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A GLOBAL MORPHISM FROM THE DOUADY SPACE TO THE CYCLE SPACE

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Abstract

We establish, for any given complex space M, a global morphism from the reduction of its Douady space to its cycle space. This morphism is an extension of the morphism defined in [1] from the subspace of the Douady space formed by all pure dimensional subspaces of M to the cycle space of M. In the case where M is projective this morphism is the classical morphism from the Hilbert scheme of M to the Chow scheme of M.

Introduction

Let M be a complex space. Then there is a natural map from its Douady space to its cycle space that maps every compact complex subspace of M to its fundamental cycle. The object of the present paper is to prove that this map is a holomorphic map from the reduction of the Douady space of M to the cycle space of M. See corollary 3.3.

This result is known in the case where M is projective since in that case there is a global morphism from the Hilbert scheme of M to the Chow scheme of M (see [6]), and in that case it is easy to see that the Douady space of M is the complex space associated with the Hilbert scheme of M and from [1] it is known that the cycle space of M is the complex space associated with the Chow scheme of M.

For a general complex space M a weaker version of this result can be found in [1], where it is proved that the restriction of the above map to the reduction of the subspace of the Douady space of M consisting of all *pure dimensional* subspaces (having no embedded components) is holomorphic.

We will obtain this result as a simple consequence of the following more general theorem. See section 1 for notations and terminology.

Main Theorem. *Let M be a complex space and let $\mathscr{X} \subset S \times M$ be a flat family of n-dimensional subspaces of M. Let $q\colon \mathscr{X} \to M$ and $\pi\colon \mathscr{X} \to S$ denote the canonical projections and for each s in S let $\mathscr{X}_s$ denote the fibre*

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of π over s and put $Z_s := q(\mathscr{X}_s)$. Then $([Z_s])_{s \in S_{\mathrm{red}}}$ is an analytic family of n-cycles in M.

In section 1 we introduce the basic definitions and preliminary results needed. Then a special case will be proved in section 2 and finally we prove the Main Theorem in section 3.

The proof is based on the same ideas as in [1].

1. Basic notions and preliminaries

In this section M will denote a (not necessarily reduced) complex space.

1.1. Basic definitions

DEFINITION 1.1. A *flat family of subspaces* of M is a pair of complex spaces $(S, \mathscr{X})$ such that $\mathscr{X}$ is a subspace of $S \times M$ and such that the natural projection $\pi : \mathscr{X} \to S$ is flat. If the projection is also a proper map then the family is called *flat and proper*.

DEFINITION 1.2. An *n-cycle* in M is a locally finite linear combination

$$Z = \sum_{i \in I} n_i Z_i$$

with coefficients in N^*, where the Z_i are globally irreducible complex subspaces of M of dimension n such that $Z_i \neq Z_j$ holds for $i \neq j$. The set

$$|Z| := \bigcup_{i \in I} Z_i$$

is called the *support* of Z. The n-cycle Z is called *compact* if its support $|Z|$ is compact.

DEFINITION 1.3. A *scale* of M is a triplet $E = (U, B, j)$ having the following properties:

(i) $U \Subset \mathsf{C}^n$ and $B \Subset \mathsf{C}^p$ are open polydisks,

(ii) j is a holomorphic embedding of an open subset M_E of M into an open neighbourhood of $\overline{U} \times \overline{B}$ in C^{n+p}.

The scale E is said to be *adapted to* an n-cycle Z if

$$j(|Z| \cap M_E) \cap (\overline{U} \times \partial B) = \emptyset.$$

The k-th symmetric group acts on $(\mathsf{C}^p)^k = \mathsf{C}^p \times \cdots \times \mathsf{C}^p$ by permutation

$$\beta(x_1, \ldots, x_k) := (x_{\beta(1)}, \ldots, x_{\beta(k)}).$$

The orbit space of this action is called the *k-th symmetric power* of $\mathbb{C}^p$ and will be denoted by $\mathrm{Sym}^k(\mathbb{C}^p)$. It is a normal complex space. (See for instance [5]).

