ *satisfying* $\rho = \mathrm{Ad}(u)|_M$.

PROOF. Suppose that ρ is a $*$-isomorphism of M onto $\hat{M}$. Then there exists by [H, Theorem 2.3] a 1-positive unitary operator of H onto $\hat{H}$ satisfying

$$\rho(x) = uxu^{-1}, x \in M.$$

Then u is completely positive by the proof as in Proposition 2.2. The unicity of u follows from Lemma 2.3.

PROPOSITION 2.5. *For* $(M, H, H_n^+, n \in \mathsf{N})$ *and* $(\hat{M}, \hat{H}, \hat{H}_n^+, n \in \mathsf{N})$, *suppose that* h *is a complete order isomorphism of* H *onto* $\hat{H}$ *with the polar decomposition* $h = u|h|$. *Then* u *is a completely positive isometry of* H *onto* $\hat{H}$. *Furthermore, if* h *as above is an o.d. homomorphism, then* h *is a complete o.d. isomorphism of* H *onto* $\hat{H}$.

PROOF. By [C, Theorem 3.3] there exists an invertible operator $k \in M^+$ such that $|h| = kJ_{H^+}kJ_{H^+}$. Therefore, $|h|$ is a complete order isomorphism, so is u. Moreover, if h is an o.d. homomorphism, then $|h|$ is an o.d. homomorphism on H. By [DY, (2.1)] $|h|$ belongs to $M \cap M'$. Since $|h| \otimes 1_n \in M \otimes M_n \cap M' \otimes I_n \subset M \otimes M_n \cap M' \otimes M_n' = M \otimes M_n \cap (M \otimes M_n)'$, where M_n' operates on M_n by matrix multiplication from the right, one sees that h_n is an o.d. homomorphism. Therefore, by [DY, (3.1)] h_n is an o.d. isomorphism. This completes the proof.

We shall next consider that a complete order isomorphism between the matrix ordered Hilbert spaces induces an isomorphism between the corresponding algebras. In [SW] Schmitt and Wittstock constructed the multiplier algebra in a matrix ordered Hilbert space as follows:

Let $(H, H_n^+, n \in \mathsf{N})$ be a matrix ordered Hilbert space. Put

$$\mathscr{M} = \{x \in B(H) | \{\mathrm{diag}(x, 1, \cdots, 1)\Xi \ \mathrm{diag}(x, 1, \cdots, 1)^J\} \in H_n^+ \\ \text{for every } \Xi \in H_n^+ \text{ and all } n \in \mathsf{N}\}.$$

Here $\mathrm{diag}(x_1, x_2, \cdots, x_n)$ denotes the $n \times n$ matrix with entries $a_{i,j} = \delta_{i,j} x_i (x_i \in B(H))$ and $\{\ \}$ denotes the Jordan product

$$\{x\xi y^J\} = \tfrac{1}{2}(xJyJ\xi + JyJx\xi), x, y \in B(H), \xi \in H.$$

It is shown in [SW] that if the completed face $(F_{\{\xi\}})^{\perp\perp}$ generated by $\xi \in H_n^+$ is projectable for all $\xi \in H_n^+, n \in \mathsf{N}$, then $\mathscr{M}$ is a von Neumann algebra and $(M, H, H_n^+, n \in \mathsf{N})$ is a matrix ordered standard form.

Proposition 2.6. *For* $(M, H, H_n^+, n \in \mathsf{N})$ *and* $(\hat{M}, \hat{H}, \hat{H}_n^+, n \in \mathsf{N})$, *if h is a complete order isomorphism of H onto $\hat{H}$, then* $\mathrm{Ad}(h)|_M$ *is an isomorphism of M onto $\hat{M}$.*

