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A SMOOTHLY BOUNDED DOMAIN IN A COMPLEX SURFACE WITH A COMPACT QUOTIENT

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Abstract

We study the classification of smoothly bounded domains in complex manifolds that cover compact sets. We prove that a smoothly bounded domain in a hyperbolic complex surface that covers a compact set is either biholomorphic to the ball or covered by the bidisc.

1. Introduction

One of the important problems in several complex variables is to study the interplay between the geometry of a domain and the structure of its automorphism group. It is known that a smoothly bounded domain in $\mathbb{C}^n$ that covers a compact set is biholomorphic to the ball. A theorem of Frankel [7] says that a bounded convex domain that covers a compact complex manifold is biholomorphic to a bounded symmetric domain. We refer readers to the recent survey [13] and references therein for the development in related subjects.

Let M be an n-dimensional complex manifold. Let $D \subset\subset M$ be a subdomain with smooth boundary (i.e., $M \setminus \overline{D}$ is non-empty and there exists a neighborhood N of $\overline{D}$ and a real-valued function $r \in C^\infty(N)$ such that $dr \neq 0$ on bD and $D = \{z \in N, r(z) < 0\}$). Let $\mathrm{Aut}(D)$ be the group of automorphisms of D. In this paper, we study the following problem: Characterize those D with the property that the quotient $D/\mathrm{Aut}(D)$ is compact (as a topological space). When M admits a C^2-smooth strictly plurisubharmonic function, the boundary bD has a strictly pseudoconvex point (at which the strictly plurisubharmonic function attains its maximum value). It then follows from the method in [28] that (under the stated hypothesis) D must be biholomorphic to the ball.

The situation becomes more intricate if the condition on M is dropped. Here is a simple example: Let $M = \Delta \times N$, where Δ is the unit disc and N is a compact Riemann surface (without boundary) of genus ≥ 2. Then M is a

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hyperbolic complex manifold which does not admit any strictly plurisubharmonic function. Let $D = \left(\frac{1}{2}\Delta\right) \times N$. Then D is a relatively compact subdomain of M with smooth boundary and compact quotient but is not biholomorphic (indeed, not homeomorphic) to a ball. This example shows that a relatively compact subdomain of a hyperbolic complex manifold with smooth boundary and compact quotient is not necessary biholomorphic to a Euclidean ball. It is because of examples like this that the result of the present paper is not just a straightforward generalization of the Euclidean case; see also the bifurcation of the proof of the main theorem in Section 3 and 4.

In this paper, we shall prove the following result:

MAIN THEOREM. *Let M be a hyperbolic complex surface. Let $D \subset\subset M$ be a subdomain with smooth boundary. If $D/\operatorname{Aut}(D)$ is compact, then either*

(i) *D is biholomorphic to a ball*

or else

(ii) *The universal covering of D is biholomorphic to a bidisc.*

This result is of interest from the viewpoint of the uniformization problem for complex manifolds. It also opens up questions regarding the classification of smooth domains with non-compact automorphism group in a complex manifold. The proof of the theorem relies on the results and techniques in [8], [9].

To conclude the introduction, we mention the following immediate application of the main theorem and of a theorem of Yang [30]:

THEOREM. *Let M be a hyperbolic complex surface and let $D \subset\subset M$ be a subdomain with smooth boundary. Suppose there exists a discrete subgroup $\Gamma \subset \operatorname{Aut}(D)$ acting freely on D such that D/Γ is a compact Kähler manifold with negative bisectional curvature. Then D is biholomorphic to the ball.*

It would be interesting to know whether there are other versions of the Main Theorem in which curvature plays a more prominent role in the hypotheses (perhaps replacing the assumption of hyperbolicity).