The k-th symmetric group acts in the same way on B^k and the orbit space $\mathrm{Sym}^k(B)$ can be naturally identified with an open subset of $\mathrm{Sym}^k(\mathbb{C}^p)$.

Assume $E = (U, B, j)$ is a scale of a complex space M adapted to an n-cycle Z in M. Then Z induces a ramified covering of a certain degree of an open neighborhood of $\overline{U}$ whose degree will be denoted by $\deg_E Z$ or k_E for short. Hence Z induces a holomorphic map

$$U \to \mathrm{Sym}^{k_E}(B).$$

For a detailed discussion see [1] or [7].

DEFINITION 1.4. Let S be a reduced complex space and let $(Z_s)_{s\in S}$ be a family of n-cycles in M.

(i) The family $(Z_s)_{s\in S}$ is called *analytic* if for every $s_0 \in S$ and every scale $E = (U, B, j)$, adapted to the n-cycle Z_{s_0}, there exists an open neighbourhood S_E of s_0 in S such that
   (a) E is adapted to Z_s for all $s \in S_E$,
   (b) $\deg_E Z_s = \deg_E Z_{s_0}$ for all $s \in S_E$,
   (c) the map $g_E\colon S_E \times U \to \mathrm{Sym}^{k_E}(B)$ is holomorphic, where $g_E(s, \cdot)\colon U \to \mathrm{Sym}^{k_E}(B)$ is the holomorphic map induced by Z_s.

(ii) The family $(Z_s)_{s\in S}$ is called a *proper analytic family of compact n-cycles* if it is analytic, every cycle is compact and for every $s_0 \in S$ and every neighborhood W of $|Z_{s_0}|$ there exists an open neighborhood S_0 of s_0 in S such that $|Z_s| \subset W$ for all $s \in S_0$.

REMARK 1.5. For a family $(Z_s)_{s\in S}$ of n-cycles in M the set

$$G := \{(s, z) \in S \times M \mid z \in |Z_s|\}$$

is usually called the *graph* of the family and it is easy to see that the topological condition in (ii) is verified if and only if the canonical projection $G \to S$ is a proper mapping.

Let Z be an irreducible component of M. Then $\mathring{Z} := Z \setminus \mathrm{Sing}(M_{\mathrm{red}})$ is a connected manifold and we have $\mathcal{O}_{\mathring{Z},z} = \mathcal{O}_{M_{\mathrm{red}},z}$ for all $z \in \mathring{Z}$. For each $z \in \mathring{Z}$ the analytic algebra $\mathcal{O}_{\mathring{Z},z}$ is regular so there exists a section

$$\begin{tikzcd} \mathcal{O}_{M,z} \arrow[r, "\mathrm{can}"'] & \mathcal{O}_{\mathring{Z},z} \arrow[l, bend right] \end{tikzcd}$$

which makes $\mathcal{O}_{M,z}$ a coherent $\mathcal{O}_{\mathring{Z},z}$-module. The rank of this module is independent of the choice of the section as can be seen in the following way.

Put

$$\mathscr{O} := \mathscr{O}_{M_{\mathrm{red}},z}$$

and let $\mathfrak{n}$ denote the nilradical of $\mathscr{O}$. Then it is easy to show that the length of the localized ring $\mathscr{O}_{\mathfrak{n}}$ is equal to the rank of $\mathscr{O}$ as a $\mathscr{O}_{\mathrm{red}}$-module.

It follows that the rank is the same for all points z in $\mathring{Z}$ and it is called the *multiplicity of the component Z in M*.

Now suppose that the complex space M is of dimension n and let M' denote the union of of the irreducible components of M whose dimension is strictly less than n. Let $\overline{M}$ denote the analytic closure of $M \setminus M'$ in M, in other words $\overline{M}$ is the smallest complex subspace of M that contains $M \setminus M'$. Then $\overline{M}$ is a complex space of pure dimension n.

Let $(\overline{M}_i)_{i\in I}$ be the family of the irreducible components of $\overline{M}$, in other words the irreducible components of dimension n of M, and put

$$[M] := \sum_{i\in I} n_i \overline{M}_i$$

where n_i is the multiplicity of $\overline{M}_i$ in M.

