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ON RØRDAM'S CLASSIFICATION OF CERTAIN C^*-ALGEBRAS WITH ONE NON-TRIVIAL IDEAL, II

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Abstract

In this paper we extend the classification results obtained by Rørdam in the paper [16]. We prove a strong classification theorem for the unital essential extensions of Kirchberg algebras, a classification theorem for the non-stable, non-unital essential extensions of Kirchberg algebras, and we characterize the range in both cases. The invariants are cyclic six term exact sequences together with the class of some unit.

In the mid-nineties Rørdam considered the classification problem for essential extensions of Kirchberg algebras ([16]). It turned out that one has to consider three cases: the stable case, the unital case, and the non-stable, non-unital case. Using the associated cyclic six term exact sequence he solved the classification problem in the first case and characterized the range of the invariant.

Rørdam's article is quite outstanding both in the general classification theory and with respect to this specific classification problem. Since Rørdam's methods are not similar to the usual methods of classification theory and some proofs are ad hoc, his methods for classifying non-simple C^*-algebras have not been generalized until recently ([5]). While Rørdam's article did solve the classification problem in the stable case, it does not seem possible to apply his argument neither to the other two cases nor to prove a lifting theorem in the stable case.

Extending Rørdam's work, the lifting theorem in the stable case and a classification theorem in the unital case were proved in [4]. In the present paper, we will present the classification results and describe the ranges of the invariants for all the three cases – in the first two cases, we will even allow for lifting of isomorphisms. Thus giving a very satisfactory answer to the classification problem for this class of C^*-algebras.

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1. Introduction

In this paper we will consider extensions of simple C^*-algebras. Let $\mathscr{E}$ denote the category of extensions $e\colon \mathfrak{B} \hookrightarrow \mathfrak{E} \twoheadrightarrow \mathfrak{A}$ of separable C^*-algebras $\mathfrak{A}$ and $\mathfrak{B}$ with the morphisms being triples (ϕ_0, ϕ_1, ϕ_2) of *-homomorphisms such that the diagram

$$\begin{array}{cccccc} e\colon & \mathfrak{B} & \hookrightarrow & \mathfrak{E} & \twoheadrightarrow & \mathfrak{A} \\ & \downarrow{\scriptstyle\phi_0} & & \downarrow{\scriptstyle\phi_1} & & \downarrow{\scriptstyle\phi_2} \\ e'\colon & \mathfrak{B}' & \hookrightarrow & \mathfrak{E}' & \twoheadrightarrow & \mathfrak{A}' \end{array}$$

commutes (note that we use the symbols $\hookrightarrow$ resp. $\twoheadrightarrow$ meaning an injective resp. surjective morphism).

Recall that a category $\mathscr{D}$ is a subcategory of the category $\mathscr{C}$, if every object in $\mathscr{D}$ is an object in $\mathscr{C}$, every morphism between objects in $\mathscr{D}$ is a morphism in $\mathscr{C}$, the identity morphism of every object in $\mathscr{D}$ is the identity morphism of that object in $\mathscr{C}$, and the composition in $\mathscr{D}$ is inherited from $\mathscr{C}$. We say that a subcategory $\mathscr{D}$ of $\mathscr{C}$ is *full*, if for all objects X and Y in $\mathscr{D}$, every morphism in $\mathscr{C}$ from X to Y is in fact a morphism in $\mathscr{D}$. When nothing else is stated, functors considered in this paper are assumed to be covariant.

Consider the subcategory of the category of C^*-algebras consisting of separable C^*-algebras with exactly one non-trivial ideal, and as morphisms we take *-homomorphisms which map the non-trivial ideal to the non-trivial ideal. This category is equivalent to the full subcategory $\mathscr{E}_0$ of $\mathscr{E}$ consisting of all essential extensions $e\colon \mathfrak{B} \hookrightarrow \mathfrak{E} \twoheadrightarrow \mathfrak{A}$, where $\mathfrak{A}$ and $\mathfrak{B}$ are separable, simple C^*-algebras – and we will freely use this identification.

Now we will consider the category $\mathscr{H}$ of cyclic six term exact sequences of countable abelian groups. We can canonically consider $\mathscr{H}$ as the subcategory of the category of chain complexes over Z with chain maps as morphisms (cf. [8, §IV.1]) – the objects are the chain complexes consisting of countable abelian groups that are exact and periodic of period six, and the morphisms are those chain maps, which are periodic of period six. In this way we may as well denote the cyclic six term exact sequence

$$\begin{array}{ccccc} M_0 & \xrightarrow{\partial_0} & M_1 & \xrightarrow{\partial_1} & M_2 \\ {\scriptstyle\partial_5}\uparrow & & & & \downarrow{\scriptstyle\partial_2} \\ M_5 & \xleftarrow[\partial_4]{} & M_4 & \xleftarrow[\partial_3]{} & M_3 \end{array}$$

by $(M_n, \partial_n)_{n\in\mathsf{Z}_6}$ (here Z_6 denotes the cyclic group of order six).

Now we define a functor $K_\circ$ from $\mathscr{E}$ to $\mathscr{H}$ as follows. For every extension $e\colon \mathfrak{B} \overset{\iota}{\hookrightarrow} \mathfrak{E} \overset{\pi}{\twoheadrightarrow} \mathfrak{A}$ in $\mathscr{E}$ we let $K_\circ(e)$ be the standard cyclic six term exact sequence in K-theory associated to e

$$\begin{array}{ccccc} K_0(\mathfrak{B}) & \xrightarrow{K_0(\iota)} & K_0(\mathfrak{E}) & \xrightarrow{K_0(\pi)} & K_0(\mathfrak{A}) \\ {\scriptstyle\delta_1}\uparrow & & & & \downarrow{\scriptstyle\delta_0} \\ K_1(\mathfrak{A}) & \xleftarrow[K_1(\pi)]{} & K_1(\mathfrak{E}) & \xleftarrow[K_1(\iota)]{} & K_1(\mathfrak{B}) \end{array}$$

We define $K_{\circ}$ on morphisms in the obvious way.