2. Preliminaries

Let Δ denote the unit disc in $\mathbb{C}$ and Δ_n the unit n-polydisc in $\mathbb{C}^n$. Let B_n be the unit ball in $\mathbb{C}^n$. Let M be a complex manifold of dimension n and let $p \in M$. Let $T_p(M)$ be the holomorphic tangent space of M at p and let $X \in T_p(M)$. Let $H(M_1, M_2)$ be the family of holomorphic mappings from the complex manifold M_1 to the complex manifold M_2. The *Kobayashi-Royden pseudometric* is defined by

$$F_M^K(p, X) = \inf\left\{1/\lambda;\ f \in H(\Delta, D),\ f(0) = z,\ f'(0) = \lambda X,\ \lambda > 0\right\}.$$

We will use $d_M^K(p, q)$ to denote the induced Kobayashi pseudo-distance of M. A complex manifold M is called (Kobayashi) *hyperbolic* if d_M^K is indeed a (non-degenerate) distance. It is *complete hyperbolic* if d_M^K is a complete distance. We refer readers to [16], [17], [18], and [14] for properties of the Kobayashi metric.

Let D_0 be a bounded domain in $\mathbb{C}^n$ and let $z_0 \in D_0$. Let $\{w_1, w_2, \ldots, w_n\}$ be local complex coordinates near $p \in M$. The *Eisenman-Kobayashi volume form* of M at p with respect to the pair (D_0, z_0) is defined by

$$V_M^E(p) = E_M(p)\left(\frac{\sqrt{-1}}{2}\right)^n dw_1 \wedge d\overline{w}_1 \wedge \ldots \wedge dw_n \wedge d\overline{w}_n,$$

where $E_M(p) = \inf\{|Jf(z_0)|^{-2};\ f \in H(D_0, M),\ f(z_0) = p\}$. Here $Jf(z_0)$ denotes the complex Jacobian determinant of f at z_0. The *Carathéodory volume form* with respect to (D_0, z_0) is defined by

$$V_M^C(p) = C_M(p)\left(\frac{\sqrt{-1}}{2}\right)^n dw_1 \wedge d\overline{w}_1 \wedge \ldots \wedge dw_n \wedge d\overline{w}_n,$$

where $C_M(p) = \sup\{|Jf(p)|^2; f \in H(M, D_0), f(p) = z_0\}$. Both of these volume forms are discussed in [5], [27], [10], [11], and [18].

These two volume forms are biholomorphically invariant. In fact, the following properties are well-known.

LEMMA 2.1. *Let M_1 and M_2 be complex manifolds.*

(1) *If $F \in H(M_1, M_2)$, then $F^*(V_{M_2}) \le V_{M_1}$ holds for either of the volume forms V;*
(2) *If $F\colon M_1 \to M_2$ is a covering map, then $F^*\big(V_{M_2}^E\big) = V_{M_1}^E$;*
(3) *For any n-dimensional complex manifold M, it holds that $V_M^E \ge V_M^C$. Moreover, if $V_M^E(p) = V_M^C(p) \ne 0$ for some $p \in M$, then M is biholomorphic to D_0.*

The second part of the last property can be found in Graham and Wu [10]; see also [27] and [23]. Similar results for complete hyperbolic manifolds appeared first in [27]. The other properties follow directly from the definitions of the volume forms. In this paper, we will use only the invariant volume forms defined with respect to (i.e., with the role of (D_0, Z_0) played by) either $(\Delta_n, 0)$ or $(B_n, 0)$. We refer readers to [5], [10], [11] for information on these invariant volume forms.

Let D be a subdomain of a complex manifold M. It is easy to see that $D/\operatorname{Aut}(D)$ is compact (as a topological space) if and only if there exists a compact subset K of D such that $\mathrm{Aut}(D) \cdot K = D$ (cf. [19]). The set K will be called a *fundamental set* for $\mathrm{Aut}(D)$.

The following result is folklore. We sketch the proof for completeness. We refer readers to [12], Chapter IX, for various notions of pseudoconvexity on complex manifolds.