DEFINITION 1.6. The n-cycle $[M]$ is called the *fundamental cycle* of M.

1.2. Preliminary results

For every positive integer l let $S^l(\mathbb{C}^p)$ be the l-th symmetric product of the complex vector space $\mathbb{C}^p$ and let $\sigma_l : (\mathbb{C}^p)^k \to S^l(\mathbb{C}^p)$ be the l-th elementary symmetric polynomial. One can interpret this mapping in the following way. Think of the elements in $\mathbb{C}^p$ as $\mathbb{C}$-linear forms on the dual space $(\mathbb{C}^p)^\star$, in other words think of them as homogeneous poynomials of degree 1 in p variables. Then $S^l(\mathbb{C}^p)$ becomes the space of homogeneous polynomials of degree l in p variables and $\sigma_l(v_1, \ldots, v_k)$ is a symmetric polynomial of degree l in p variables for every $l \in \{1, \ldots k\}$ and for every $(v_1, \ldots, v_k) \in (\mathbb{C}^p)^k$.

PROPOSITION 1.7. *The mapping*

$$S : \mathrm{Sym}^k(\mathbb{C}^p) \to \bigoplus_{l=1}^{k} S^l(\mathbb{C}^p)$$

induced by $(\sigma_1, \ldots, \sigma_k)$ is a proper holomorphic embedding.

PROOF. See [1] or [7].

THEOREM 1.8 (Douady). *Let M be a complex space. Then there exists a complex space $\mathscr{D} = \mathscr{D}(M)$ and a subspace $\mathscr{X} \subset \mathscr{D} \times M$, called the universal (flat and proper) family, having the following properties:*

(i) *The canonical projection* $\pi : \mathscr{X} \to \mathscr{D}$ *is a flat and proper map.*

(ii) (Universal property) *If* $(S, \mathscr{Z})$ *is a flat and proper family of subspaces of* M*, then there exists a unique holomorphic map* $f : S \to \mathscr{D}$ *such that*

$$\mathscr{Z} = S \times_{\mathscr{D}} \mathscr{X}.$$

PROOF. See [3].

The set of all compact n-cycles in X will be denoted by $\mathscr{C}_n(M)$ and the set of all compact cycles in M will be denoted by $\mathscr{C}(M)$. Obviously $\mathscr{C}(M)$ is the disjoint union of the family $\big(\mathscr{C}_j(M)\big)_{j\geq 0}$.

THEOREM 1.9 (Barlet). *The set* $\mathscr{C}_n(M)$ *carries a reduced complex structure such that the following conditions are fulfilled:*

(i) *The family* $(X)_{X\in\mathscr{C}_n(M)}$ *is a proper analytic family of compact n-cycles.*

(ii) (Universal property) *For every analytic family* $(Z_s)_{s\in S}$ *of compact n-cycles the map*

$$S \to \mathscr{C}_n(M), \quad s \mapsto Z_s$$

is holomorphic.

PROOF. See [1].

The complex spaces $\mathscr{D}(M)$ and $\mathscr{C}(M)$ are called the Douady *space* and the (Barlet) *cycle space* of M.

2. The case of dimension zero

THEOREM 2.1. *Let* T *be a reduced complex space and let* B *be a relatively compact open polydisk in* $\mathbb{C}^p$*. Let* $\mathscr{Z} \subset T \times B$ *be a flat and proper family of* 0*-dimensional subspaces of* B*. Then the associated family of* 0*-cycles in* B *is analytic.*

PROOF. For each t in T let Y_t denote the image of the fiber $\mathscr{Z}_t$ by the canonical projection $q : \mathscr{Z} \to B$, so that

$$\mathscr{Z}_t = \{t\} \times Y_t.$$

Let $\mathscr{A}^t$ be the structure sheaf of the zero-dimensional subspace Y_t of B and let X_t be the fundamental cycle of Y_t. Then X_t is given by the formula

$$X_t := \sum_{y\in Y_t} n_y(t)\{y\}$$

where $n_y(t) := \dim_{\mathbb{C}} \mathscr{A}^t_y$.