Let $\mathscr{E}_K$ be the full subcategory of $\mathscr{E}_0$ consisting of all essential extensions $e\colon \mathfrak{B} \hookrightarrow \mathfrak{E} \twoheadrightarrow \mathfrak{A}$, where $\mathfrak{A}$ and $\mathfrak{B}$ are Kirchberg algebras satisfying the UCT (a Kirchberg algebra is a separable, nuclear, simple, purely infinite C^*-algebra). By Zhang's dichotomy ([18]), $\mathfrak{B}$ is stable.

Rørdam proves in [16, Proposition 4.6] the following:

Proposition 1.1. *Let $e\colon \mathfrak{B} \hookrightarrow \mathfrak{E} \twoheadrightarrow \mathfrak{A}$ be an object in $\mathscr{E}_K$. Then*

(i) *$\mathscr{E}$ is unital if and only if $\mathfrak{A}$ is unital and the Busby map $\tau\colon \mathfrak{A} \to \mathcal{Q}(\mathfrak{B})$ is unital.*

(ii) *$\mathscr{E}$ is stable if and only if $\mathfrak{A}$ is non-unital (i.e. $\mathfrak{A}$ is stable).*

(iii) *$\mathscr{E}$ is neither unital nor stable if and only if $\mathfrak{A}$ is unital but the Busby map $\tau\colon \mathfrak{A} \to \mathcal{Q}(\mathfrak{B})$ is not unital.*

The purpose of this paper is to look at some functor F in each of these cases, and look at the following questions (here, we are using the language promoted by Elliott in [6]):

1. Is F a *classification functor*, i.e. do we have that $\mathsf{F}(e) \cong \mathsf{F}(e')$ implies $e \cong e'$ in $\mathscr{E}_K$ (for all extensions e and e' in the class considered)?
2. Is F a *strong classification functor*, i.e. does there for each isomorphism $\alpha\colon \mathsf{F}(e) \to \mathsf{F}(e')$ exist an isomorphism $\Phi\colon e \to e'$ such that $\mathsf{F}(\Phi) = \alpha$ (for all extensions e and e' in the class considered)? Clearly, this implies 1.
3. What is the range of the invariant F (in this context, this is only interesting when the answer to 1. is positive)?

2. Main results

The main theorems of this paper will be stated in this section. The proofs are in Sections 4, 5, and 6, while a few results needed in the proofs are in Section 3.

In [4, Theorem 11] there is a rather general metatheorem, which – in certain cases – allows us to deduce from a strong classification functor on stable algebras a classification functor on the unital algebras. With some mild extra conditions, we will prove that this is in fact a strong classification functor on the unital algebras:

THEOREM 2.1. *Let $\mathscr{C}$ be a subcategory of the category of C^*-algebras which is closed under tensoring by* $\mathrm{Mat}_2(\mathsf{C})$ *and $\mathcal{K}$ and contains the canonical embeddings $\kappa_1\colon \mathfrak{A} \to \mathrm{Mat}_2(\mathfrak{A})$ and $\kappa\colon \mathfrak{A} \to \mathfrak{A} \otimes \mathcal{K}$ as morphisms for every C^*-algebra $\mathfrak{A}$ in $\mathscr{C}$. Assume that there is a functor $\mathsf{F}\colon \mathscr{C} \to \mathscr{D}$ satisfying*

- *For every C^*-algebra $\mathfrak{A}$ in $\mathscr{C}$, the embeddings $\kappa_1\colon \mathfrak{A} \to \mathrm{Mat}_2(\mathfrak{A})$ and $\kappa\colon \mathfrak{A} \to \mathfrak{A} \otimes \mathcal{K}$ induce isomorphisms $\mathsf{F}(\kappa_1)$ and $\mathsf{F}(\kappa)$.*
- *For all stable C^*-algebras $\mathfrak{A}$ and $\mathfrak{B}$ in $\mathscr{C}$, every isomorphism from $\mathsf{F}(\mathfrak{A})$ to $\mathsf{F}(\mathfrak{B})$ is induced by an isomorphism from $\mathfrak{A}$ to $\mathfrak{B}$.*
- *There exists a functor G from $\mathscr{D}$ to the category of abelian groups such that $\mathsf{G} \circ \mathsf{F} = K_0$*

Then the following holds

(i) *Assume that $\mathfrak{A}$ and $\mathfrak{B}$ are unital, properly infinite, separable algebras in $\mathscr{C}$, and that there is an isomorphism $\alpha\colon \mathsf{F}(\mathfrak{A}) \to \mathsf{F}(\mathfrak{B})$, such that $\mathsf{G}(\alpha)([\mathbf{1}_{\mathfrak{A}}]_0) = [\mathbf{1}_{\mathfrak{B}}]_0$. Then $\mathfrak{A}$ and $\mathfrak{B}$ are *-isomorphic.*

(ii) *In addition to* (i) *assume that every *-isomorphism between algebras in $\mathscr{C}$ is a morphism in $\mathscr{C}$, and that for every C^*-algebra $\mathfrak{C}$ in $\mathscr{C}$, $\mathsf{F}(\mathrm{Ad}\, u|_{\mathfrak{C}}) = \mathrm{id}_{\mathsf{F}(\mathfrak{C})}$ for every unitary u in $\mathcal{M}(\mathfrak{C})$. Then there exists an isomorphism $\phi\colon \mathfrak{A} \to \mathfrak{B}$ in $\mathscr{C}$ such that $\mathsf{F}(\phi) = \alpha$.*

If $\mathfrak{A} \otimes \mathcal{K}$ and $\mathfrak{B} \otimes \mathcal{K}$ have the cancellation property, then we can omit the assumption on properly infiniteness and separability in (i) *and* (ii).

Rørdam proved in [16, Theorem 5.3] that $K_{\bigcirc}$ is a classification functor for stable extensions in $\mathscr{E}_K$. Using Bonkat's thesis [2] and Kirchberg's isomorphism theorem for ideal filtrated KK-theory (see e.g. [9]), it was proven in [4] that $K_{\bigcirc}$ is in fact a strong classification functor for stable extensions in $\mathscr{E}_K$.

THEOREM 2.2. *The functor $K_{\bigcirc}$ restricted to the stable extensions in $\mathscr{E}_K$ is a strong classification functor.*

Moreover, Rørdam also characterized the range in this case ([16, Proposition 5.4]). We have added the (almost) trivial fact, that the extensions can be chosen to be essential.