Proof. Suppose that h is a complete order isomorphism of H onto $\hat{H}$. We show that $hMh^{-1} \subset \hat{M}$. Choose an element $x \in M$. We then obtain for all

$$\Xi = \begin{bmatrix} \xi_{11} & \cdots & \xi_{1n} \\ \vdots & & \vdots \\ \xi_{n1} & \cdots & \xi_{nn} \end{bmatrix} \in \hat{H}_n^+$$

$$\{\mathrm{diag}(hxh^{-1}, 1, \cdots, 1)\Xi \ \mathrm{diag}(hxh^{-1}, 1, \cdots, 1)^{\hat{J}}\} =$$

$$= \frac{1}{2}\left(\begin{bmatrix} hxh^{-1}\hat{J}hxh^{-1}\hat{J}\xi_{11} & hxh^{-1}\xi_{12} & \ldots & hxh^{-1}\xi_{1n} \\ \hat{J}hxh^{-1}\hat{J}\xi_{21} & \xi_{22} & \ldots & \xi_{2n} \\ \vdots & \vdots & & \vdots \\ \hat{J}hxh^{-1}\hat{J}\xi_{n1} & \xi_{n2} & \ldots & \xi_{nn} \end{bmatrix}\right.$$

$$\left. + \begin{bmatrix} \hat{J}hxh^{-1}\hat{J}hxh^{-1}\xi_{11} & hxh^{-1}\xi_{12} & \ldots & hxh^{-1}\xi_{1n} \\ \hat{J}hxh^{-1}\hat{J}\xi_{21} & \xi_{22} & \ldots & \xi_{2n} \\ \vdots & \vdots & & \vdots \\ \hat{J}hxh^{-1}\hat{J}\xi_{n1} & \xi_{n2} & \ldots & \xi_{nn} \end{bmatrix}\right)$$

$$= \begin{bmatrix} hxJxJh^{-1}\xi_{11} & hxh^{-1}\xi_{12} & \ldots & hxh^{-1}\xi_{1n} \\ hJxJh^{-1}\xi_{21} & \xi_{22} & \ldots & \xi_{2n} \\ \vdots & \vdots & & \vdots \\ hJxJh^{-1}\xi_{n1} & \xi_{n2} & \ldots & \xi_{nn} \end{bmatrix}$$

$$= h_n \ \mathrm{diag}(x, 1, \cdots, 1)J_n \ \mathrm{diag}((x, 1, \cdots, 1)J_n h_n^{-1}\Xi,$$

which belongs to $\hat{H}_n^+$ because h and h^{-1} are completely positive. This implies $hMh^{-1} \subset \hat{M}$. By the symmetric argument we obtain the converse inclusion.

THEOREM 2.7. *With $(M, H, H_n^+, n \in \mathsf{N})$, let $(\hat{H}, \hat{H}_n^+, n \in \mathsf{N})$ be a matrix ordered Hilbert space. Suppose that h is an order isomorphism of H onto $\hat{H}$. Then the following conditions are equivalent:*

1) *h is a complete o.d. isomorphism.*

2) *There exists a von Neumann algebra $\hat{M}$ such that $(\hat{M}, \hat{H}, \hat{H}_n^+, n \in \mathsf{N})$ is a matrix ordered standard form, and $\mathrm{Ad}(h)|_M$ is a $*$-isomorphism of M onto $\hat{M}$.*

PROOF. 1) $\Rightarrow$ 2): We show that every completed face G in $\hat{H}_n^+$ is projectable for each n. Since h is completely positive, there exists a closed face F in H_n^+ such that $h_n F = G$. By virtue of the matrix ordered standard form F is a selfdual cone in the closed linear span $[F]$ of F. Since h_n is an o.d. homomorphism, G is a selfdual cone in $[G]$. Hence G is projectable. Indeed, every element $\eta = P_G \xi \in P_G \hat{H}_n^+ (\xi \in \hat{H}_n^+)$ has the form

$$\eta = \eta_1 - \eta_2 + i(\eta_3 - \eta_4)$$

such that $\eta_1 \perp \eta_2, \eta_3 \perp \eta_4, \eta_i \in G$. If $i \geq 2$ then $(\eta, \eta_i) = (\xi, \eta_i) \geq 0$, so $\eta_i = 0$. Hence $\eta = \eta_1$. Thus we see the existence of $\hat{M}$ by [SW, Theorem 4.3]. Consider the polar decomposition $h = u|h|$. By assumption $|h|$ is an o.d. homomorphism, it follows from [DY, (2.1)] that $|h|$ belongs to $M \cap M'$. Then we have $\mathrm{Ad}(h) = \mathrm{Ad}(u)$ on M. Applying Proposition 2.6, we obtain the desired result.