The canonical projection $\pi\colon \mathscr{X} \to T$ is flat and finite so $\pi_*\mathcal{O}_{\mathscr{X}}$ is a locally free $\mathcal{O}_T$-module. Without loss of generality we may assume T connected and consequently $\pi_*\mathcal{O}_{\mathscr{X}}$ of constant rank which will be denoted by k. For every t in T we then have

$$(\pi_*\mathcal{O}_{\mathscr{X}})_t = \bigoplus_{y\in Y_t} \mathcal{O}_{\mathscr{X},(t,y)}$$

and

$$\mathscr{A}_y^t = \mathcal{O}_{\mathscr{X},(t,y)} \otimes_{\mathcal{O}_{\mathscr{X},(t,y)}} \mathsf{C}$$

for every y in Y_t. It follows that

$$\dim_{\mathsf{C}} \mathscr{A}_y^t = \dim_{\mathcal{O}_{T,t}} \mathcal{O}_{\mathscr{X},(t,y)}$$

and that

$$\sum_{y\in Y_t} n_y(t) = k$$

for every t in T. Hence $X_t \in \operatorname{Sym}^k(B)$ for all t in T and the proof of the theorem consists of showing that the mapping

$$T \to \operatorname{Sym}^k(B), \qquad t \mapsto X_t$$

is holomorphic.

As before let $q\colon \mathscr{X} \to B$ denote the canonical projection and write $q = (q_1, \dots q_p)$. Each q_j is a global holomorphic function on $\mathscr{X}$ so multiplication by q_j defines a $\mathcal{O}_T$-linear endomorphism

$$\pi_*\mathcal{O}_{\mathscr{X}} \to \pi_*\mathcal{O}_{\mathscr{X}}$$

and for each t in T it induces a C-linear endomorphism

$$L_j\colon \mathscr{A}^t \to \mathscr{A}^t.$$

In terms of the decomposition

$$\mathscr{A}^t = \bigoplus_{y\in Y_t} \mathscr{A}_y^t$$

we obviously have for every j that

$$L_j\left(\sum_{y\in Y_t} v_y\right) = \sum_{y\in Y_t} q_j(t,y)v_y$$

so for each y in Y_t every non-zero element of $\mathscr{A}_y^t$ is an eigenvector for L_j corresponding to the eigenvalue $q_j(t,y)$.

Now let t be a fixed point in T. Since the $\mathcal{O}_{T,t}$-algebra $(\pi_*\mathcal{O}_{\mathcal{X}})_t$ is a free $\mathcal{O}_{T,t}$-module the polynomial algebra $(\pi_*\mathcal{O}_{\mathcal{X}})_t[X_1, \dots, X_p]$ is a free $\mathcal{O}_{T,t}[X_1, \dots, X_p]$-module. Put

$$P := q_1 X_1 + \cdots + q_p X_p$$

and consider it as an element in $\mathcal{O}_{\mathcal{X}}(\mathcal{X})[X_1, \dots, X_p]$. Multiplication by P defines a $\mathcal{O}_{T,t}[X_1, \dots, X_p]$-linear endomorphism

$$\mathcal{L}\colon (\pi_*\mathcal{O}_{\mathcal{X}})_t[X_1, \dots, X_p] \to (\pi_*\mathcal{O}_{\mathcal{X}})_t[X_1, \dots, X_p]$$

whose characteristic polynomial is of the form

$$\lambda^k + \alpha_1\lambda^{k-1} + \cdots + \alpha_{k-1}\lambda + \alpha_k$$

where α_j is a homogeneous polynomial of degree j in $\mathcal{O}_{T,t}[X_1, \dots, X_p]$.

