MATH. SCAND. 115 (2014), 303–319

EQUATIONS RELATED TO SUPERDERIVATIONS ON PRIME SUPERALGEBRAS

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Abstract

In this paper we investigate equations related to superderivations on prime superalgebras. We prove the following result. Let $D = D_0 + D_1$ be a nonzero superderivation on a prime associative superalgebra $\mathscr{A}$ satisfying the relations $D_i(x)[D_i(x), x]_s = 0$, $[D_i(x), x]_s D_i(x) = 0$ for all $x \in \mathscr{A}$, $i = 0, 1$. Then one of the following is true: (a) $\mathscr{A}_1 = 0$ and $D(\mathscr{A}_0) \subseteq Z(\mathscr{A})$ or (b) $D(\mathscr{A}_0) = 0$ and $\mathscr{A}$ is commutative or (c) $D^2 = 0$. The research is a generalization of the results in [10] and [4] by using the theory of superalgebras.

1. Introduction and preliminaries

This research is motivated by the work of Vukman [10], where the author considers the relation $[D(x), x]\, D(x) = 0$ on noncommutative prime rings and Banach algebras. We will consider this identity on the field of an associative superalgebra.

Throughout the paper, by an algebra $\mathscr{A}$ we shall mean an algebra over a fixed unital commutative ring Φ and we assume that Φ contains the element $\frac{1}{2}$ (i.e., $1 + 1$ is an invertible element). Recall that a derivation D on $\mathscr{A}$ is a Φ-linear map $D : \mathscr{A} \to \mathscr{A}$ such that $D(xy) = D(x)y + xD(y)$ for all $x, y \in \mathscr{A}$.

Over the last few decades there has been a considerable interest in associative superalgebras, especially concerning their Lie and Jordan structures (for example, we refer the reader to [5], [6] and the references therein). For the sake of the completeness we will write some basic definitions which we will need in our further investigations. Let $\mathscr{A}$ be an associative superalgebra, that is a Z_2-graded associative algebra. This means that there exist Φ-submodules $\mathscr{A}_0$ and $\mathscr{A}_1$ of $\mathscr{A}$ such that $\mathscr{A} = \mathscr{A}_0 \oplus \mathscr{A}_1$ and $\mathscr{A}_0\mathscr{A}_0 \subseteq \mathscr{A}_0$ ($\mathscr{A}_0$ is a subalgebra of $\mathscr{A}$), $\mathscr{A}_0\mathscr{A}_1 \subseteq \mathscr{A}_1$, $\mathscr{A}_1\mathscr{A}_0 \subseteq \mathscr{A}_1$ ($\mathscr{A}_1$ is an $\mathscr{A}_0$-bimodule), and $\mathscr{A}_1\mathscr{A}_1 \subseteq \mathscr{A}_0$. We say that $\mathscr{A}_0$ is the even part and $\mathscr{A}_1$ is the odd part of $\mathscr{A}$. An element $x \in \mathscr{A}_i$, $i = 0$ or $i = 1$, is said to be homogeneous of degree i. In this case we write $|x| = i$. The set of all homogeneous elements of $\mathscr{A}$ will be denoted by $\mathscr{H}(\mathscr{A})$. If $\mathscr{A}_1 = 0$, then $\mathscr{A}$ is called a trivial superalgebra. For $x, y \in \mathscr{H}(\mathscr{A})$

Received 2 September 2012.

we shall write $[x, y]_s = xy - (-1)^{|x||y|} yx$ for the supercommutator of x and y. We say that $\mathscr{A}$ is supercommutative if $[x, y]_s = 0$ for all homogeneous elements $x, y \in \mathscr{A}$. The usual commutator $xy - yx$ of $x, y \in \mathscr{A}$ will be denoted by $[x, y]$. Of course, $[x, y]_s = [x, y]$ if at least one of the elements x and y is homogeneous of degree 0. An ideal $\mathscr{I}$ of $\mathscr{A}$ is said to be graded if $\mathscr{I} = \mathscr{I}_0 \oplus \mathscr{I}_1$, where $\mathscr{I}_0 = \mathscr{I} \cap \mathscr{A}_0$ and $\mathscr{I}_1 = \mathscr{I} \cap \mathscr{A}_1$. An associative superalgebra $\mathscr{A}$ is called prime if the product of any two nonzero graded ideals in $\mathscr{A}$ is nonzero, and is called semiprime if it does not contain nonzero nilpotent graded ideals. A semiprime associative superalgebra $\mathscr{A}$ is also semiprime as an algebra. In this case the even part $\mathscr{A}_0$ is semiprime as well. On the other hand, a prime associative superalgebra $\mathscr{A}$ is not necessarily a prime algebra. In this case either $\mathscr{A}$ is prime as an algebra or $\mathscr{A}_0$ is a prime algebra. Note that a prime superalgebra is also a semiprime superalgebra. We refer the reader to [8] for more details.

A superderivation of degree 0 is a Φ-linear map $D_0 : \mathscr{A} \to \mathscr{A}$ such that $D_0(\mathscr{A}_0) \subseteq \mathscr{A}_0$, $D_0(\mathscr{A}_1) \subseteq \mathscr{A}_1$, and $D_0(xy) = D_0(x)y + xD_0(y)$ for all $x, y \in \mathscr{H}(\mathscr{A})$. This is actually a derivation on $\mathscr{A}$. A superderivation of degree 1 is a Φ-linear map $D_1 : \mathscr{A} \to \mathscr{A}$ such that $D_1(\mathscr{A}_0) \subseteq \mathscr{A}_1$, $D_1(\mathscr{A}_1) \subseteq \mathscr{A}_0$, and $D_1(xy) = D_1(x)y + (-1)^{|x|} x D_1(y)$ for all $x, y \in \mathscr{H}(\mathscr{A})$. A superderivation $D : \mathscr{A} \to \mathscr{A}$ is a sum of superderivations D_0 and D_1.

Let $\mathscr{A}$ be a semiprime associative superalgebra. Since $\mathscr{A}$ is then semiprime as an algebra, one can construct the extended centroid of $\mathscr{A}$. For this construction and basic properties of the extended centroid we refer the reader to [1, Chapter 2]. By $C = C_0 \oplus C_1$ we shall denote the extended centroid of a prime associative superalgebra. The semiprime associative superalgebra is prime if and only if all homogeneous elements of the extended centroid are invertible. We refer the reader to [6] for more details on the extended centroid of a prime associative superalgebra which will be used as the main tool in the research of Theorem 1.