2) $\Rightarrow$ 1): Suppose that $\mathrm{Ad}(h)|_M$ is a $*$-isomorphism of M onto $\hat{M}$. Since h is an order isomorphism, we have $hJh^{-1} = \hat{J}$. It follows from the proof of Proposition 2.2 that h is completely positive. Then $hxh^{-1} = (h^{-1})^* x h^*$, $x \in M$. Hence h^*h belongs to M', so does $|h|$. Since $|h| = kJ_{H^+}kJ_{H^+}$ for some invertible positive element $k \in M$, we have $k \in M'$. Therefore, [DY, (2,1)] shows that h is an o.d. homomorphism, so h is a complete o.d. isomorphism by Proposition 2.5. This completes the proof.

For a matrix ordered standard form $(M, H, H_n^+, n \in \mathsf{N})$, let A be a von Neumann subalgebra of M. Put for $n \in \mathsf{N}$

$$P_n(A) = \{[\xi_{i,j}] \in H_n | \sum_{i,j=1}^n a_i J_{H^+} a_j J_{H^+} \xi_{i,j} \in H^+ \text{ for } a_1, \cdots, a_n \in A\}.$$

One easily sees that $H_n^+ = P_n(M), n \in \mathsf{N}$. We also have that if $P_n(M) = P_n(A)$ for a subalgebra A of M and $n \in \mathsf{N}$ then

$$P_n(A) = \overline{\mathrm{co}}\{[a_i J_{H^+} a_j J_{H^+} \xi] \in H_n | a_1, \cdots, a_n \in A, \xi \in H^+\}.$$

Here $\overline{\mathrm{co}}$ denotes the closed convex hull. We obtain the following theorem:

THEOREM 2.8. *For* $(M, H, H_n^+, n \in \mathsf{N})$ *and* $(\hat{M}, \hat{H}, \hat{H}_n^+, n \in \mathsf{N})$*, let* u *be a* 1-*positive isometry of* H^+ *onto* $\hat{H}^+$*. Suppose that* A *is a von Neumann subalgebra of* M *satisfying* $uAu^{-1} \subset \hat{M}$ *and* $P_n(A) = H_n^+$ *for all* $n \in \mathsf{N}$*. Then* u *is completely positive, and* $\mathrm{Ad}(u)$ *is a* $*$-*isomorphism of* M *onto* $\hat{M}$.

PROOF. Let $a_i \in A, \xi \in H^+$. We have

$$u_n[a_iJa_jJ\xi] = [ua_iJa_jJ\xi] = [ua_iu^{-1}\hat{J}ua_ju^{-1}\hat{J}u\xi],$$

which belongs to $\hat{H}_n^+$ by assumption. Hence u is completely positive, so we get the proof applying Proposition 2.4.

As an example of the above theorem we have obtained the following fact:

EXAMPLE. For $(M, H, H_n^+, n \in \mathsf{N})$, let M be an injective factor (or a semifinite injective von Neumann algebra) and let H be separable. Then there exists an abelian von Neumann subalgebra A of M such that $H_n^+ = P_n(A)$ for every $n \in \mathsf{N}$ (see [M1, Theorem 2.4]).