For each t in T the polynomial

$$(*) \qquad \lambda^k + \alpha_1(t)\lambda^{k-1} + \cdots + \alpha_{k-1}(t)\lambda + \alpha_k(t)$$

is the characteristic polynomial of the $\mathbb{C}[X_1, \dots, X_p]$-linear endomorphism

$$\mathcal{L}(t)\colon \mathcal{A}^t[X_1, \dots, X_p] \to \mathcal{A}^t[X_1, \dots, X_p]$$

defined by multiplication by the polynomial

$$P(t) := q_1(t, \cdot)X_1 + \cdots + q_p(t, \cdot)X_p.$$

From our considerations above concerning the endomorphisms $L_1, \dots, L_p$ we see that the characteristic polynomial (*) has k different roots in $\mathbb{C}[X_1, \dots, X_p]$, namely

$$q_1(t, y)X_1 + \cdots + q_p(t, y)X_p,$$

where y ranges over the k points in Y_t (counted with multiplicities). Hence $\alpha_1(t), \dots, \alpha_k(t)$ are the symmetric polynomials of these roots and this proves that the holomorphic map

$$T \to \bigoplus_{j=1}^{k} S_j(\mathbb{C}^p), \qquad t \mapsto (\alpha_1(t), \dots, \alpha_k(t))$$

is the composition of the map

$$T \to \mathrm{Sym}^k(B), \qquad t \mapsto X_t$$

and the holomorphic embedding $\mathrm{Sym}^k(B) \to \bigoplus_{j=1}^{k} S_j(\mathbb{C}^p)$ described in section 1. It then follows that the former map is holomorphic and the theorem is proved.

3. The general case

For the proof of our Main Theorem (stated in the introduction) we need the following results.

LEMMA 3.1. *Let S be a reduced complex space, let U be an open polydisk in $\mathbf{C}^n$, let Σ be a closed subset of $S \times U$ such that $\Sigma \cap (\{s\} \times U)$ is contained in a nowhere dense analytic subset of $\{s\} \times U$ for all s in S and let*

$$g\colon S \times U \setminus \Sigma \to \mathbf{C}$$

be a holomorphic function. If the restriction of g to $\{s\} \times U \setminus \Sigma$ extends holomorphically to $\{s\} \times U$ for all s in S, then g extends holomorphically to $S \times U$.

PROOF. The proof is based on Cauchy's formula. See [1] for details.

LEMMA 3.2. *Let A and B be local Noetherian rings and $\rho\colon A \to B$ be a local homomorphism such that B is a flat A-module via ρ. Let $\mathfrak{a}$ be a proper ideal of A and let $\mathfrak{b}$ denote the ideal generated by $\rho(\mathfrak{a})$ in B. Then for every finitely generated B-module M the following conditions are equivalent:*

(i) *M is a flat B-module.*

(ii) *M is a flat A-module and $M/\mathfrak{b}M$ is flat $B/\mathfrak{b}$-module.*

PROOF. See [2].

PROOF OF THE MAIN THEOREM. Since base change respects flatness we may assume without loss of generality that the space S is reduced. Let s_0 be a point in S and let $E = (U, B, j)$ be a scale of M adapted to Z_{s_0}, i.e.

$$j^{-1}(\overline{U} \times \partial B) \cap Z_{s_0} = \emptyset.$$

Since $j^{-1}(\overline{U} \times \partial B)$ is compact there exists an open neighbourhood S_0 of s_0 in S such that

$$j^{-1}(\overline{U} \times \partial B) \cap Z_s = \emptyset$$

for all s in S_0. Then

$$Y := (\mathrm{id}_{S_0} \times j)((S_0 \times j^{-1}(U \times B)) \cap \mathscr{Z})$$

is a complex n-dimensional subspace of $S_0 \times U \times B$ that satisfies the following conditions:

- The canonical projection $f\colon Y \to S_0$ is flat.
- The canonical projection $\pi\colon Y \to S_0 \times U$ is finite.

For each s in S_0 the fiber of f over s will be denoted by Y_s and $\{s\} \times U$ will often be identified with U to simplify the presentation.

Denote by Σ the set of all points in $S_0 \times U$ where the coherent $\mathcal{O}_{S_0 \times U}$-module $\pi_* \mathcal{O}_Y$ is not free. Then Σ is a thin analytic subset of $S_0 \times U$. See [4] for instance.