Let us see in some more details the background and the motivation of Theorem 1. Let $\mathscr{R}$ be an associative ring with a center $Z(\mathscr{R})$. A map $F : \mathscr{R} \to \mathscr{R}$ is called centralizing on $\mathscr{R}$ if $[F(x), x] \in Z(\mathscr{R})$ holds for all $x \in \mathscr{R}$. In a special case, when $[F(x), x] = 0$ is fulfilled for all $x \in \mathscr{R}$, a map F is called commuting on $\mathscr{R}$. A classical result of Posner (Posner's second theorem) [9] asserts that the existence of a nonzero derivation $D : \mathscr{R} \to \mathscr{R}$, where $\mathscr{R}$ is a prime ring, which is centralizing on $\mathscr{R}$, forces $\mathscr{R}$ to be commutative. Posner's second theorem in general cannot be proved for semiprime rings. It is also well known and easy to prove that if D is a commuting derivation on a semiprime ring $\mathscr{R}$, then D maps $\mathscr{R}$ into $Z(\mathscr{R})$. From the result of Deng and Bell [2] it follows that if $\mathscr{R}$ is a $n!$-torsion free semiprime ring, where $n > 1$ is some fixed integer and $D : \mathscr{R} \to \mathscr{R}$ a derivation satisfying the relation $[D(x), x^n] = 0$

for all $x \in \mathscr{R}$, then D maps $\mathscr{R}$ into $Z(\mathscr{R})$ (see also [7]). Vukman [11] has proved the following result: Let $m \geq 1, n \geq 1$ be some fixed integers and let $\mathscr{R}$ be a $2mn(m+n-1)!$-torsion free semiprime ring. Suppose that there exist derivations $D, G : \mathscr{R} \to R$ such that $D(x^m)x^n + x^n G(x^m) = 0$ is fulfilled for all $x \in \mathscr{R}$. Then D and G map $\mathscr{R}$ into $Z(\mathscr{R})$. Moreover, $D + G = 0$. This result has been fairly generalized by A. Fošner, M. Fošner, and Vukman [3].

If D is a derivation on a noncommutative prime ring $\mathscr{R}$ such that the map $x \mapsto D(x)^2$ is commuting on $\mathscr{R}$, then one cannot prove in general that $D = 0$, as shows the following example. Let $\mathscr{R}$ be a ring of all 2×2 matrices over the field F and let $D(x) = [a, x], a \notin Z(\mathscr{R})$, be an inner derivation. Then a simple calculation shows that the map $x \mapsto D(x)^2$ is commuting on $\mathscr{R}$, but $D \neq 0$ since $a \notin Z(\mathscr{R})$. Here we are actually investigating the relation $[D(x)^2, x] = 0, x \in \mathscr{R}$, which can be written in the form

$$[D(x), x]D(x) + D(x)[D(x), x] = 0, \qquad x \in \mathscr{R}.$$

And the next natural question is what we can say about the Lie version of the above relation.

In [4] the authors investigated the following identity

$$[[D(x), x], D(x)] = 0, \qquad x \in \mathscr{R}.$$

They proved that if D is a derivation on a 2-torsion free semiprime ring satisfying the above relation for all $x \in \mathscr{R}$, then D maps $\mathscr{R}$ into $Z(\mathscr{R})$. And the aim of the following paper is to investigate the superalgebra version of the above mentioned results. In particular, we will observe superderivations $D = D_0 + D_1$ on an associative superalgebra $\mathscr{A}$ satisfying the relations $D_i(x)[D_i(x), x]_s = 0$ and $[D_i(x), x]_s D_i(x) = 0$ for all $x \in H(\mathscr{A})$, $i = 0, 1$.

Let us write our main result.

THEOREM 1. *Let $\mathscr{A}$ be a prime associative superalgebra and let $D = D_0 + D_1$ be a nonzero superderivation on $\mathscr{A}$. Suppose that D_0 and D_1 satisfy the relations*

$$D_i(x)[D_i(x), x]_s = 0 \qquad \text{and} \qquad [D_i(x), x]_s D_i(x) = 0 \tag{1}$$

for all $x \in H(\mathscr{A})$, $i = 0, 1$. Then one the following statements are true:

(a) $\mathscr{A}_1 = 0$ *and* $D(\mathscr{A}_0) \subseteq Z(\mathscr{A})$ *or*

(b) $D(\mathscr{A}_0) = 0$ *and* $\mathscr{A}$ *is commutative or*

(c) $D^2 = 0$.

Before proving Theorem 1, let us write some basic characteristics of prime associative superalgebras.

LEMMA 1 ([6, Lemma 2.1]). *Let* $\mathscr{A} = \mathscr{A}_0 \oplus \mathscr{A}_1$ *be a prime associative superalgebra.*

(i) *If* $a \in \mathscr{A}$ *is such that* $a\mathscr{A}_1 = 0$ (*or* $\mathscr{A}_1 a = 0$)*, then* $a = 0$ *or* $\mathscr{A}$ *is a trivial superalgebra.*

(ii) *If* $a_1\mathscr{A}_1 a_1 = 0$*, where* $a_1 \in \mathscr{A}_1$*, then* $a_1 = 0$.

(iii) *If* $a_0 \in \mathscr{A}_0$ *and* $a_1 \in \mathscr{A}_1$ *are such that* $a_0\mathscr{A}_i a_1 = a_1\mathscr{A}_i a_0 = 0$*, where* $i = 0$ *or* $i = 1$*, then* $a_0 = 0$ *or* $a_1 = 0$.

(iv) *If* $a \in \mathscr{A}$ *and* $a_1 \in \mathscr{A}_1$ *are such that* $a_1\mathscr{A}_i a = 0$ *(or* $a\mathscr{A}_i a_1 = 0$*), where* $i = 0$ *or* $i = 1$*, then* $a_1\mathscr{A}_0 a_1 = 0$ *or* $a = 0$.

(v) *If* $a \in \mathscr{A}$ *and* $a_0 \in \mathscr{A}_0$ *are such that* $a_0\mathscr{A}_i a = 0$ *(or* $a\mathscr{A}_i a_0 = 0$*), where* $i = 0$ *or* $i = 1$*, then* $a_0\mathscr{A}_1 a_0 = 0$ *or* $a = 0$.