THEOREM 2.3. *The range of $K_{\bigcirc}$ restricted to the stable extensions in $\mathscr{E}_K$ is all the objects in $\mathscr{H}$.*

With this version of the metatheorem (Theorem 2.1), we are able to prove that the classification functor in [4, Corollary 12] is in fact a strong classification functor.

THEOREM 2.4. *The functor $(e\colon \mathfrak{B} \hookrightarrow \mathfrak{E} \twoheadrightarrow \mathfrak{A}) \mapsto (K_{\bigcirc}(e), [\mathbf{1}_{\mathfrak{E}}]_0)$ restricted to the unital extensions in $\mathscr{E}_K$ is a strong classification functor.*

Using Rørdam's range result in the stable case, we characterize the range in the unital case.

THEOREM 2.5. *The range of the functor* $(e\colon \mathfrak{B} \hookrightarrow \mathfrak{E} \twoheadrightarrow \mathfrak{A}) \mapsto (K_{\circ}(e), [\mathbf{1}_{\mathfrak{E}}]_0)$ *restricted to the unital extensions in* $\mathscr{E}_K$ *is all the objects* $(M_n, \partial_n)_{n\in\mathsf{Z}_6}$ *in* $\mathscr{H}$ *together with one distinguished element* $m_1 \in M_1$.

Using the methods invented by Rørdam in [16] and a more recent result of Elliott and Kucerovsky [7], we are able to arrive at a classification functor in the non-stable, non-unital case. There is no obvious way to deduce from our proof that this is a strong classification functor (though, we believe that this is the case).

THEOREM 2.6. *The functor* $(e\colon \mathfrak{B} \hookrightarrow \mathfrak{E} \twoheadrightarrow \mathfrak{A}) \mapsto (K_{\circ}(e), [\mathbf{1}_{\mathfrak{A}}]_0)$ *restricted to the non-stable, non-unital extensions in* $\mathscr{E}_K$ *is a classification functor.*

Using Rørdam's range result in the stable case, we are also able to characterize the range in the non-stable, non-unital case.

THEOREM 2.7. *The range of the functor* $(e\colon \mathfrak{B} \hookrightarrow \mathfrak{E} \twoheadrightarrow \mathfrak{A}) \mapsto (K_{\circ}(e), [\mathbf{1}_{\mathfrak{A}}]_0)$ *restricted to the non-stable, non-unital extensions in* $\mathscr{E}_K$ *is all the objects* $(M_n, \partial_n)_{n\in\mathsf{Z}_6}$ *in* $\mathscr{H}$ *together with one distinguished element* $m_2 \in M_2$.

QUESTION 1. Is the functor $(e\colon \mathfrak{B} \hookrightarrow \mathfrak{E} \twoheadrightarrow \mathfrak{A}) \mapsto (K_{\circ}(e), [\mathbf{1}_{\mathfrak{A}}]_0)$ in the preceding theorem a strong classification functor?

QUESTION 2. To what extent do we have lifting of homomorphisms?

QUESTION 3. Is it possible to prove a uniqueness theorem of automorphisms, isomorphisms, or homomorphisms (by including some sort of K-theory with coefficients)?

While this paper gives a (strong) classification result for the purely infinite Cuntz-Krieger algebras with exactly one ideal, the classification result is unknown for general Cuntz-Krieger algebras. In [14] the purely infinite Cuntz-Krieger algebras are classified up to stable isomorphism by an invariant naturally extending $K_{\circ}$.

QUESTION 4. Is the functor given in [14] a strong classification functor (of the stabilized Cuntz-Krieger algebras)?

With a positive answer to this question, we would get a strong classification result for Cuntz-Krieger algebras from Theorem 2.1.

3. Prerequisites

In this section we will state some results and prove some lemmas we will need in the proofs of the main theorems.

REMARK 3.1. Recall that the multiplier algebra $\mathcal{M}(\mathfrak{A})$ of a C^*-algebra $\mathfrak{A}$ is the largest unital C^*-algebra which contains $\mathfrak{A}$ as an essential ideal, i.e. if $\mathfrak{A}$ is embedded as an essential ideal in a unital C^*-algebra $\mathfrak{B}$, then the embedding $\mathfrak{A} \hookrightarrow \mathcal{M}(\mathfrak{A})$ can be uniquely extended to an (injective) *-homomorphism $\mathfrak{B} \to \mathcal{M}(\mathfrak{A})$ – moreover, this embedding is unital.

If $\mathfrak{A}$ and $\mathfrak{B}$ are C^*-algebras, then we have canonical embeddings $\mathfrak{A} \otimes_* \mathfrak{B} \subseteq \mathcal{M}(\mathfrak{A}) \otimes_* \mathcal{M}(\mathfrak{B}) \subseteq \mathcal{M}(\mathfrak{A} \otimes_* \mathfrak{B})$ (see e.g. [13, Lemma 11.12]). By the above comments, the latter embedding is unital.

We call a functor F on the category of C^*-algebras *stable* if $\mathsf{F}(\kappa)$ is an isomorphism whenever $\kappa\colon \mathfrak{A} \to \mathfrak{A} \otimes \mathcal{K}$ is the canonical embedding. The next lemma is well known (follows easily from [1, Proposition 12.2.2]).

LEMMA 3.2. *Let $\mathfrak{A}$ be a C^*-algebra, let u be a unitary in $\mathcal{M}(\mathfrak{A})$, and let F be a stable, homotopy invariant functor on the category of C^*-algebras (e.g. K_0 or K_1). Then* $\mathsf{F}(\operatorname{Ad} u|_{\mathfrak{A}}) = \mathrm{id}_{\mathsf{F}(\mathfrak{A})}$.