Let us first show that $\Sigma \cap (\{s\} \times U)$ is a thin analytic subset of $\{s\} \times U$ for all $s \in S_0$. For $s \in S_0$ and $u \in U$, put $M := (\pi_* \mathcal{O}_Y)_{(s,u)}$, $B := \mathcal{O}_{S_0 \times U,(s,u)}$, $A := \mathcal{O}_{S_0,s}$ and let $\mathfrak{a}$ be the maximal ideal of A. Then by lemma 3.2 we get:

$(\pi_* \mathcal{O}_{Y_s})_u$ is a free $\mathcal{O}_{U,u}$-module if and only if $(\pi_* \mathcal{O}_Y)_{(s,u)}$ is a free $\mathcal{O}_{S \times U,(s,u)}$-module.

It follows that $\Sigma \cap (\{s\} \times U)$ is a thin analytic subset of $\{s\} \times U$ for all $s \in S_0$.

For $s \in S_0$, denote by Y_s' the union of those irreducible components of Y_s whose dimension is strictly less than n and denote by $\overline{Y}_s$ the analytic closure of $Y_s \setminus Y_s'$ in Y_s. Let us show that $\pi(Y_s \setminus \overline{Y}_s) \subset \Sigma$.

If $y \in Y_s \setminus \overline{Y}_s$ then $\mathcal{O}_{Y_s,y}$ is not a free module over $\mathcal{O}_{U,\pi(y)}$ since $\dim_y Y_s < n = \dim U$. It follows that $\left(\pi_* \mathcal{O}_{Y_s}\right)_{\pi(y)}$ is not a free $\mathcal{O}_{U,\pi(y)}$-module so by lemma 3.2 the $\mathcal{O}_{S_0 \times U,\pi(y)}$-module $(\pi_* \mathcal{O}_Y)_{\pi(y)}$ is not free. Hence $\pi(y) \in \Sigma$. This shows that $\pi(Y_s \setminus \overline{Y}_s) \subset \Sigma$ for all $s \in S_0$.

Put $T := S_0 \times U \setminus \Sigma$. Then

$$\pi^{-1}(T) \subset T \times B$$

is a flat and proper family of 0-dimensional subspaces of B and by theorem 2.1 the corresponding family of fundamental cycles is an analytic family of 0-cycles in B. Let

$$g\colon T \to \mathrm{Sym}^k(B)$$

be the holomorphic map defined by that family, where k is the rank of the $\mathcal{O}_{S_0 \times U}$-module $\pi_* \mathcal{O}_Y$. For every s in S_0 the holomorphic map

$$g(s, \cdot)\colon \{s\} \times U \setminus \Sigma \to \mathrm{Sym}^k(B)$$

extends to the holomorphic map induced by the fundamental cycle $[Z_s]$ in the scale E. From this and lemma 3.1 we deduce that the map g extends to a holomorphic map

$$\hat{g}\colon S_0 \times U \to \mathrm{Sym}^k(B)$$

such that $\hat{g}(s, \cdot)\colon U \to \mathrm{Sym}^k(B)$ is the holomorphic map induced by $[Z_s]$ in the scale E. Hence $([Z_s])_{s \in S}$ is an analytic family of n-cycles and the proof is completed.

COROLLARY 3.3. *Let M be a complex space. Then the mapping*

$$\mathscr{D}(M)_{\text{red}} \to \mathscr{C}(M), \qquad Z \to [Z]$$

is holomorphic.

PROOF. It is easy to see that the complex subspaces of M belonging to the same connected component of $\mathscr{D}(M)$ are all of the same dimension. Thus we only have to show that for any flat and proper family of subspaces of a certain dimension in M the corresponding analytic family of fundamental cycles is also proper.

Let $(S, \mathscr{Z})$ be a flat and proper family of n dimensional subspaces of M. To show that the corresponding analytic family of the fundamental n-cycles is proper we may assume, without loss of generality, that the space S is reduced and (globally) irreducible. Put $d := \dim S$ and let $\pi: \mathscr{Z} \to S$ be the canonical projection. Since π is flat the equality

$$\dim_z \mathscr{Z} = \dim_z \mathscr{Z}_{\pi(z)} + d$$

holds for every $z \in \mathscr{Z}$. It follows easily that the graph of the corresponding analytic family is the union of all irreducible components of dimension $n + d$ of $\mathscr{Z}$. The restriction of π to this subspace of $\mathscr{Z}$ being proper the proof is completed in virtue of the remark following definition 1.4.

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