(vi) *If* $a_0 \in \mathscr{A}_0$ *and* $a_1 \in \mathscr{A}_1$ *are such that* $a_0x_1a_1 = a_1x_1a_0$ *for all* $x_1 \in \mathscr{A}_1$*, then* $a_0xa_1 = a_1xa_0$ *for all* $x \in \mathscr{A}$.

(vii) *If* $[\mathscr{A}_0, \mathscr{A}_1] = 0$*, then either* $\mathscr{A}$ *is commutative* (*as an algebra*) *or it is a trivial superalgebra.*

(viii) $\mathscr{A}$ *is supercommutative if and only if* $\mathscr{A}$ *is a trivial superalgebra and commutative* (*as an algebra*).

In the proof of our main theorem we will also need the following lemmas.

LEMMA 2. *Let* $\mathscr{A}$ *be an associative superalgebra and* $a_0 \in \mathscr{A}_0$*,* $b_1 \in \mathscr{A}_1$ *such that* $a_0x_0b_1 + b_1x_0a_0 = 0$ *for all* $x_0 \in \mathscr{A}_0$*. Then* $b_1x_0(a_0z_1b_1 - b_1z_1a_0) = 0$ *and* $(a_0z_1b_1 - b_1z_1a_0)x_0b_1 = 0$ *for all* $x_0 \in \mathscr{A}_0$ *and* $z_1 \in \mathscr{A}_1$.

PROOF. It is easy to see that from $a_0x_0b_1 + b_1x_0a_0 = 0$ we get

$$(-b_1x_0a_0)z_1b_1 = a_0(x_0b_1z_1)b_1 = -b_1(x_0b_1z_1)a_0$$

for every $z_1 \in \mathscr{A}_1$. Hence,

$$b_1x_0(a_0z_1b_1 - b_1z_1a_0) = 0$$

for every $x_0 \in \mathscr{A}_0$ and $z_1 \in \mathscr{A}_1$. Similarly we can show that

$$(a_0z_1b_1 - b_1z_1a_0)x_0b_1 = 0.$$

for every $x_0 \in \mathscr{A}$ and $z_1 \in \mathscr{A}_1$. The proof is completed.

LEMMA 3. *Let* $\mathscr{A}$ *be an associative superalgebra and* $a_1, b_1 \in \mathscr{A}_1$ *such that* $a_1x_0b_1 + b_1x_0a_1 = 0$ *for all* $x_0 \in \mathscr{A}_0$*. Then* $(a_1x_1b_1 - b_1x_1a_1)x_0b_1 = 0$ *for all* $x_0 \in \mathscr{A}_0$ *and* $x_1 \in \mathscr{A}_1$.

PROOF. According to our assumption we have

$$a_1(x_1b_1x_0)b_1 = -b_1x_1(b_1x_0a_1) = b_1x_1(a_1x_0b_1)$$

for all $x_0 \in \mathscr{A}_0$ and $x_1 \in \mathscr{A}_1$. Thus,

$$(a_1x_1b_1 - b_1x_1a_1)x_0b_1 = 0.$$

The proof is completed.

LEMMA 4. *Let $\mathscr{A}$ be a nontrivial prime associative superalgebra and $D : \mathscr{A} \to \mathscr{A}$ a nonzero superderivation of degree 0. Suppose that*

$$D(x_0)\mathscr{A}_1D(x_0) = 0$$

for all $x_0 \in \mathscr{A}_0$. Then either $D(x_0)\mathscr{A}_1D(y_0) = 0$ for all $x_0, y_0 \in \mathscr{A}_0$ or $\mathscr{A}$ is commutative.

PROOF. According to our assumptions we have

$$(D(x_0)\mathscr{A}_1D(y_0))\mathscr{A}_0D(x_0) = 0$$

for all $x_0, y_0 \in \mathscr{A}_0$. Since $D(x_0)x_1D(y_0) = -D(y_0)x_1D(x_0)$ for all $x_0, y_0 \in \mathscr{A}_0$, $x_1 \in \mathscr{A}_1$, we also have

$$D(x_0)\mathscr{A}_0(D(x_0)\mathscr{A}_1D(y_0)) = 0.$$

From the proof of Lemma 1 (iii) we obtain

$$D(x_0)\mathscr{A}_1D(y_0)\mathscr{A}D(x_0)\mathscr{A}_1D(y_0) = 0$$

for all $x_0, y_0 \in \mathscr{A}_0$. The primeness of $\mathscr{A}$ yields $D(x_0)\mathscr{A}_1D(y_0) = 0$ for all $x_0, y_0 \in \mathscr{A}$. Thereby the proof is completed.

2. Proof of the main result

In this section we will prove our main theorem. We will split the proof into two steps. First we will observe the case when D is a superderivation of degree 0. In the second step we will prove Theorem 1 in the case when D is a superderivation of degree 1.

PROPOSITION 1. *Let $\mathscr{A}$ be a prime associative superalgebra and let $D : \mathscr{A} \to \mathscr{A}$ be a nonzero superderivation of degree 0 satisfying the relations (1). Then $\mathscr{A}$ is a trivial superalgebra and $D(\mathscr{A}_0) \subseteq Z(\mathscr{A}_0)$.*

PROOF. Write $x+y+z, x, y, z \in \mathcal{H}(\mathcal{A})$, instead of x in $D(x)[D(x), x]_s = 0$. We arrive at

$$0 = D(x)[D(y), z]_s + D(x)[D(z), y]_s + D(y)[D(x), z]_s + D(y)[D(z), x]_s + D(z)[D(x), y]_s + D(z)[D(y), x]_s. \tag{2}$$

Let $x_0 \in \mathcal{A}_0$. According to our assumptions we have

$$0 = [[D(x_0), x_0]_s, D(x_0)] = [[D(x_0), x_0], D(x_0)]$$

for all $x_0 \in \mathcal{A}_0$. Note that $\mathcal{A}_0$ is a semiprime algebra. Using [4, Theorem 1] we obtain

$$[D(\mathcal{A}_0), \mathcal{A}_0] = 0. \tag{3}$$

Regarding the assumptions we have

$$D(x_1)[x_1, D(x_1)]_s = [x_1, D(x_1)]_s D(x_1) = 0$$

for all $x_1 \in \mathcal{A}_1$. Note that $[x_1, D(x_1)]_s = D(x_1^2)$ and $x_1^2 \in \mathcal{A}_0$. Using (3) it follows that

$$D(x_1)x_0[x_1, D(x_1)]_s = 0$$

and

$$[x_1, D(x_1)]_s x_0 D(x_1) = 0$$

for all $x_0 \in \mathcal{A}_0$, $x_1 \in \mathcal{A}_1$. Using Lemma 1 (iii) we get

