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# USING EDGE-INDUCED AND VERTEX-INDUCED SUBHYPERGRAPH POLYNOMIALS

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## Abstract

For a hypergraph $\mathscr{H}$, we consider the edge-induced and vertex-induced subhypergraph polynomials and study their relation. We use this relation to prove that both polynomials are reconstructible, and to prove a theorem relating the Hilbert series of the Stanley-Reisner ring of the independent complex of $\mathscr{H}$ and the edge-induced subhypergraph polynomial. We also consider reconstruction of some algebraic invariants of $\mathscr{H}$.


## 1. Introduction

To every hypergraph $\mathscr{H}$ one can associate several subhypergraph enumerating polynomials. In this note we consider two of these polynomials: the vertex-induced subhypergraph polynomial $P_{\mathscr{H}}(x,y)$ enumerating vertex-induced subhypergraphs of $\mathscr{H}$, and the edge-induced subhypergraph polynomial $S_{\mathscr{H}}(x, y)$. Precise definitions will be given in §2. These and several other polynomials were extensively studied for graphs, see [1], [8], [4], [5] and their citations. The notion has been naturally generalized to hypergraphs, see White [14].

L. Borzacchini, et al. [5] studied the relation between these and other subgraph enumerating polynomials. He earlier proved that both are reconstructible, i.e. they can be derived from the subgraph enumerating polynomials of vertex-deleted subgraphs, see [3], [4]. A. Goodarzi [9] used $S_{\mathscr{H}}(x, y)$ to compute the Hilbert series of the Stanley-Resiner ring of the independent complex of $\mathscr{H}$. More precisely, if $R$ is such a ring, then its Hilbert series $H_R(t)$ is given by

$$H_R(t) = \frac{S_{\mathscr{H}}(t, -1)}{(1-t)^n} \tag{1}$$

where $n$ is the number of vertices in $\mathscr{H}$.

In section 2, we define the polynomials, and then prove that

$$S_{\mathscr{H}}(x, y) = (1-x)^n P_{\mathscr{H}}\left(\frac{x}{1-x}, 1+y\right).$$

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In section 3, we use this relation to give a short and elementary proof of (1). One may compare our proof with the technical proof in [9]. In section 4, generalizing Borzacchini's results [3], [4], we prove that both polynomials are reconstructible for hypergraphs. We also reformulate the reconstruction problems of some algebraic invariants of the independent complex of $\mathscr{H}$, where their graph counterpart is proven by Dalili, Faridi and Traves in [6]. That is, we consider reconstructibility of the Hilbert series, the $f$-vector, the (multi-)graded Betti numbers and some graded Betti tables of the independent complex of $\mathscr{H}$.

## 2. Preliminaries

A hypergraph is a pair $\mathscr{H} = (V, E)$ where $V$ is a set of elements called vertices and $E \subseteq 2^V$ is a set of distinct subsets of $V$ called edges such that for any two edges $\varepsilon_1, \varepsilon_2 \in E$, we have $\varepsilon_1 \subseteq \varepsilon_2 \Rightarrow \varepsilon_1 = \varepsilon_2$. A hypergraph $\mathscr{H}$ is called finite if the vertex set $V$ is finite. We say $\mathscr{H}$ is a $d$-hypergraph if $|\varepsilon| = d$ for each $\varepsilon \in E$, where $|\varepsilon|$ is the cardinality of $\varepsilon$. A graph is a 2-hypergraph. In this note we always consider finite hypergraphs.

Let $\mathscr{H} = (V, E)$ be hypergraph, $W \subseteq V$ and $L \subset E$. We say that $\mathscr{L} = (W, L)$ an *edge-induced subhypergraph* of $\mathscr{H}$ if $W = \cup_{\varepsilon \in L} \varepsilon$. We say that $\mathscr{H}_W = (W, L)$ is *vertex-induced subhypergraph* if $L$ is the largest subset of $E$ such that $L \subseteq 2^W$.

Let $\mathscr{H}$ be a hypergraph. The *edge-induced subhypergraph polynomial* $S_{\mathscr{H}}(x, y)$ is defined by

$$S_{\mathscr{H}}(x, y) = \sum_{i,j} \theta_{ij} x^i y^j, \tag{2}$$

where $\theta_{00} = 1$ and for $i, j \geq 0$, $\theta_{ij}$ is the number of edge-induced subhypergraphs of $\mathscr{H}$ with $i$ vertices and $j$ edges. Similarly, the *vertex-induced subhypergraph polynomial* $P_{\mathscr{H}}(x, y)$ of $\mathscr{H}$ is defined by

$$P_{\mathscr{H}}(x, y) = \sum_{i,j} \beta_{ij} x^i y^j, \tag{3}$$

where $\beta_{00} = 1$ and for $i, j \geq 0$, $\beta_{ij}$ is the number of vertex-induced subhypergraphs of $\mathscr{H}$ with $i$ vertices and $j$ edges.

We recall some simple properties of these polynomials. In what follows, $F_{\mathscr{H}}(x, y)$ refers to any one of the two polynomials.