We say that a sub-C^*-algebra $\mathfrak{B}$ of a C^*-algebra $\mathfrak{A}$ is *full* if the ideal generated by $\mathfrak{B}$ is $\mathfrak{A}$. We say that a projection p in $\mathcal{M}(\mathfrak{A})$ is *full* if the hereditary corner $p\mathfrak{A}p$ is a full sub-C^*-algebra of $\mathfrak{A}$. L. G. Brown proved the corollary below for the contravariant functor $\operatorname{Ext}(-)$ ([3, Corollary 2.10]). Using the previous lemma in Brown's proof, we get the analogous result for K-theory:

COROLLARY 3.3. *Let $\mathfrak{A}$ and $\mathfrak{B}$ be σ-unital C^*-algebras, and assume that $\mathfrak{B}$ is a full hereditary subalgebra of $\mathfrak{A}$. Then the inclusion map $\iota\colon \mathfrak{B} \hookrightarrow \mathfrak{A}$ induces isomorphisms $K_0(\iota)$ and $K_1(\iota)$ in K-theory.*

LEMMA 3.4. *Let $\mathscr{C}$ be a subcategory of the category of C^*-algebras, and let $\mathsf{F}\colon \mathscr{C} \to \mathscr{D}$ be a functor. Assume that*

- *For every $\mathfrak{A}$ in $\mathscr{C}$, the C^*-algebra* $\operatorname{Mat}_2(\mathfrak{A})$ *is an object in $\mathscr{C}$, the canonical embedding $\kappa_1\colon \mathfrak{A} \to$* $\operatorname{Mat}_2(\mathfrak{A})$ *is a morphism in $\mathscr{C}$, and $\mathsf{F}(\kappa_1)$ is an isomorphism.*
- *For every $\mathfrak{A}$ in $\mathscr{C}$ and every unitary u in $\mathcal{M}(\mathfrak{A})$,* $\operatorname{Ad} u|_{\mathfrak{A}}$ *is an automorphism in $\mathscr{C}$ and* $\mathsf{F}(\operatorname{Ad} u|_{\mathfrak{A}}) = \mathrm{id}_{\mathsf{F}(\mathfrak{A})}$.

Let $\mathfrak{A}$ and $\mathfrak{B}$ be C^*-algebras in $\mathscr{C}$, let $\varphi\colon \mathfrak{A} \to \mathfrak{B}$ be a morphism in $\mathscr{C}$, let $v \in \mathcal{M}(\mathfrak{B})$ be a partial isometry satisfying $v^*v\varphi(a) = \varphi(a) = \varphi(a)v^*v$ for all $a \in \mathfrak{A}$, and define $\psi\colon \mathfrak{A} \to \mathfrak{B}$ by $\psi(a) = v\varphi(a)v^*$. Then ψ is a *-homomorphism. If ψ is a morphism in $\mathscr{C}$, then $\mathsf{F}(\varphi) = \mathsf{F}(\psi)$.

PROOF. Clearly, ψ is linear and *-preserving. Also

$$\psi(aa') = v\varphi(aa')v^* = v\varphi(a)v^*v\varphi(a')v^* = \psi(a)\psi(a')$$

for all $a, a' \in \mathfrak{A}$.

Assume ψ is a morphism in $\mathscr{C}$. Let

$$\begin{aligned} u = \begin{pmatrix} v & \mathbf{1} - vv^* \\ \mathbf{1} - v^*v & v^* \end{pmatrix} &\in \operatorname{Mat}_2(\mathcal{M}(\mathfrak{B})) = \operatorname{Mat}_2(\mathsf{C}) \otimes \mathcal{M}(\mathfrak{B}) \\ &\subseteq \mathcal{M}(\operatorname{Mat}_2(\mathsf{C}) \otimes \mathfrak{B}) = \mathcal{M}(\operatorname{Mat}_2(\mathfrak{B})), \end{aligned}$$

and recall from Remark 3.1 that the above inclusion is a unital embedding. Then $u^* = \begin{pmatrix} v^* & \mathbf{1}-v^*v \\ \mathbf{1}-vv^* & v \end{pmatrix}$, and a short calculation shows that u is a unitary in $\mathcal{M}(\operatorname{Mat}_2(\mathfrak{B}))$. Let $\kappa_1 \colon \mathfrak{B} \to \operatorname{Mat}_2(\mathfrak{B})$ be the canonical embedding. Then

$$\operatorname{Ad} u \circ \kappa_1 \circ \varphi(a) = u \begin{pmatrix} \varphi(a) & 0 \\ 0 & 0 \end{pmatrix} u^* = \begin{pmatrix} v\varphi(a)v^* & 0 \\ 0 & 0 \end{pmatrix} = \kappa_1 \circ \psi(a)$$

for all $a \in \mathfrak{A}$. So by the assumption, we have that

$$\begin{aligned} \mathsf{F}(\kappa_1) \circ \mathsf{F}(\psi) &= \mathsf{F}(\kappa_1 \circ \psi) = \mathsf{F}(\operatorname{Ad} u|_{\operatorname{Mat}_2(\mathfrak{B})} \circ \kappa_1 \circ \varphi) \\ &= \mathsf{F}(\operatorname{Ad} u|_{\operatorname{Mat}_2(\mathfrak{B})}) \circ \mathsf{F}(\kappa_1) \circ \mathsf{F}(\varphi) = \mathsf{F}(\kappa_1) \circ \mathsf{F}(\varphi). \end{aligned}$$

Since $\mathsf{F}(\kappa_1)$ is an isomorphism, $\mathsf{F}(\psi) = \mathsf{F}(\varphi)$.

LEMMA 3.5. *Let $\mathfrak{I}$ be a non-trivial ideal in the C^*-algebra $\mathfrak{A}$, and assume that $\mathfrak{H}$ is a hereditary sub-C^*-algebra of $\mathfrak{A}$. Then we have the following commutative diagram:*

$$\begin{array}{ccccccccc} 0 & \longrightarrow & \mathfrak{H} \cap \mathfrak{I} & \hookrightarrow & \mathfrak{H} & \twoheadrightarrow & \mathfrak{H}/(\mathfrak{H} \cap \mathfrak{I}) & \longrightarrow & 0 \\ & & \downarrow & & \downarrow & & \downarrow & & \\ 0 & \longrightarrow & \mathfrak{I} & \hookrightarrow & \mathfrak{A} & \twoheadrightarrow & \mathfrak{A}/\mathfrak{I} & \longrightarrow & 0 \end{array}$$

with exact rows, and the vertical arrows being the inclusions. Moreover, $\mathfrak{H} \cap \mathfrak{I}$ and $\mathfrak{H}/(\mathfrak{H} \cap \mathfrak{I})$ are hereditary sub-C^*-algebras of $\mathfrak{I}$ and $\mathfrak{A}/\mathfrak{I}$, resp.