$$[x_1, D(x_1)]_s \mathcal{A} D(x_1) = 0.$$

In particular,

$$[x_1, D(x_1)]_s \mathcal{A} [x_1, D(x_1)]_s = 0$$

for all $x_1 \in \mathcal{A}_1$. The primeness of $\mathcal{A}$ yields

$$[D(x_1), x_1]_s = 0$$

for all $x_1 \in \mathcal{A}_1$. Write $x_1 + y_1$, $x_1, y_1 \in \mathcal{A}_1$, instead of x_1 in this relation. It follows that

$$[x_1, D(y_1)]_s + [y_1, D(x_1)]_s = 0$$

for all $x_1, y_1 \in \mathcal{A}_1$. In particular,

$$[x_1 x_0, D(y_1)]_s + [y_1, D(x_1 x_0)]_s = 0$$

for all $x_0 \in \mathscr{A}_0$, $x_1, y_1 \in \mathscr{A}_1$, which implies

$$\begin{aligned} 0 &= x_1[x_0, D(y_1)]_s + [x_1, D(y_1)]_s x_0 \\ &\quad + [y_1, D(x_1)]_s x_0 - D(x_1)[y_1, x_0]_s \\ &\quad + [y_1, x_1]_s D(x_0) - x_1[y_1, D(x_0)]_s \\ &= x_1[x_0, D(y_1)]_s - D(x_1)[y_1, x_0]_s \\ &\quad + [y_1, x_1]_s D(x_0) - x_1[y_1, D(x_0)]_s. \end{aligned}$$

Multiplying the obtained identity by $z_0 \in \mathscr{A}_0$ on the left side we obtain

$$\begin{aligned} 0 &= z_0 x_1[x_0, D(y_1)]_s - z_0 D(x_1)[y_1, x_0]_s \\ &\quad + z_0[y_1, x_1]_s D(x_0) - z_0 x_1[y_1, D(x_0)]_s. \end{aligned}$$

On the other hand we have

$$\begin{aligned} 0 &= z_0 x_1[x_0, D(y_1)]_s - D(z_0 x_1)[y_1, x_0] \\ &\quad + [y_1, z_0 x_1]_s D(x_0) - z_0 x_1[y_1, D(x_0)]_s \end{aligned}$$

for all $x_0, z_0 \in \mathscr{A}_0$, $x_1, y_1 \in \mathscr{A}_1$. Comparing both identities we arrive at

$$\begin{aligned} 0 &= -z_0 D(x_1)[y_1, x_0] + D(z_0 x_1)[y_1, x_0] \\ &\quad + z_0[y_1, x_1]_s D(x_0) - [y_1, z_0 x_1]_s D(x_0) \\ &= D(z_0) x_1[y_1, x_0]_s - [y_1, z_0]_s x_1 D(x_0) \end{aligned}$$

for all $x_0, z_0 \in \mathscr{A}_0$, $x_1, y_1 \in \mathscr{A}_1$. Assume first that $C_1 = 0$. Then from [6, Theorem 3.5 (i)] we get

$$D(z_0) x_1[y_1, x_0]_s = [y_1, z_0]_s x_1 D(x_0) = 0$$

for all $x_0, z_0 \in \mathscr{A}_0$, $x_1, y_1 \in \mathscr{A}_1$. According to Lemma 1 (iii) we have $D(\mathscr{A}_0) = 0$ or $[\mathscr{A}_1, \mathscr{A}_0] = 0$. In the second case we obtain that $\mathscr{A}$ is commutative (as an algebra) or $\mathscr{A}$ is a trivial superalgebra (by Lemma 1 (vii)). If $\mathscr{A}$ is commutative then $[D(x_1), x_1] = 0$. Since $[D(x_1), x_1]_s = 0$ as well, we have $D(x_1)x_1 = 0$ for all $x_1 \in \mathscr{A}_1$. Thus, $D(x_1)\mathscr{A}x_1 = 0$, which yields $D = 0$ or $\mathscr{A}$ is a trivial superalgebra.

If $D(\mathscr{A}_0) = 0$ we have $D(x_1)y_1 + x_1 D(y_1) = 0$ for all $x_1, y_1 \in \mathscr{A}_1$. In particular, $D(D(x_1)y_1 + x_1 D(y_1)) = 0$, which implies $D(x_1)D(y_1) = 0$. Thus, $D(x_1)\mathscr{A}_1 D(y_1) = 0$ and consequently $D = 0$, a contradiction.

Suppose now that $C_1 \neq 0$. Using [6, Lemma 3.5 (ii)] we arrive at

$$D(z_0)x[y_1, x_0] - [y_1, z_0]x D(x_0) = 0 \tag{4}$$

for all $x \in \mathscr{A}$ and $x_0, z_0 \in \mathscr{A}_0$, $x_1, y_1 \in \mathscr{A}_1$. Pick a nonzero $\lambda_1 \in C_1$ and choose an essential graded ideal $\mathscr{I}$ of $\mathscr{A}$ such that $\lambda_1 \mathscr{I} \subseteq \mathscr{A}$. Since $\lambda_1 x_0 \in \mathscr{A}_1$ for every $x_0 \in \mathscr{I}_0$ we have from (4)

$$\begin{aligned} 0 &= D(z_0)x[\lambda_1 x_0, x_0] - [\lambda_1 x_0, z_0]xD(x_0) \\ &= \lambda_1(D(z_0)x[x_0, x_0] - [x_0, z_0]xD(x_0)) \\ &= \lambda_1[x_0, z_0]xD(x_0). \end{aligned}$$

Since $\lambda_1 \neq 0$ it follows that $[x_0, z_0]xD(x_0) = 0$ for all $z_0 \in \mathscr{A}_0$, $x_0 \in \mathscr{I}_0$, $x \in \mathscr{A}$. Note that the primeness of $\mathscr{A}$ bring us $D(\mathscr{I}_0) = 0$ or $[\mathscr{I}_0, \mathscr{A}_0] = 0$. In both cases it is easy to verify that $D(\mathscr{A}_0) = 0$ or $[\mathscr{A}_0, \mathscr{A}_0] = 0$. Since we already proved that the first case can not occur, we have, according to the second case, $[\lambda_1 \mathscr{I}_1, \mathscr{A}_0] = 0$, which yields $[\mathscr{I}_1, \mathscr{A}_0] = 0$. Note that this implies $[\mathscr{A}_1, \mathscr{A}_0] = 0$. Thus, $\mathscr{A}$ is commutative (as an algebra) or $\mathscr{A}$ is a trivial superalgebra. In case of commutativity the same procedure as above shows us that $D = 0$ or $\mathscr{A}_1 = 0$. Again, because D is a nonzero superderivation, the superalgebra $\mathscr{A}$ is trivial. Thereby the proof is completed.