LEMMA 2.2. *Let D be a subdomain of a hyperbolic complex manifold M. Assume that $D/\mathrm{Aut}(D)$ is compact. Then D is complete hyperbolic. In particular, if the boundary bD is smooth, then D is Levi pseudoconvex.*

PROOF. Let $\{p_j\}$ be a Cauchy sequence with respect to d_D^K, the Kobayashi distance of D. Let K be a fundamental set for $\mathrm{Aut}(D)$. Then there exists a sequence $\{g_j\} \subset \mathrm{Aut}(D)$ such that $g_j(p_j) \in K$. Passing to a subsequence, we may assume that $g_j(p_j) \to q \in K$. Let $\epsilon > 0$ be so small that $\big\{z \in D,\ d_D^K(z, q) \leq \epsilon\big\}$ is a relatively compact subset of D. Since $d_D^K(g_j(p_k), q) \leq d_D^K(g_j(p_k), g_j(p_j)) + d_D^K(g_j(p_j), q) = d_D^K(p_k, p_j) + d_D^K(g_j(p_j), q)$, there exists a positive integer N such that $d_D^K(g_N(p_k), q) \leq \epsilon$ for all $k \geq N$. Therefore there exists a subsequence $\{g_N(p_{k_l})\}$ of $\{g_N(p_k)\}$ and a point $q' \in D$ such that $d_D^K\big(p_{k_l}, g_N^{-1}(q')\big) = d_D^K(g_N(p_{k_l}), q') \to 0$ as $l \to \infty$. Hence $\{p_k\}$ also converges to $g_N^{-1}(q')$. Thus D is complete hyperbolic.

For the second assertion, let $p \in bD$. Choose a complex coordinate (U, ϕ) at p such that $\phi(U) = B_n$ and $\phi(p) = 0$. Then $U \cap D$, hence $\phi(U \cap D)$, is taut in the sense of H. Wu [29]. Therefore, $\phi(U \cap D) \subset \mathbb{C}^n$ is pseudoconvex ([29], Theorem F). It then follows that bD is Levi pseudoconvex near p when bD is smooth (cf. [18], Chapter 3).

LEMMA 2.3 (Montel). *Let D be a relatively compact subset of a hyperbolic complex manifold M. Let N be a complex manifold. Then for any sequence $\{f_j\} \subset H(N, D)$, there exists a subsequence $\{f_{j_k}\}$ that converges local uniformly to a holomorphic mapping $f \colon N \to \overline{D}$.*

We refer the reader to [29] for a detailed treatment of this lemma and of normal families of holomorphic mappings between complex manifolds. Lemma 2.3 is a generalization of the classical Montel theorem. The proof is essentially the same as the classical one (cf. [1], §5.5): the hyperbolicity of M implies that $\{f_j\}$ is equicontinuous ([29], [24]), then a diagonalization argument concludes the proof.

We will also need the following version of a classical result of H. Cartan (cf. [20], Theorem 5.4). The proof is again essentially the same as the classical one. Nonetheless, we sketch the proof here for the reader's convenience.

LEMMA 2.4 (H. Cartan). *Let D be a relatively compact subdomain of a hyperbolic complex manifold M. Suppose that a sequence $\{f_j\} \subset \mathrm{Aut}(D)$*

converges local uniformly on D to $f\colon D \to \overline{D}$. Then either $f \in \mathrm{Aut}(D)$ or $f(D) \subset bD$.

PROOF. Assume that $q = f(p) \in D$ for some $p \in D$. Then there exist neighborhoods U of p and V of q such that $\overline{U}, \overline{V} \subset D$ and $f_j(\overline{U}) \subset \overline{V}$ for sufficiently large j. Let $g_j = f_j^{-1}$. It follows from Montel's theorem (Lemma 2.3) that, after passing to a subsequence, $\{g_j\}$ converges local uniformly to g on D. It is easy to see that $g(f(z)) = \lim g_j(f_j(z)) = z$ for $z \in \overline{U}$. Since the Jacobian determinant Jf_j converges local uniformly on D to $Jf \not\equiv 0$, it follows from the Hurwitz's theorem that $Jf \neq 0$ on D. Therefore, f is locally biholomorphic. Thus $f(D) \subset D$. Furthermore, $g(f(z)) = z$ on D. Similarly, $f(g(z)) = z$. Thus $f \in \mathrm{Aut}(D)$.