If, in addition, $\mathfrak{I}$ and $\mathfrak{A}/\mathfrak{I}$ are simple, $\mathfrak{H}$ is a full sub-C^*-algebra of $\mathfrak{A}$, and $\mathfrak{H} \cap \mathfrak{I}$ is non-zero, then $\mathfrak{H} \cap \mathfrak{I}$ and $\mathfrak{H}/(\mathfrak{H} \cap \mathfrak{I})$ are full hereditary sub-C^*-algebras of $\mathfrak{I}$ and $\mathfrak{A}/\mathfrak{I}$, resp.

PROOF. The short, straightforward proof is left to the reader.

4. The classification theorem – the unital case

In this section we will extend the proofs in [4] to prove the metatheorem, Theorem 2.1, and the classification result in the unital case, Theorem 2.4.

PROOF OF THEOREM 2.1. The first part is proved in [4, Theorem 11]. So assume that $\mathfrak{A}$, $\mathfrak{B}$, and α are as in the theorem. Assume furthermore that for every C^*-algebra $\mathfrak{C}$ in $\mathscr{C}$, we have $\mathsf{F}(\operatorname{Ad} u|_{\mathfrak{C}}) = \mathrm{id}_{\mathsf{F}(\mathfrak{C})}$ for all u in $\mathcal{M}(\mathfrak{C})$.

In the proof of [4, Theorem 11] we found a *-isomorphism $\phi\colon \mathfrak{A}\otimes\mathcal{K} \to \mathfrak{B}\otimes\mathcal{K}$ such that $\mathsf{F}(\phi) = \mathsf{F}(\kappa')\circ\alpha\circ\mathsf{F}(\kappa)^{-1}$, where $\kappa\colon \mathfrak{A} \to \mathfrak{A}\otimes\mathcal{K}$ and $\kappa'\colon \mathfrak{B} \to \mathfrak{B}\otimes\mathcal{K}$ are the canonical embeddings (corresponding to the minimal projection e_{11}). We found a partial isometry $v \in \mathcal{M}(\mathfrak{B} \otimes \mathcal{K})$ such that $v^*v = \mathbf{1}_{\mathcal{M}(\mathfrak{B}\otimes\mathcal{K})}$ and $vv^* = \mathbf{1}_{\mathcal{M}(\mathfrak{B})} \otimes e_{11}$, and we showed that $\psi(x) = v(\phi \circ \kappa)(x)v^*$ for $x \in \mathfrak{A}$, is a *-isomorphism of $\mathfrak{A}$ onto $\mathfrak{B}\otimes e_{11} \cong \mathfrak{B}$. So there exists a unique *-isomorphism $\psi_0\colon \mathfrak{A} \to \mathfrak{B}$ such that $\psi = \kappa' \circ \psi_0$. We claim that $\mathsf{F}(\psi_0) = \alpha$.

By Lemma 3.4

$$\mathsf{F}(\kappa') \circ \mathsf{F}(\psi_0) = \mathsf{F}(\psi) = \mathsf{F}(\phi \circ \kappa) = \mathsf{F}(\kappa') \circ \alpha.$$

Since $\mathsf{F}(\kappa')$ is an isomorphism, it follows that $\mathsf{F}(\psi_0) = \alpha$.

PROOF OF THEOREM 2.4. This is a direct consequence of Lemma 3.2 and Theorem 2.1. We only need to prove that $K_{\bigcirc}(\operatorname{Ad} u|_{\mathfrak{E}}) = \mathrm{id}_{K_{\bigcirc}(\mathfrak{E})}$ for every extension $e\colon \mathfrak{B} \hookrightarrow \mathfrak{E} \twoheadrightarrow \mathfrak{A}$ in $\mathscr{E}_K$ and every unitary $u \in \mathcal{M}(\mathfrak{E})$. Since $\mathfrak{B}$ is an essential ideal in $\mathcal{M}(\mathfrak{E})$, we have that $\mathcal{M}(\mathfrak{E}) \subseteq \mathcal{M}(\mathfrak{B})$ (and, just as in Remark 3.1, this embedding is unital). The quotient map from $\mathfrak{E}$ to $\mathfrak{A}$ can be extended to a surjective *-homomorphism $\mathcal{M}(\mathfrak{E}) \to \mathcal{M}(\mathfrak{A})$ by [12, Proposition 3.12.10], which of course is unital. Now Lemma 3.2 directly implies that $K_{\bigcirc}(\operatorname{Ad} u|_{\mathfrak{E}}) = \mathrm{id}_{K_{\bigcirc}(\mathfrak{E})}$ (in the above settings).

5. The range results

In this section we prove the (slight) improvement of Rørdam's range result in the stable case, Theorem 2.3. Using this result, we prove the range results in the two other cases, Theorems 2.5 and 2.7.

PROOF OF EXTRA ASSERTION IN THEOREM 2.3. In [16, Proposition 5.4] Rørdam proves everything except that the extension can be chosen to be essential. Since the functors K_0 and K_1 are split exact, $K_{\bigcirc}(e)$ degenerates into two split exact sequences for every trivial extension e in $\mathscr{E}_K$. So if the given sequence in the theorem does not consist of two split exact sequences $0 \to M_{0+i} \to M_{1+i} \to M_{2+i} \to 0$, for $i = 0, 3$, then the extension constructed by Rørdam is necessarily essential.

So assume that $0 \to M_{0+i} \to M_{1+i} \to M_{2+i} \to 0$, for $i = 0, 3$, are split exact sequences. Then there exist stable Kirchberg algebras $\mathfrak{A}$ and $\mathfrak{B}$ in the UCT class with $G_0 \cong K_0(\mathfrak{B})$, $G_2 \cong K_0(\mathfrak{A})$, $G_3 \cong K_1(\mathfrak{B})$, $G_5 \cong K_1(\mathfrak{A})$. Because $\mathfrak{B}$ is stable and $\mathfrak{A}$ is separable, there exists an essential trivial extension $e\colon \mathfrak{B} \hookrightarrow \mathfrak{E} \twoheadrightarrow \mathfrak{A}$ (represent $\mathfrak{A}$ on $\mathcal{B}$, use $\mathfrak{B} \cong \mathfrak{B} \otimes \mathcal{K}$ and $\mathfrak{B} \otimes \mathcal{K} \subseteq \mathcal{M}(\mathfrak{B}) \otimes_* \mathcal{B} \subseteq \mathcal{M}(\mathfrak{B} \otimes \mathcal{K})$ to create an injective *-homomorphism $\tau\colon \mathfrak{A} \to \mathcal{M}(\mathfrak{B})$ with the range not intersecting $\mathfrak{B}$). Because e is trivial, $K_{\bigcirc}(e) \cong (M_n, \partial_n)_{n \in \mathsf{Z}_6}$.