PROPOSITION 2. *Let $\mathscr{A}$ be a prime associative superalgebra and let $D : \mathscr{A} \to \mathscr{A}$ be a nonzero superderivation of degree 1 satisfying the relations* (1). *Then $D(\mathscr{A}_0) = 0$ and $\mathscr{A}$ is commutative, or $D^2 = 0$.*

PROOF. Write $x = x_0 \in \mathscr{A}_0$ and $y = z = y_0 \in \mathscr{A}_0$ in (2). We obtain

$$0 = D(x_0)[D(y_0), y_0]_s + D(y_0)[D(x_0), y_0]_s + D(y_0)[D(y_0), x_0]_s. \tag{5}$$

Multiplying the identity on the right side by $D(y_0)$ we get

$$0 = D(y_0)([D(x_0), y_0]_s + [D(y_0), x_0]_s)D(y_0) \tag{6}$$

for all $x_0, y_0 \in \mathscr{A}_0$ since $[D(y_0), y_0]_s D(y_0) = 0$. In particular,

$$0 = D(y_0)([D(x_0), y_0]_s + [D(y_0), x_0]_s)y_0 D(y_0)^2. \tag{7}$$

If we write $x_0 y_0$ instead of x_0 in the above relation (6), we get

$$\begin{aligned} 0 &= D(y_0)([D(x_0)y_0 + x_0 D(y_0), y_0]_s + [D(y_0), x_0 y_0]_s)D(y_0) \\ &= D(y_0)([D(x_0), y_0]_s y_0 + [x_0, y_0]_s D(y_0) + [D(y_0), x_0]_s y_0)D(y_0). \end{aligned}$$

In particular,

$$0 = D(y_0)([D(x_0), y_0]_s y_0 + [x_0, y_0]_s D(y_0) + [D(y_0), x_0]_s y_0)D(y_0)^2.$$

Using (7) it follows that

$$0 = D(y_0)[x_0, y_0]_s D(y_0)^3 \tag{8}$$

for all $x_0, y_0 \in \mathscr{A}_0$. Let $x_1 \in \mathscr{A}_1$. Note that $x_1 D(y_0) \in \mathscr{A}_0$. Then we have

$$0 = D(y_0)[x_1 D(y_0), y_0]_s D(y_0)^3 = D(y_0)[x_1, y_0]_s D(y_0)^4$$

for all $y_0 \in \mathscr{A}_0$ and $x_1 \in \mathscr{A}_1$. Using this relation together with (8) we arrive at

$$0 = D(y_0)[\mathscr{A}, y_0]_s D(y_0)^4$$

for all $y_0 \in \mathscr{A}_0$. This can be written as

$$0 = (D(y_0)y_0)zD(y_0)^4 - D(y_0)z(y_0 D(y_0)^4) \tag{9}$$

for all $z \in \mathscr{A}$. Analogously we can prove that

$$0 = D(y_0)^4[\mathscr{A}, y_0]_s D(y_0),$$

which can be written as

$$0 = D(y_0)^4 z(y_0 D(y_0)) - (D(y_0)^4 y_0)zD(y_0) \tag{10}$$

for all $z \in \mathscr{A}$.

Case 1. Suppose that $D(y_0)^4 \neq 0$ for some $y_0 \in \mathscr{A}_0$. According to (9) and [6, Theorem 3.3] there exists $0 \neq \lambda_0 \in C_0$ such that

$$D(y_0)y_0 = \lambda_0 D(y_0). \tag{11}$$

In particular we have $D(y_0)^4 y_0 = \lambda_0 D(y_0)^4$. Putting this in (10) we obtain

$$0 = D(y_0)^4 z(y_0 D(y_0) - \lambda_0 D(y_0)),$$

for all $z \in \mathscr{A}$. The primeness of $\mathscr{A}$ implies $D(y_0)^4 = 0$ or $y_0 D(y_0) = \lambda_0 D(y_0)$. Since $D(y_0)^4 \neq 0$ we have $y_0 D(y_0) = \lambda_0 D(y_0)$. By (11) we obtain

$$[D(y_0), y_0]_s = 0.$$

By (5) we get

$$D(y_0)[D(x_0), y_0]_s + D(y_0)[D(y_0), x_0]_s = 0 \tag{12}$$

for all $x_0 \in \mathscr{A}_0$. In particular,

$$\begin{aligned} 0 &= D(y_0)[D(y_0x_0), y_0]_s + D(y_0)[D(y_0), y_0x_0]_s \\ &= D(y_0)[D(y_0)x_0 + y_0 D(x_0), y_0]_s + D(y_0)y_0[D(y_0), x_0]_s \\ &= D(y_0)^2[x_0, y_0]_s + D(y_0)y_0[D(x_0), y_0]_s + D(y_0)y_0[D(y_0), x_0]_s \\ &= D(y_0)^2[x_0, y_0]_s \end{aligned}$$

for all $x_0 \in \mathscr{A}_0$. Therefore,

$$D(y_0)^2 \mathscr{A}_0 [x_0, y_0]_s = 0 \tag{13}$$

for all $x_0 \in \mathscr{A}_0$. Now putting $x = x_1 \in \mathscr{A}_1$ and $y = z = y_0 \in \mathscr{A}_0$ in (2) we obtain

$$D(y_0)[D(x_1), y_0]_s + D(y_0)[D(y_0), x_1]_s = 0. \tag{14}$$

In particular,

$$\begin{aligned} 0 &= D(y_0)[D(y_0 x_1), y_0]_s + D(y_0)[D(y_0), y_0 x_1]_s \\ &= D(y_0)[D(y_0)x_1 + y_0 D(x_1), y_0]_s + D(y_0) y_0 [D(y_0), x_1]_s \\ &= D(y_0)^2 [x_1, y_0]_s + D(y_0) y_0 [D(x_1), y_0]_s + D(y_0) y_0 [D(y_0), x_1]_s \\ &= D(y_0)^2 [x_1, y_0]_s. \end{aligned} \tag{15}$$