We end this section with the following simple lemma.

LEMMA 2.5. *Let $\pi\colon \widetilde{M} \to M$ be a covering map of complex manifolds. Let D be a subdomain of M. If $\widetilde{M}$ is simply connected and D is a retract of M (i.e., there exists a continuous map $R\colon M \to D$ such that $R|_D$ is the identity map), then $\pi^{-1}(D)$ is simply connected.*

PROOF. Let $\tilde{\gamma}$ be a closed path in $\pi^{-1}(D)$. Let $\widetilde{G} = \widetilde{G}(s,t)$ be the homotopy of $\tilde{\gamma}$ with $\tilde{\gamma}(0)$ in $\widetilde{M}$. Let $G = R \circ \pi \circ \widetilde{G}$. It follows from the covering homotopy theorem (see [26]) that G has a unique lifting $\widehat{G}$ to $\pi^{-1}(D)$ such that $\widehat{G}(s,0) = \tilde{\gamma}(s)$. Therefore, $\tilde{\gamma}$ is homotopic to a constant path in $\pi^{-1}(D)$.

3. Proof of the Main Theorem, Part I

In this section, we prove the main theorem when the boundary bD of D contains a strictly pseudoconvex point. In fact, we have

PROPOSITION 3.1. *Let D be a relatively compact subdomain of an n-dimensional hyperbolic complex manifold M. If $D/\mathrm{Aut}(D)$ is compact and bD is smooth and strictly pseudoconvex near a point $p \in bD$, then D is biholomorphic to B_n.*

PROOF. The proof uses the ideas in [27] and follows similar lines. We only indicate the major steps here. Let $\{p_j\} \subset D$ be a sequence converging to the point p. Let $\{g_j\}$ be a sequence in $\mathrm{Aut}(D)$ such that $g_j^{-1}(p_j) \in K$, the fundamental set of $\mathrm{Aut}(D)$. Passing to a subsequence, we may assume that $g_j^{-1}(p_j) \to q \in K$. It then follows that $d_M^K(g_j(q), p) \le d_D^K(g_j(q), p_j) + d_M^K(p_j, p) = d_D^K(q, g_j^{-1}(p_j)) + d_M^K(p_j, p) \to 0$ as $j \to \infty$. Therefore $g_j(q) \to p$.

Since bD can be approximated by a biholomorphic image of B_n to the third order at p, it follows from the preceding paragraph and the arguments in [27] that $V_D^C(q)/V_D^E(q) = 1$, where the Eisenman-Kobayashi and Carathéodory volume forms are defined with respect to $(B_n, 0)$. Therefore, by Lemma 2.1(3), D is biholomorphic to B_n.

We have therefore established the main theorem in this case.

4. Proof of the Main Theorem, Part II

In this section, we prove the main theorem in the case when the boundary bD of D does not contain any strictly pseudoconvex point. Recall that, when the domain D sits in the complex Euclidean space, then the boundary bD *always* has a strongly pseudoconvex—indeed a strongly convex—point (see [18]). It is the recognition of this possible failure that gives us our handle on the situation in a general complex manifold.

In this case, it follows from Lemma 2.2 that bD is Levi pseudoconvex. Since the boundary bD does not contain any strictly pseudoconvex point, it must be Levi flat, i.e., the Levi form vanishes identically on bD. (The assumption that the domain D sits in a complex manifold of dimension 2 comes into play here.) The proof of this case is analogous to proofs appearing in [8], [9]. Since there are some essential modifications, we provide the details below.

Throughout this section, the invariant volume forms are defined with respect to $(\Delta_2, 0)$.

