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CORRECTION OF AN ERROR IN THE PAPER "CHARACTERIZATION OF PERFECT INVOLUTION GROUPS"

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The purpose of the present note is to point out a gap in the proof of Lemma 4 in [1] and to indicate how it can be mended. In the proof of Lemma 4 in [1], the following situation is encountered: E is a finite subset of R^k; φ is a real-valued function on E; $(\pi_t)_{t\in E}$ is a family of probability measures on E such that for $t \in E$,

$$t = \int u\, d\pi_t(u) \qquad \text{and} \qquad \varphi(t) \le \int \varphi\, d\pi_t.$$

As in [1], we denote by S the set of those $t \in E$ such that π_t is ε_t, the Dirac measure. The problem is to construct, given $x \in E$, a probability measure μ on S such that

$$x = \int t\, d\mu(t) \qquad \text{and} \qquad \varphi(x) \le \int \varphi\, d\mu.$$

In [1], one defines a sequence $(\mu_n)_{n\ge 0}$ of probability measures on E by $\mu_0 = \varepsilon_x$ and $\mu_{n+1} = \sum_{t\in E} \mu_n(\{t\})\pi_t$; one then chooses an accumulation point μ of (μ_n). It is claimed that $\mu = \sum_{t\in E} \mu(\{t\})\pi_t$. This conclusion is unwarranted. Indeed, it would be true if the whole sequence (μ_n) converged to μ. However, all that we know is that some subsequence (μ_{n_k}) of (μ_n) converges to μ. In this case, all that we get from $\mu_{n+1} = \sum_{t\in E} \mu_n(\{t\})\pi_t$ by inserting $n = n_k$ and going to the limit is $\lim_{k\to\infty} \mu_{n_k+1} = \sum_{t\in E} \mu(\{t\})\pi_t$, which is not good enough since the sequence (μ_{n_k+1}) might have a limit different from that of (μ_{n_k}). To repair this, let $\mathscr{D}$ be the set of those subsets D of E, containing S, such that there is a probability measure μ on D such that $x = \int t\, d\mu(t)$ and $\varphi(x) \le \int \varphi\, d\mu$. Since E is a finite set, we can choose $D \in \mathscr{D}$ minimal with respect to the inclusion ordering. If $D = S$, we have the desired measure μ on S. Suppose $D \ne S$; we shall derive a contradiction. Choose $t \in D \setminus S$. Since

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$t \notin S$, we have $\pi_t \neq \varepsilon_t$ by definition, that is, $\pi_t(\{t\}) < 1$. From $t = \int u\,d\pi_t(u)$, by subtracting $\pi_t(\{t\})t$ from both sides and dividing by $1 - \pi_t(\{t\})$, we get

$$t = \int u\,d\varrho(u)$$

where ϱ is the probability measure $\bigl(1 - \pi_t(\{t\})\bigr)^{-1}\pi_t|(D \setminus \{t\})$. Similarly,

$$\varphi(t) \leq \int \varphi\,d\varrho.$$

Define $D^* = D \setminus \{t\}$. Define a probability measure μ^* on D^* by $\mu^*(\{u\}) = \mu(\{u\}) + \mu(\{t\})\varrho(\{u\})$ for $u \in D^*$. Then

$$\begin{aligned} x &= \int_D u\,d\mu(u) = \int_{D^*} u\,d\mu(u) + \mu(\{t\})t \\ &= \int_{D^*} u\,d\mu(u) + \mu(\{t\}) \int_{D^*} u\,d\varrho(u) = \int_{D^*} u\,d\mu^*(u) \end{aligned}$$

and (similarly) $\varphi(x) \leq \int_{D^*} \varphi\,d\mu^*$. The existence of a probability measure μ^* with these properties shows that $D^* \in \mathscr{D}$, in contradiction with the minimality of D.

REFERENCES

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