For a matrix ordered Hilbert space $(H, H_n^+, n \in \mathsf{N})$, we shall write $P(H^+)$ for the 1-positive maps on H. Put

$$CPU(H^+) = \{u \in P(H^+) | u \text{ is a completely positive unitary}\}.$$

Moreover, for a matrix ordered standard form $(M, H, H_n^+, n \in \mathsf{N})$, put

$$CPU^\circ(H^+) = \{uJuJ | u \text{ is a unitary in } M\}.$$

One easily sees that $CPU(H^+)$ is a topological group under the strong operator topology. Since H_n^+ is generated by the elements $[a_iJa_jJ\xi]_{i,j=1}^n \times (a_1, \cdots, a_n \in M, \xi \in H^+)$, $uJuJ$ is completely positive. One then sees that $CPU^\circ(H^+) \subset CPU(H^+)$. In the following proposition we shall show that there exists a one-to-one correspondence between $CPU(H^+)$ (resp. $CPU^\circ(H^+)$) and a group of the automorphisms Aut(M) of M (resp. the inner automorphisms $\mathrm{Int}(M)$).

PROPOSITION 2.9. *Keep the notation above. The map:* $u \mapsto \mathrm{Ad}(u)$ *is a homeomorphism of* $CPU(H^+)$ *onto* $\mathrm{Aut}(M)$*. In addition,* $CPU^\circ(H^+)$ *is homeomorphic to* $\mathrm{Int}(M)$.

PROOF. Applying Proposition 2.4, Theorem 2.7 and [H, Proposition 3.5] we have that $CPU(H^+)$ is homeomorphic to $\mathrm{Aut}(M)$. It is now clear that $CPU^\circ(H^+)$ is isomorphic to $\mathrm{Int}\, M)$. This completes the proof.

In the above discussion, if $uJuJ = vJvJ$ for unitaries $u, v \in M$, then $v^*u = Jvu^*J \in M'$. Then there exists a unitary w in the center of M such that $u = vw$.

In the rest of this section, we examine the results of Dang and Yamamuro [D, DY, Y2] in the framework of the completely positive maps.Using their results,we immediately obtain the similar properties.

For $\xi, \eta \in H$, put

$$P(H^+, \xi, \eta) = \{h \in P(H^+) | h\xi = \eta\}.$$

PROPOSITION 2.10 (see [D, (2)]). *For* $(M, H, H_n^+, n \in \mathsf{N})$*, the following conditions are equivalent:*

1) *For all cyclic and separating vectors* $\xi, \eta \in H^+$*, the set of all complete o.d. homomorphisms in* $CP(H^+, \xi, \eta)$ *coincides with the set of all extreme points in* $CP(H^+, \xi, \eta)$*.*

2) *For all cyclic and separating vectors* $\xi, \eta \in H^+$*, the set of all o.d. homomorphisms in* $P(H^+, \xi, \eta)$ *coincides with the set of all extreme points in* $P(H^+, \xi, \eta)$*.*

3) M *is abelian.*

PROOF. That 1) $\Rightarrow$ 2) and 2) $\Leftrightarrow$ 3) follow from [D, (1) and (2)]. If M is abelian, then every 1-positive map on H is completely positive by [M1, Corollary 1.6]. Hence 3) $\Rightarrow$ 1).

PROPOSITION 2.11 (see [DY, (3.4)]). *For* $(M, H, H_n^+, n \in \mathsf{N})$*, the following conditions are equivalent:*

1) *Every complete o.d. isomorphism on* H *is normal.*

2) *Every* $*$*-automorphism of* M *is identical on the center of* M*.*

PROOF. 1) $\Rightarrow$ 2): Let ρ be a $*$-automorphism of M. By Proposition 2.4 there exists a completely positive unitary u on H satisfying $\rho(x) = uxu^*, x \in M$. For an invertible positive element $a \in M \cap M'$, put $h = ua$. Then h is a complete o.d. isomorphism by [Y1, (3.4)]. By assumption we have $a^2 = ua^2u^*$. It follows that $ux = xu$ for every $x \in M \cap M'$.