Let $r = r(z)$ be a defining function for bD (i.e., there exists a neighborhood N of $\overline{D}$ such that $r \in C^\infty(N)$, $dr \neq 0$ on bD, and $D = \{z \in N, r(z) < 0\}$). By choosing $\epsilon_1 > \epsilon_2 > 0$ sufficiently small, we may be sure that the domains $D_j = \{z \in D, r(z) < -\epsilon_j\}$, $j = 1, 2$, satisfy

(1) $D_2 \supset\supset D_1 \supset\supset K$ (where K is the fundamental set for $\mathrm{Aut}(D)$);

and

(2) Both D_1 and D_2 are retracts of D.

Let $\pi\colon \widetilde{D} \to D$ be the universal covering of D. Set $\widetilde{D}_j = \pi^{-1}(D_j)$, $j = 1, 2$. By Lemma 2.5, the domains $\widetilde{D}_j$, $j = 1, 2$, are simply connected.

Let $\{p_k\}_{k=1}^\infty$ be a sequence in D converging to a point $p \in bD$. Then there exists $\{g_k\} \subset \mathrm{Aut}(D)$ such that $g_k^{-1}(p_k) \in K$, the fundamental set for $\mathrm{Aut}(D)$. Passing to a subsequence, we may assume that $\{g_k^{-1}(p_k)\}$ converges to a point q in K. As in the proof of Part I, $q_k = g_k(q)$ converges to p. By Lemmas 2.3 and 2.4, we may further assume that $\{g_k\}$ converges local uniformly on D to a holomorphic map $g\colon D \to bD$.

Let $V = g(D_2)$. It follows from (the proof of) Lemma 4.1 in [9] (see also Lemma 3.3 in [8]) that V is a hyperbolic, locally closed open Riemann surface in M. (The hyperbolicity of V follows from that of M.) Since every open Riemann surface is Stein (cf. [21], Theorem 3.10.13) and every holomorphic line bundle over an open Riemann surface is trivial (cf. [6], Theorem 30.3), it follows from [4] and [25], Corollary 1 that there exists a biholomorphic mapping Ψ from an open neighborhood W of V to an open neighborhood U of $V \times \{0\}$ in $V \times \mathbb{C}$ such that $\Psi(z) = (z, 0)$ for $z \in V$. We may assume that $U \subseteq V \times \Delta$.

Let $\pi_1 \colon \Delta \to V$ be the universal covering of V. Let $\pi_2 \colon \Delta \times \mathbb{C} \to V \times \mathbb{C}$ be defined by $\pi_2(z, w) = (\pi_1(z), w)$. Define $\Omega = \pi_2^{-1}(\Psi(W \cap D))$. Then

$$\Omega = \{(z', w') \in \Delta \times \mathbb{C};\, \pi_2(z', w') \in U, \tilde{r}(z', w') < 0\}$$

where $\tilde{r}(z', w') = r(\Psi^{-1}(\pi_2(z', w')))$. It is easy to see that $b\Omega$ is smooth and Levi flat in a neighborhood of $\Delta \times \{0\}$.

Let $\tilde{q} \in \pi^{-1}(q)$ and $p' \in \pi_2^{-1}(\Psi(p))$. After a unitary transformation, we may assume that $p' = (0, 0)$. Since π_2 is locally one-to-one, there exist unique liftings q'_k of $\Psi(q_k)$ for sufficiently large k such that $q'_k \to p'$. Since $\widetilde{D}_1$ is simply-connected and $g_k \circ \pi(\widetilde{D}_1) = g_k(D_1) \subseteq W \cap D$ for sufficiently large k, there exist unique liftings $\tilde{g}_k$ and $\tilde{g} \colon \widetilde{D}_1 \to \Omega$ of $\Psi \circ g_k \circ \pi$ and $\Psi \circ g \circ \pi$ respectively such that $\tilde{g}_k(q) = q'_k$ and $\tilde{g}(q) = p'$. Let $\widehat{D}_1$ be a relatively compact subdomain of $\widetilde{D}_1$ such that $\pi(\widehat{D}_1) \supset K$. Choose $\delta \in (0, 1)$ sufficiently closed to 1 such that $\tilde{g}(\widehat{D}_1) \subset\subset \Delta_\delta \times \{0\}$. Let $\epsilon > 0$ and $U_{\delta\epsilon} = \Delta_\delta \times \Delta_\epsilon$. Then $\tilde{g}_k(\widehat{D}_1) \subset \Omega \cap U_{\delta\epsilon}$ for sufficiently large k. It follows from Lemma 2.1 that, for sufficiently large k,