Proof of Theorems 2.5 and 2.7. Let $(M_n, \partial_n)_{n \in \mathsf{Z}_6}$ be an object in $\mathscr{H}$. From Rørdam's range result, Theorem 2.3, we know that there exists an essential stable extension $e\colon \mathfrak{B} \hookrightarrow \mathfrak{E} \twoheadrightarrow \mathfrak{A}$ in $\mathscr{E}_K$, such that we have an isomorphism $\alpha = (\alpha_n)_{n \in \mathsf{Z}_6}\colon K_{\bigcirc}(e) \to (M_n, \partial_n)_{n \in \mathsf{Z}_6}$. For notational convenience, we may assume that $\mathfrak{B}$ is an ideal in $\mathfrak{E}$, and that $\mathfrak{A}$ is the quotient $\mathfrak{E}/\mathfrak{B}$.

If a C^*-algebra has a full, properly infinite projection, then every element of K_0 is of the form $[p]_0$ for a full, properly infinite projection p in the algebra. Since $\mathfrak{A}$ is purely infinite, there exists a full, properly infinite projection q in $\mathfrak{A}$ such that $[q]_0 = 0$ in $K_0(\mathfrak{A})$. Because of [16, Proposition 4.1] we can lift it to a projection $q_0 \in \mathfrak{E}$. By [16, Proposition 4.5] q_0 is full and properly infinite.

Now we prove Theorem 2.5. Let $m_1 \in M_1$ be given. Then there exists a full, properly infinite projection p in $\mathfrak{E}$ such that $\alpha_1([p]_0) = m_1$. Then $p\mathfrak{E}p$ is a full, hereditary sub-C^*-algebra of $\mathfrak{E}$ and $p\mathfrak{E}p \cap \mathfrak{B} = p\mathfrak{B}p$ (note that $p\mathfrak{B}p$ is not unital, because $p \notin \mathfrak{B}$). By Brown's Theorem ([3, Theorem 2.8]) we have $p\mathfrak{E}p \otimes \mathcal{K} \cong \mathfrak{E} \otimes \mathcal{K}$. The functor $- \otimes \mathcal{K}$ preserves the ideal lattice, therefore $p\mathfrak{E}p$ has exactly one non-trivial ideal, say $\mathfrak{I}$. The ideal $\mathfrak{B}$ is the only non-trivial ideal in $\mathfrak{E}$, so by ([11, Theorem 3.2.7]) $\mathfrak{I} = p\mathfrak{E}p \cap \mathfrak{B}$. Consequently, $\mathfrak{I} = p\mathfrak{B}p$. So $p\mathfrak{B}p$ and $p\mathfrak{E}p/p\mathfrak{B}p$ are simple, and from Lemma 3.5 we have the commutative diagram

$$\begin{array}{ccccccccc}
0 & \longrightarrow & p\mathfrak{B}p & \hookrightarrow & p\mathfrak{E}p & \twoheadrightarrow & p\mathfrak{E}p/p\mathfrak{B}p & \longrightarrow & 0 \\
 & & \big\downarrow{\scriptstyle \iota|_{p\mathfrak{B}p}} & & \big\downarrow{\scriptstyle \iota} & & \big\downarrow{\scriptstyle \bar{\iota}} & & \\
0 & \longrightarrow & \mathfrak{B} & \hookrightarrow & \mathfrak{E} & \twoheadrightarrow & \mathfrak{A} & \longrightarrow & 0
\end{array}$$

where the vertical maps are embeddings as full, hereditary sub-C^*-algebras. Let e' denote the extension $e'\colon p\mathfrak{B}p \hookrightarrow p\mathfrak{E}p \twoheadrightarrow p\mathfrak{E}p/p\mathfrak{B}p$.

From Brown's Theorem for K-theory (Corollary 3.3) we get isomorphisms in the following diagram:

$$\begin{array}{ccccccc}
K_i(p\mathfrak{B}p) & \longrightarrow & K_i(p\mathfrak{E}p) & \longrightarrow & K_i(p\mathfrak{E}p/p\mathfrak{B}p) & \longrightarrow & K_{i+1}(p\mathfrak{B}p) \\
{\scriptstyle \cong}\big\downarrow{\scriptstyle K_i(\iota|_{p\mathfrak{B}p})} & & {\scriptstyle \cong}\big\downarrow{\scriptstyle K_i(\iota)} & & {\scriptstyle \cong}\big\downarrow{\scriptstyle K_i(\bar{\iota})} & & {\scriptstyle \cong}\big\downarrow{\scriptstyle K_{i+1}(\iota|_{p\mathfrak{B}p})} \\
K_i(\mathfrak{B}) & \longrightarrow & K_i(\mathfrak{E}) & \longrightarrow & K_i(\mathfrak{A}) & \longrightarrow & K_{i+1}(\mathfrak{B})
\end{array}$$

for $i = 0, 1$. Clearly $\alpha \circ K_{\bigcirc}(\iota)$ is an isomorphism from $K_{\bigcirc}(e')$ to $(M_n, \partial_n)_{n \in \mathsf{Z}_6}$ mapping $[p]_0 \in K_0(p\mathfrak{E}p)$ to $m_1 \in M_1$. Clearly $p\mathfrak{E}p$ is a unital C^*-algebra with unit p. The C^*-algebras $p\mathfrak{B}p$ and $p\mathfrak{E}p/p\mathfrak{B}p$ are separable, nuclear, and belong to the UCT class (by Brown's Theorem ([3, Theorem 2.8]) and [1, 22.3.5(a)]). Also they are purely infinite ([10, Proposition 4.17]). We have already seen that e' is essential, so e' is a unital extension in $\mathscr{E}_K$.