It follows that

$$0 = D(y_0)^2 [x_1 x_0, y_0]_s = D(y_0)^2 x_1 [x_0, y_0]_s + D(y_0)^2 [x_1, y_0]_s x_0$$

for all $x_0 \in \mathscr{A}_0$ and $x_1 \in \mathscr{A}_1$. Therefore,

$$D(y_0)^2 \mathscr{A}_1 [x_0, y_0]_s = 0.$$

Using (13) we have

$$D(y_0)^2 \mathscr{A} [x_0, y_0]_s = 0.$$

Consequently, $D(y_0)^2 = 0$ or $[\mathscr{A}_0, y_0]_s = 0$. Since $D(y_0)^4 \neq 0$ it follows that $[\mathscr{A}_0, y_0]_s = 0$. Using (15) we obtain

$$D(y_0)^2 \mathscr{A} [x_1, y_0]_s = 0.$$

Therefore we arrive at

$$[\mathscr{A}_1, y_0]_s = 0,$$

which together with $[\mathscr{A}_0, y_0]_s = 0$ implies

$$[\mathscr{A}, y_0]_s = 0.$$

By (12) and (14) we get

$$D(y_0)[D(y_0), x_1]_s = D(y_0)[D(y_0), x_0]_s = 0$$

for all $x_0 \in \mathscr{A}_0$ and $x_1 \in \mathscr{A}_1$. In particular,

$$0 = D(y_0)[D(y_0), zw]_s = D(y_0) z [D(y_0), w]_s$$

for all $z, w \in \mathscr{H}(\mathscr{A})$. Consequently,

$$[D(y_0), \mathscr{A}]_s = 0.$$

From (2) we arrive at

$$D(y_0)([D(x_1), z_1]_s + [D(z_1), x_1]_s) = 0$$

for all $x_1, z_1 \in \mathscr{A}_1$. It follows that

$$D(y_0)\mathscr{A}([D(x_1), z_1]_s + [D(z_1), x_1]_s) = 0.$$

Hence, $[D(x_1), z_1]_s + [D(z_1), x_1]_s = 0$, which yields

$$[D(y_0 x_1), z_1]_s + [D(z_1), y_0 x_1]_s = 0$$

for all $x_1, z_1 \in \mathscr{A}_1$. Therefore, $D(y_0)[x_1, z_1]_s = 0$ for all $x_1, z_1 \in \mathscr{A}_1$. Since $D(y_0)^4 \neq 0$ we have $[x_1, z_1]_s = 0$, which implies $[\mathscr{A}_0, \mathscr{A}_1] = 0$. It follows that $\mathscr{A}$ is commutative (as an algebra) or $\mathscr{A}$ is a trivial superalgebra. Note that in the first case $\mathscr{A}$ is a trivial superalgebra as well. But if $\mathscr{A}$ is trivial, then $D = 0$, a contradiction.

Case 2. Now assume that $D(y_0)^4 = 0$ for all $y_0 \in \mathscr{A}_0$. A complete linearization of this identity gives us

$$\sum_{\pi \in S_4} D(z_{\pi(1)})D(z_{\pi(2)})D(z_{\pi(3)})D(z_{\pi(4)}) = 0,$$

where $z_1, z_2, z_3, z_4 \in \mathscr{A}_0$. Now let $y_0 = z_1 = z_2 = z_3$ and $x_0 = z_4$. Then we have

$$\begin{aligned} 0 = {} & D(x_0)D(y_0)^3 + D(y_0)D(x_0)D(y_0)^2 \\ & + D(y_0)^2 D(x_0)D(y_0) + D(y_0)^3 D(x_0). \end{aligned} \tag{16}$$

Write $x_0 y_0$ instead of x_0 in the above relation. Then

$$\begin{aligned} 0 = {} & D(x_0)y_0 D(y_0)^3 + D(y_0)D(x_0)y_0 D(y_0)^2 + D(y_0)x_0 D(y_0)^3 \\ & + D(y_0)^2 D(x_0)y_0 D(y_0) + D(y_0)^2 x_0 D(y_0)^2 + D(y_0)^3 x_0 D(y_0) \\ & + D(y_0)^3 D(x_0)y_0. \end{aligned}$$

In particular,

$$\begin{aligned} 0 = {} & D(x_0)D(y_0)^4 y_0 + D(y_0)D(x_0)D(y_0)^3 y_0 + D(y_0)x_0 D(y_0)^4 \\ & + D(y_0)^2 D(x_0)D(y_0)^2 y_0 + D(y_0)^2 x_0 D(y_0)^3 + D(y_0)^3 x_0 D(y_0)^2 \\ & + D(y_0)^3 D(x_0)y_0 D(y_0). \end{aligned}$$

Using $D(y_0)^3D(x_0)[y_0, D(y_0)]_s = 0$ by (5) we have

$$0 = (D(x_0)D(y_0)^3 + D(y_0)D(x_0)D(y_0)^2 + D(y_0)^2D(x_0)D(y_0) + D(y_0)^3D(x_0))D(y_0)y_0 + D(y_0)^2x_0D(y_0)^3 + D(y_0)^3x_0D(y_0)^2.$$

From (16) we arrive at

$$0 = D(y_0)^2x_0D(y_0)^3 + D(y_0)^3x_0D(y_0)^2$$

for all $x_0, y_0 \in \mathscr{A}_0$. According to Lemma 2 we have $D(y_0)^3 = 0$ for all $y_0 \in \mathscr{A}_0$ or

$$D(y_0)^2x_1D(y_0)^3 = D(y_0)^3x_1D(y_0)^2$$

for all $y_0 \in \mathscr{A}_0$ and $x_1 \in \mathscr{A}_1$. If the last relation holds true we have

$$D(y_0)^3x_1D(y_0)^3 = D(y_0)^4x_1D(y_0)^2 = 0$$

for all $y_0 \in \mathscr{A}_0$ and $x_1 \in \mathscr{A}_1$. Using Lemma 1 (ii) it follows that $D(y_0)^3 = 0$ for all $y_0 \in \mathscr{A}_0$. A complete linearization of this identity gives us

$$\sum_{\pi\in S_4} D(z_{\pi(1)})D(z_{\pi(2)})D(z_{\pi(3)}) = 0,$$

where $z_1, z_2, z_3 \in \mathscr{A}_0$. Now let $y_0 = z_1 = z_2$ and $x_0 = z_3$. It follows that

$$D(x_0)D(y_0)^2 + D(y_0)D(x_0)D(y_0) + D(y_0)^2D(x_0) = 0 \tag{17}$$

for all $x_0, y_0 \in \mathscr{A}_0$. If we write x_0y_0 instead of x_0 in this relation, we get

$$0 = D(x_0)y_0D(y_0)^2 + D(y_0)D(x_0)y_0D(y_0) + D(y_0)x_0D(y_0)^2 + D(y_0)^2D(x_0)y_0 + D(y_0)^2x_0D(y_0).$$