$$V^C_{\widehat{D}_1}(\tilde{q}) \geq \tilde{g}^*_k\big(V^C_{\Omega \cap U_{\delta\epsilon}}\big)(q'_k)$$

and

$$\begin{aligned} V^E_{\widetilde{D}}(\tilde{q}) &= \pi^*\big(V^E_D\big)(q) = g^*_k \circ \pi^*\big(V^E_D\big)(q_k) \\ &\leq \Psi^* \circ g^*_k \circ \pi^*\big(V^E_{\pi_2(\Omega \cap U_{\delta\epsilon})}\big)(q_k) = \tilde{g}^*_k\big(V^E_{\Omega \cap U_{\delta\epsilon}}\big)(q'_k). \end{aligned}$$

The last equation follows from $\pi_2 \circ \tilde{g}^*_k = \Psi \circ g_k \circ \pi$ and Lemma 2.1 (2). Therefore, after choosing a complex coordinate of $\widetilde{D}$ near $\tilde{q}$, we have

$$\frac{V^C_{\widehat{D}_1}(\tilde{q})}{V^E_{\widetilde{D}}(\tilde{q})} \geq \frac{C_{\Omega \cap U_{\delta\epsilon}}(q'_k)}{E_{\Omega \cap U_{\delta\epsilon}}(q'_k)}.$$

Using Theorem 8 in [3], we can prove that $V^C_{\widehat{D}_1}(\tilde{q})/V^E_{\widetilde{D}}(\tilde{q}) \geq 1$ (see the proof of Lemma 4.2 in [9] for details). Exhausting $\widetilde{D}_1$ by $\widehat{D}_1$, we have $V^C_{\widetilde{D}_1}(\tilde{q})/V^E_{\widetilde{D}}(\tilde{q}) \geq 1$. Letting $\epsilon_1 \to 0$, since D_1 and $\widetilde{D}_1$ exhaust D and $\widetilde{D}$ respectively, we have $V^C_{\widetilde{D}}(\tilde{q})/V^E_{\widetilde{D}}(\tilde{q}) \geq 1$. Therefore $V^C_{\widetilde{D}}(\tilde{q}) = V^E_{\widetilde{D}}(\tilde{q})$. Hence $\widetilde{D}$ is biholomorphic to the bidisc.

5. Concluding remarks

(1) The main theorem remains true if the boundary bD of the domain D is piecewise smooth. The proof is essentially the same: if one of the regular boundary points is strictly pseudoconvex, then the proof is the same as that of Part I; if none of the regular boundary points is strictly pseudoconvex, then the proof reduces to that of Part II (note that by Lemma 3.2 in [9] the variety V in Part II lies on a single defining hypersurface). A related result of Pinchuk [22] states that any bounded homogeneous domain in $\mathbb{C}^n$ with piecewise smooth boundary is biholomorphic to a product of balls.

(2) It was proved in [9] that a simply-connected domain D in $\mathbb{C}^2$ with generic piecewise smooth, Levi flat boundary and non-compact automorphism group is biholomorphic to the bidisc. This result was proved earlier by K.-T. Kim [15] for the case when the domain D is convex. (See [28] for related results.) Combining the arguments in this paper with those in [9], one can prove the following generalization: Let M be a hyperbolic complex surface and $D \subset\subset M$ a subdomain with a generic piecewise smooth, Levi flat boundary. If $\mathrm{Aut}(D)$ is non-compact, then the universal covering of D is biholomorphic to the bidisc. The proof of this generalization goes as follows: when one of the singular boundary points of the domain D is a boundary accumulation point, it follows from the same lines of the arguments in Section 5 of [9] that the domain D is biholomorphic to the bidisc; otherwise, the proof is the same as that of Part II in the present paper.

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