Now we consider the non-stable, non-unital case (Theorem 2.7). Let $m_2 \in M_2$ be given. Then there exists a full, properly infinite projection p in $\mathfrak{A}$ such that $\alpha_2([p]_0) = m_2$. We can lift p to a positive element $x \in \mathfrak{E}$ (e.g. [15, 2.2.10]). It is easy to see that $\overline{x\mathfrak{E}x}/(\overline{x\mathfrak{E}x} \cap \mathfrak{B}) \cong (\overline{x\mathfrak{E}x} + \mathfrak{B})/\mathfrak{B} = p\mathfrak{A}p$. Let $\mathfrak{B}' = \overline{x\mathfrak{E}x} \cap \mathfrak{B}$, $\mathfrak{E}' = \overline{x\mathfrak{E}x}$, and $\mathfrak{A}' = p\mathfrak{A}p$, and let e' denote the extension $e' \colon \mathfrak{B}' \hookrightarrow \mathfrak{E}' \twoheadrightarrow \mathfrak{A}'$.

Clearly $\mathfrak{E}'$ is a full hereditary sub-C^*-algebra of $\mathfrak{E}$ ($x \in \mathfrak{E}'$ and $x \notin \mathfrak{B}$ since $p \neq 0$). Since $\mathfrak{B}$ is essential in $\mathfrak{E}$ and $x \neq 0$, there exists $b \in \mathfrak{B}$ such that $bx \neq 0$. So $xb^*bx = (bx)^*bx \neq 0$, and hence $\mathfrak{B}' \neq \{0\}$. Being hereditary subalgebras of simple C^*-algebras, $\mathfrak{B}'$ and $\mathfrak{A}'$ are simple. By [11, Theorem 3.2.7] we see that $\mathfrak{B}'$ is the only non-trivial ideal in $\mathfrak{E}'$. From Lemma 3.5 we have the commutative diagram

$$\begin{array}{ccccccccc}
0 & \longrightarrow & \mathfrak{B}' & \hookrightarrow & \mathfrak{E}' & \twoheadrightarrow & \mathfrak{A}' & \longrightarrow & 0 \\
 & & \big\downarrow{\scriptstyle \iota|_{\mathfrak{B}'}} & & \big\downarrow{\scriptstyle \iota} & & \big\downarrow{\scriptstyle \bar{\iota}} & & \\
0 & \longrightarrow & \mathfrak{B} & \hookrightarrow & \mathfrak{E} & \twoheadrightarrow & \mathfrak{A} & \longrightarrow & 0
\end{array}$$

where the vertical maps are embeddings as full, hereditary subalgebras.

As above, Brown's Theorem for K-theory shows that $K_{\bigcirc}(\iota)$ is an isomorphism. So $\alpha \circ K_{\bigcirc}(\iota)$ is an isomorphism from $K_{\bigcirc}(e')$ to $(M_n, \partial_n)_{n \in \mathsf{Z}_6}$ mapping $[p]_0 \in K_0(\mathfrak{A}')$ to $m_2 \in M_2$. Clearly $\mathfrak{A}'$ is a unital C^*-algebra with unit p. As above, we see that $\mathfrak{A}'$ and $\mathfrak{B}'$ are Kirchberg algebras from the UCT class. We have already seen that e' is essential, so e' is an extension in $\mathscr{E}_K$. So if e' is non-unital, then we are done.

Assume that e' is unital. Let e_0 denote the direct sum extension $e_0 \colon \mathfrak{B}' \hookrightarrow \mathfrak{B}' \oplus \mathfrak{A}' \twoheadrightarrow \mathfrak{A}'$. Let τ' and τ_0 be the Busby invariant of e' and e_0, resp. Then τ' is unital, but τ_0 is non-unital. Let $\tau'' = \tau' \oplus \tau_0$. Then τ'' is an essential, non-unital extension, and $[\tau''] = [\tau' \oplus \tau_0] = [\tau']$ in $\operatorname{Ext}(\mathfrak{A}', \mathfrak{B}')$. Hence by [16, Proposition 2.1] the cyclic six term exact sequences corresponding to τ'' and τ' are congruent, i.e. there exists an isomorphism which is the identity on the K-theory of the ideal and the quotient. This assures us that – when composed with the isomorphism from $K_{\bigcirc}(e')$ to $(M_n, \partial_n)_{n \in \mathsf{Z}_6}$ – the class of the unit in $p\mathfrak{A}p$, $[p]_0 \in K_0(p\mathfrak{A}p)$, will be mapped onto m_2. By the above remarks, e'' is a non-stable, non-unital extension in $\mathscr{E}_K$.

6. The classification theorem – the non-stable, non-unital case

In this section we prove the classification theorem in the non-stable, non-unital case, Theorem 2.6. First we need to recall some notation and some facts from [16].

All C^*-algebras in this section are assumed to be separable and nuclear – so in particular, we always have $\mathrm{Ext}(\mathfrak{A},\mathfrak{B}) = KK^1(\mathfrak{A},\mathfrak{B})$. If $\mathfrak{B}$ is a stable C^*-algebra and $\mathfrak{A}$ is a C^*-algebra, then we will denote the class of essential extensions $e\colon \mathfrak{B} \hookrightarrow \mathfrak{E} \twoheadrightarrow \mathfrak{A}$ by $\mathcal{E}\mathrm{xt}(\mathfrak{A},\mathfrak{B})$. For each injective *-homomorphism $\varphi\colon \mathfrak{A}_1 \to \mathfrak{A}_2$ and for each (essential) extension $e\colon \mathfrak{B} \hookrightarrow \mathfrak{E} \twoheadrightarrow \mathfrak{A}_2$ in $\mathcal{E}\mathrm{xt}(\mathfrak{A}_2,\mathfrak{B})$, there exists a unique extension $\varphi\cdot e\colon \mathfrak{B} \hookrightarrow \mathfrak{E}' \twoheadrightarrow \mathfrak{A}_1$ in $\mathcal{E}\mathrm{xt}(\mathfrak{A}_1,\mathfrak{B})$, where $\mathfrak{E}'$ is a sub-C^*-algebra of $\mathfrak{E}$, making the following diagram commute:

$$\begin{array}{rccccc} \varphi\cdot e\colon & \mathfrak{B} & \hookrightarrow & \mathfrak{E}' & \twoheadrightarrow & \mathfrak{A}_1 \\ & {\scriptstyle \mathrm{id}_{\mathfrak{B}}}\Big\| & & \Big\downarrow & & \Big\downarrow{\scriptstyle \varphi} \\ e\colon & \mathfrak{B} & \hookrightarrow & \mathfrak{E} & \twoheadrightarrow & \mathfrak{A}_2 \end{array}$$