Using $D(y_0)^2D(x_0)[y_0, D(y_0)]_s = 0$ by (5) it follows that

$$0 = (D(x_0)D(y_0)^2 + D(y_0)D(x_0)D(y_0) + D(y_0)^2D(x_0))D(y_0)y_0 + D(y_0)^2x_0D(y_0)^2.$$

Using (17) we get

$$D(y_0)^2x_0D(y_0)^2 = 0$$

and, by semiprimeness of $\mathscr{A}_0$,

$$D(y_0)^2 = 0$$

for all $y_0 \in \mathscr{A}_0$. Therefore,

$$D(x_0)D(y_0) + D(y_0)D(x_0) = 0 \tag{18}$$

for all $x_0, y_0 \in \mathscr{A}_0$. If we write $x_0 y_0$ instead of x_0 in the above relation, we obtain

$$D(x_0)y_0D(y_0) + D(y_0)D(x_0)y_0 + D(y_0)x_0D(y_0) = 0$$

for all $x_0, y_0 \in \mathscr{A}_0$. Note that from (18) we get

$$D(x_0)y_0D(y_0) - D(x_0)D(y_0)y_0 + D(y_0)x_0D(y_0) = 0$$

for all $x_0, y_0 \in \mathscr{A}_0$. Multiplying this identity on the left side by $D(x_0)$ we arrive at

$$D(x_0)D(y_0)x_0D(y_0) = 0.$$

Again using (18) we obtain

$$D(y_0)D(x_0)x_0D(y_0) = 0$$

for all $x_0, y_0 \in \mathscr{A}_0$. Similarly we can show that

$$D(y_0)x_0D(x_0)D(y_0) = 0.$$

Hence,

$$D(y_0)D(z_0)x_0D(x_0)D(y_0) = 0$$

for all $x_0, y_0, z_0 \in \mathscr{A}_0$. It is easy to verify that

$$\begin{aligned} 0 &= D(y_0)D(z_0)z_0D(x_0)D(y_0) + D(y_0)D(z_0)x_0D(z_0)D(y_0) \\ &= -D(y_0)D(z_0)z_0D(y_0)D(x_0) + D(y_0)D(z_0)x_0D(z_0)D(y_0) \\ &= D(y_0)D(z_0)x_0D(z_0)D(y_0). \end{aligned}$$

Thus,

$$D(y_0)D(z_0)x_0D(y_0)D(z_0) = 0$$

for all $x_0, y_0, z_0 \in \mathscr{A}_0$. Therefore,

$$D(y_0)D(z_0) = 0$$

for all $y_0, z_0 \in \mathscr{A}_0$ by the semiprimeness of $\mathscr{A}_0$. In particular,

$$0 = D(x_0y_0)D(x_0) = D(x_0)y_0D(x_0),$$

which yields

$$D(x_0)\mathscr{A}_0 D(x_0) = 0 \tag{19}$$

for all $x_0 \in \mathscr{A}_0$.

Let $x_0 \in \mathscr{A}_0$ and $y_1, z_1, w_1 \in \mathscr{A}_1$. We have

$$0 = D(y_1z_1)D(x_0) = D(y_1)z_1D(x_0) - y_1D(z_1)D(x_0).$$

Putting $D(y_0) = y_1 \in \mathscr{A}_1$ in the above relation we obtain

$$0 = D(D(y_0))z_1D(x_0) - D(y_0)D(z_1)D(x_0) = D(D(y_0))z_1D(x_0).$$

Analogously we can prove that $D(x_0)z_1D(D(y_0)) = 0$ for all $x_0 \in \mathscr{A}_0$ and $y_1, z_1, w_1 \in \mathscr{A}_1$. Using Lemma 1 (iii) we arrive at $D(\mathscr{A}_0) = 0$ or $D(D(\mathscr{A}_0)) = 0$.

Subcase 2.1. First suppose that $D(\mathscr{A}_0) = 0$. Then we have

$$0 = D(x_1y_1) = D(x_1)y_1 - x_1D(y_1) \tag{20}$$

for all $x_1, y_1 \in \mathscr{A}_1$. Hence,

$$0 = D(x_1)x_0y_1 - x_1x_0D(y_1) \tag{21}$$

for all $x_0 \in \mathscr{A}_0$ and $x_1, y_1 \in \mathscr{A}_1$. In particular, $0 = D(x_1)x_0x_1 - x_1x_0D(x_1)$ for all $x_0 \in \mathscr{A}_0$ and $x_1 \in \mathscr{A}_1$. In the case $C_1 = 0$ we have $0 = D(x_1)x_0x_1 = x_1x_0D(x_1)$ for all $x_0 \in \mathscr{A}_0$ by [6, Theorem 3.5 (i)]. Using Lemma 1 (iii) it follows that $D(\mathscr{A}_1) = 0$ or $\mathscr{A}$ is a trivial superalgebra. In both cases $D = 0$, a contradiction.

Assume that $C_1 \neq 0$. By [6, Theorem 3.5 (ii)] we have $0 = D(x_1)xx_1 - x_1xD(x_1)$ for all $x \in \mathscr{A}$ and $x_1 \in \mathscr{A}_1$. Suppose that $x_1 \neq 0$. Hence, $D(x_1) = \lambda_1x_1$ for some $\lambda_1 \in C_1$. Using (20) we obtain

$$0 = x_1(\lambda_1y_1 - D(y_1)) \tag{22}$$

for all $y_1 \in \mathscr{A}_1$. Note that this yields

$$0 = x_1\mathscr{A}_0(\lambda_1y_1 - D(y_1)).$$

Analogously we can show that

$$0 = (\lambda_1y_1 - D(y_1))\mathscr{A}_0x_1.$$

Therefore, $D(y_1) = \lambda_1y_1$ for all $y_1 \in \mathscr{A}_1$ by Lemma 1 (iii). If we write $x = x_0 \in \mathscr{A}_0$ and $y = z = y_1 \in \mathscr{A}_1$ in (2), we get