2) $\Rightarrow$ 1): Let h be a complete o.d. isomorphism on H with the polar decomposition $h = u|h|$. We then have by Theorem 2.7 that $\mathrm{Ad}(u)|_M$ is a $*$-automorphism of M. Since $|h|$ belongs to the center of M, we have $u|h|u^* = |h|$ by assumption. Therefore,

$$h^*h = |h|u^*u|h| = |h|^2 = u|h|^2u^* = hh^*.$$

This completes the proof.

PROPOSITION 2.12 (see [Y2, Theorem]). *For* $(M, H, H_n^+, n \in \mathsf{N})$*, suppose that* $H_n^+ (n \in \mathsf{N})$ *is a selfdual cone related to a cyclic and separating vector* ξ_0 *in* H *for* M*, and* $J = J_{\xi_0}$*. Then the following conditions are equivalent:*

1) *Every complete order isomorphism* h *such that* $h\xi_0 = \xi_0$ *is a complete o.d. isomorphism.*

2) *Every order isomorphism* h *such that* $h\xi_0 = \xi_0$ *is an o.d. isomorphism.*

3) ξ_0 *is a trace vector.*

PROOF. We remark the following fact:

For $x \in M$, put $\delta = x + JxJ$. Applying the theorem of a derivation on a homogeneous cone (see [C, Theorem 3.4]), we have for all $n \in \mathsf{N}$ and $t \in \mathsf{R}$

$$(e^{t\delta} \otimes 1_n)H_n^+ = (e^{t\delta \otimes 1_n})H_n^+ \subset H_n^+$$

by virtue of the standard form $(M_n(M), H_n, J_n, H_n^+)$ and

$$\delta \otimes 1_n = x \otimes 1_n + J_n(x \otimes 1_n)J_n.$$

This means $e^{t\delta}$ is a complete o.d. isomorphism. Then by the proof of [Y2, Theorem] we obtain the desired result.

3. Completely order homomorphism between matrix ordered Hilbert spaces

In this section we shall describe what happens with homomorphisms which are not bijective. To do this, we need the results of [M2]. We considered the relationship between a completely positive projection on a matrix ordered Hilbert space and a normal conditional expectation with respect to a faithful normal state on the related von Neumann algebra, and showed that each of them induces the other. We immediately obtain the following property by [M2, Lemma 1]:

(1) *For matrix ordered standard forms* $(M, H, H_n^+, n \in \mathsf{N})$ *and* $(\hat{M}, \hat{H}, \hat{H}_n^+, n \in \mathsf{N})$, *suppose that* h *is a complete order homomorphism of* H *into* $\hat{H}$ *with the support projection* e *and the range projection* f, *and* $h_n H_n^+$ *is a selfdual cone in the range space of* h_n *for every* $n \in \mathsf{N}$. *If* e *and* f *are completely positive, then there exist von Neumann algebras* A *and* B *such that* $(A, eH, e_n H_n^+, n \in \mathsf{N})$ *and* $(B, f\hat{H}, f_n \hat{H}_n^+, n \in \mathsf{N})$ *are matrix ordered standard forms, and* $h|_{eH}$ *is a complete order isomorphism of* eH *onto* $f\hat{H}$.

The next property follows from Proposition 2.6 and [M2, Theorem 3].

(2) *With the notations as in* (1), *suppose that* e *and* f *contain cyclic and separating fixed vectors in* H^+ *and* $\hat{H}^+$ *for* M *and* $\hat{M}$, *respectively. If* $N = M \cap \{e\}'$ *and* $\hat{N} = \hat{M} \cap \{f\}'$, *then* $N|_{eH} = eM|_{eH} = A$ *and* $\hat{N}|_{f\hat{H}} = f\hat{M}|_{f\hat{H}} = B$, *and there exists uniquely an isomorphism* ρ *of* N *onto* $\hat{N}$ *such that* $\rho(x)|_{f\hat{H}} = \mathrm{Ad}(h|_{eH})(x|_{eH})$ *for all* $x \in N$.

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