For each isomorphism $\psi\colon \mathfrak{B}_1 \to \mathfrak{B}_2$ and for each $e\colon \mathfrak{B}_1 \hookrightarrow \mathfrak{E} \twoheadrightarrow \mathfrak{A}$ in $\mathcal{E}\mathrm{xt}(\mathfrak{A},\mathfrak{B}_1)$ there exists a unique extension $e\cdot\psi\colon \mathfrak{B}_2 \hookrightarrow \mathfrak{E} \twoheadrightarrow \mathfrak{A}$ in $\mathcal{E}\mathrm{xt}(\mathfrak{A},\mathfrak{B}_2)$ making the diagram commute:

$$\begin{array}{rccccc} e\colon & \mathfrak{B}_1 & \hookrightarrow & \mathfrak{E} & \twoheadrightarrow & \mathfrak{A} \\ & {\scriptstyle \psi}\Big\downarrow & & {\scriptstyle \mathrm{id}_{\mathfrak{E}}}\Big\| & & {\scriptstyle \mathrm{id}_{\mathfrak{A}}}\Big\| \\ e\cdot\psi\colon & \mathfrak{B}_2 & \hookrightarrow & \mathfrak{E} & \longrightarrow & \mathfrak{A} \end{array}$$

Let $x_{\mathfrak{A},\mathfrak{B}}\colon \mathcal{E}\mathrm{xt}(\mathfrak{A},\mathfrak{B}) \to \mathrm{Ext}(\mathfrak{A},\mathfrak{B})$ be the canonical map, and let $\gamma_0\colon KK(\mathfrak{A},\mathfrak{B}) \to \mathrm{Hom}(K_0(\mathfrak{A}), K_0(\mathfrak{B}))$ denote the map from the UCT.

PROOF OF THEOREM 2.6. Assume that $e_1\colon \mathfrak{B}_1 \hookrightarrow \mathfrak{E}_1 \twoheadrightarrow \mathfrak{A}_1$ and $e_2\colon \mathfrak{B}_2 \hookrightarrow \mathfrak{E}_2 \twoheadrightarrow \mathfrak{A}_2$ are non-stable, non-unital extensions in $\mathscr{E}_K$ and that $(\alpha_n)_{n\in\mathbb{Z}_6}\colon K_{\bigcirc}(e_1) \to K_{\bigcirc}(e_2)$ is an isomorphism satisfying $\alpha_2([\mathbf{1}_{\mathfrak{A}_1}]_0) = [\mathbf{1}_{\mathfrak{A}_2}]_0$.

By the proof of [16, Theorem 3.2], there exist invertible elements $a \in KK(\mathfrak{A}_1,\mathfrak{A}_2)$ and $b \in KK(\mathfrak{B}_1,\mathfrak{B}_2)$ such that $x_{\mathfrak{A}_1,\mathfrak{B}_1}(e_1)\cdot b = a\cdot x_{\mathfrak{A}_2,\mathfrak{B}_2}(e_2)$ in $\mathrm{Ext}(\mathfrak{A}_1,\mathfrak{B}_2)$ and $\gamma_0(a) = \alpha_2$. By Kirchberg-Phillips' classification theorem (see e.g. [17, Theorem 8.4.1]) there exist *-isomorphisms $\varphi\colon \mathfrak{A}_1 \to \mathfrak{A}_2$ and $\psi\colon \mathfrak{B}_1 \to \mathfrak{B}_2$ such that $KK(\varphi) = a$ and $KK(\psi) = b$. Hence, by [16, Proposition 1.1], $x_{\mathfrak{A}_1,\mathfrak{B}_2}(e_1\cdot\psi) = x_{\mathfrak{A}_1,\mathfrak{B}_2}(\varphi\cdot e_2)$.

Since φ and ψ are isomorphisms, e_1 is isomorphic to $e_1\cdot\psi$ and e_2 is isomorphic to $\varphi\cdot e_2$. Note that $e_1\cdot\psi$ and $\varphi\cdot e_2$ are non-unital essential extensions of $\mathfrak{A}_1$ by $\mathfrak{B}_2$. Since $\mathfrak{B}_2$ is a stable, purely infinite, simple C^*-algebra, by [7, Theorem 17], $e_1\cdot\psi$ and $\varphi\cdot e_2$ are purely large. Let τ_1 and τ_2 be the Busby invariant associated to $e_1\cdot\psi$ and $\varphi\cdot e_2$, resp. Then $[\tau_1] = [\tau_2]$ in

$\mathrm{Ext}(\mathfrak{A}_1, \mathfrak{B}_2)$. Since $e_1 \cdot \psi$ and $\varphi \cdot e_2$ are non-unital, purely large, essential extensions and $[\tau_1] = [\tau_2]$ in $\mathrm{Ext}(\mathfrak{A}_1, \mathfrak{B}_2)$, by [7, Corollary 16] there exists a unitary $u \in \mathcal{M}(\mathfrak{B}_2)$ such that

$$\mathrm{Ad}(\pi(u)) \circ \tau_1 = \tau_2. \tag{1}$$

Let $\mathfrak{E}'_i = \pi^{-1}(\tau_i(\mathfrak{A}_1))$, for $i = 1, 2$, where $\pi : \mathcal{M}(\mathfrak{B}_2) \to \mathcal{M}(\mathfrak{B}_2)/\mathfrak{B}_2$ is the quotient map. Then $\mathfrak{E}_1 \cong \mathfrak{E}'_1$ and $\mathfrak{E}_2 \cong \mathfrak{E}'_2$. By Equation (1), $uxu^* \in \mathfrak{E}'_2$ for all $x \in \mathfrak{E}'_1$ and $u^*yu \in \mathfrak{E}'_1$ for all $y \in \mathfrak{E}'_2$. Then $\mathfrak{E}'_1 \ni x \mapsto uxu^* \in \mathfrak{E}'_2$ is a *-isomorphism, so e_1 is isomorphic to e_2.

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