$$0 = D(y_1)[D(y_1), x_0]_s = \lambda_1^2 y_1[y_1, x_0]_s.$$

In particular, we have

$$0 = y_1 \mathscr{A}_0 [y_1, x_0]_s.$$

Since $C_1 \neq 0$ we also have

$$0 = y_1 \lambda_1 z_1 [y_1, x_0]_s$$

for all $z_1 \in \mathscr{I}_1$, where $\mathscr{I}$ is some essential ideal of $\mathscr{A}$ such that $\lambda_1 \mathscr{I} \subseteq \mathscr{A}$. Note that this yields

$$0 = y_1 \mathscr{I}_1 [y_1, x_0]_s.$$

Consequently,

$$0 = y_1 \mathscr{I} [y_1, x_0]_s$$

for all $x_0 \in \mathscr{A}_0$ and $y_1 \in \mathscr{A}_1$. The primeness of $\mathscr{A}$ yields $[\mathscr{A}_1, \mathscr{A}_0]_s = 0$ or $\mathscr{A}_1 = 0$. Since $D \neq 0$ the last case can not occur. Therefore, $\mathscr{A}$ is commutative (it can not be trivial, since $D \neq 0$) and we are done in this case.

Subcase 2.2. Suppose that $D(\mathscr{A}_0) \neq 0$ and $D(D(\mathscr{A}_0)) = 0$. Hence,

$$0 = D(D(x_1 y_1)) = D(D(x_1)) y_1 + x_1 D(D(y_1))$$

for all $x_1, y_1 \in \mathscr{A}_1$. Multiplying this relation on the right side by $z_1 D(x_0)$ and using (19) we arrive at $0 = x_1 D(D(y_1)) z_1 D(x_0)$. Hence, $D(D(y_1)) z_1 D(x_0) = 0$ by Lemma 1 (i). Again using (19) it follows that

$$D(D(y_1)) \mathscr{A} D(x_0) = 0.$$

Thus, $D(\mathscr{A}_0) = 0$ or $D(D(\mathscr{A}_1)) = 0$. Since $D(\mathscr{A}_0) \neq 0$ we have $D(D(\mathscr{A})) = 0$, as desired. The proof is completed.

The next example will show that there exist nontrivial noncommutative superalgebras $\mathscr{A}$ and nonzero superderivations $D : \mathscr{A} \to \mathscr{A}$ of degree 1 such that $D(x)[D(x), x]_s = 0$ and $[D(x), x]_s D(x) = 0$ for all $x \in \mathscr{A}$ and $D^2 = 0$.

EXAMPLE 1. Let $\mathscr{A} = M_2(\mathsf{C})$ a prime superalgebra with Z_2-grading

$$\mathscr{A}_0 = \begin{bmatrix} \mathsf{C} & 0 \\ 0 & \mathsf{C} \end{bmatrix} \qquad \text{and} \qquad \mathscr{A}_1 = \begin{bmatrix} 0 & \mathsf{C} \\ \mathsf{C} & 0 \end{bmatrix}.$$

Let us fix $A_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ and define a map $D : \mathscr{A} \to \mathscr{A}$ by

$$D(X) = [A_1, X]_s, \qquad X \in \mathscr{A}.$$

Note that D is a superderivation of degree 1. It is also easy to verify that D satisfies (1) and $D^2 = 0$.

Proof of Theorem 1. Note that D is a sum of a superderivation of degree 0 and a superderivation of degree 1. According to Proposition 1 and Proposition 2 the result follows.

Conjecture. In this paper we deal with prime superalgebras. We do not know if the same result holds true for semiprime superalgebras. Below we give a nontrivial example of a superderivation on a semiprime superalgebra which satisfies the conjectures of our main theorem.

Example 2. Let $p \neq q$ be be two prime numbers, $\mathscr{A} = M_2(\mathsf{Z}_{pq})$, and consider $\mathscr{A}$ as a superalgebra with

$$\mathscr{A}_0 = \begin{bmatrix} \mathsf{Z}_{pq} & 0 \\ 0 & \mathsf{Z}_{pq} \end{bmatrix} \qquad \text{and} \qquad \mathscr{A}_1 = \begin{bmatrix} 0 & \mathsf{Z}_{pq} \\ \mathsf{Z}_{pq} & 0 \end{bmatrix}.$$

Let

$$A_1 = \begin{bmatrix} 0 & p \\ q & 0 \end{bmatrix} \in \mathscr{A}_1.$$

Define a superderivation $D : \mathscr{A} \to \mathscr{A}$ by

$$D(X) = [A_1, X]_s, \qquad X \in H(\mathscr{A}).$$

It is easy to show that

$$D(X)[D(X), X]_s = 0 \qquad \text{and} \qquad [D(X), X]_s D(X) = 0$$

for all $X \in H(\mathscr{A})$. On the other hand, $\mathscr{A}$ is not commutative or a trivial superalgebra.

REFERENCES

1. Beidar, K. I., Martindale III, W. S., and Mikhalev, A. V., *Rings with generalized identities*, Marcel Dekker, New York 1996.
2. Deng, Q., and Bell, H. E., *On derivations and commutativity in semiprime rings*, Comm. Algebra 23 (1995), 3705–3713.
3. Fošner, A., Fošner, M., and Vukman, J., *An identity with derivations on rings and Banach algebras*, Demonstratio Math. 41 (2008), 525–530.
4. Fošner, A., Fošner, M., and Vukman, J., *Identities with derivations in rings*, Glas. Mat. Ser. III 46 (2011), 339–349.
5. Fošner, M., *Jordan superderivations*, Comm. Algebra 31 (2003), 4533–4545.
6. Fošner, M., *On the extended centroid of prime associative superalgebras with applications to superderivations*, Comm. Algebra 32 (2004), 689–705.
7. Lee, T.-K., *Semiprime rings with differential identities*, Bull. Inst. Math. Acad. Sinica 20 (1992), 27–38.
8. Montaner, F., *On the Lie structure of associative superalgebras*, Comm. Algebra 26 (1998), 2337–2349.

9. Posner, E. C., *Derivations in prime rings*, Proc. Amer. Math. Soc. 8 (1957), 1093–1100.
10. Vukman, J., *A resut concerning derivations in noncommutative Banach algebras*, Glas. Mat. Ser. III 26 (1991), 83–88.
11. Vukman, J., *Identities with derivations on rings and Banach algebras*, Glas. Mat. Ser. III 40 (2005), 189–199.

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