(1) If the hypergraph $\mathscr{H}$ has connected components $\mathscr{H}_1, \ldots, \mathscr{H}_m$, we have $F_{\mathscr{H}}(x, y) = \prod_{i=1}^{m} F_{\mathscr{H}_i}(x, y)$. We also have $F_{\mathscr{H}}(0, y) = 1$. If $E = \emptyset$, then $F_{\mathscr{H}}(x, y) = (1 + x)^n$.

(2) $\sum_j \beta_{ij} = \binom{n}{i}$ and $\sum_i \theta_{ij} = \binom{m}{j}$ where $m$ is the number of edges in $\mathscr{H}$.

(3) $S_{\mathcal{H}}(x,0)$ is a subgraph polynomial of the 0-subhypergraphs, i.e. isolated vertices. $P_{\mathcal{H}}(x,0)$ the polynomial of the independent subsets, i.e. sets of vertices having no edges in common.

(4) If $\mathcal{H}$ is a $d$-complete hypergraph, then $P_{\mathcal{H}}(x,y) = \sum_{i=0}^{n}\binom{n}{i}x^i y^{\binom{i}{d}}$.

The following proposition is a generalization of Borzacchini [3]. Even though he considered graphs, the proofs can easily be generalized to hypergraphs.

PROPOSITION 2.1. *Let $\mathcal{H}$ be a hypergraph on n vertices. Then*

$$S_{\mathcal{H}}(x,y) = (1-x)^n P_{\mathcal{H}}\left(\frac{x}{1-x}, 1+y\right).$$

PROOF. To every vertex-induced subhypergraph with $i$ vertices and $l$ edges there are $\binom{l}{j}$ hypergraphs with $i$ vertices and $j$ edges. Moreover, those obtained from different vertex-induced subhypergraphs are different since they contain different vertex sets. On the other hand, to every edge-induced subhypergraph with $l$ vertices and $j$ edges we can construct $\binom{n-l}{i-j}$ hypergraphs with $i$ vertices and $j$ edges. So

$$\sum_{l=0}\beta_{i,j+l}\binom{j+l}{j} = \sum_{l=0}^{i}\theta_{i-l,j}\binom{n-(i-l)}{l}. \tag{4}$$

Setting $r = j+l$ and $s = i-l$, substituting this in (4) and multiplying it by $x^i y^j$, we obtain:

$$\sum_{i,j}x^i y^j\left[\sum_{l=0}\beta_{i,j+l}\binom{j+l}{j}\right] = \sum_{i,j}x^i y^j\left[\sum_{l=0}^{i}\theta_{i-l,j}\binom{n-(i-l)}{l}\right],$$

$$\sum_{i,j}x^i y^j\left[\sum_{r}\beta_{ir}\binom{r}{j}\right] = \sum_{s,l,j}x^{s+l}y^j\left[\sum_{l=0}^{i}\theta_{sj}\binom{n-s}{l}\right],$$

$$\sum_{i,r}\beta_{ir}x^i\left[\sum_{j}\binom{r}{j}y^j\right] = \sum_{s,j}\theta_{sj}x^s y^j\left[\sum_{l}x^j\binom{n-s}{l}\right],$$

$$\sum_{i,r}\beta_{ir}x^i(1+y)^r = \sum_{s,j}\theta_{sj}x^s y^j(1+x)^{n-s},$$

$$P_{\mathcal{H}}(x,y+1) = (1+x)^n\sum_{s,j}\theta_{sj}\left(\frac{x}{1+x}\right)^s y^j,$$

$$P_{\mathcal{H}}(x,y+1) = (1+x)^n S_{\mathcal{H}}\left(\frac{x}{1+x}, y\right).$$

By change of variable, we obtain $S_{\mathscr{H}}(x, y) = (1-x)^n P_{\mathscr{H}}\big(\frac{x}{1-x}, 1+y\big)$.

COROLLARY 2.2. *Let $\mathscr{H}$ be a hypergraph on n vertices. Then*

$$P_{\mathscr{H}}(x, y) = (1+x)^n S_{\mathscr{H}}\left(\frac{x}{1+x}, y-1\right).$$

## 3. $P_{\mathscr{H}}(x, y)$ and $S_{\mathscr{H}}(x, y)$ in Algebra

A *simplicial complex* $\Delta$ on a vertex set $V = \{v_1, \dots, v_n\}$ is a set of subsets of $V$, called faces or simplices such that $\{v_i\} \in \Delta$ for each $i$ and every subset of a face is itself a face. If $B \subset V$, the restriction of $\Delta$ to $B$ is a simplicial complex defined by $\Delta(B) = \{\delta \in \Delta \mid \delta \subseteq B\}$. The dimension of a face $\delta \in \Delta$ is $|\delta| - 1$. Let $f_i = f_i(\Delta)$ denote the number of faces of $\Delta$ of dimension $i$. Setting $f_{-1} = 1$, the sequence $f(\Delta) = (f_{-1}, f_0, f_1, \dots, f_{d-1})$ is called the $f$-vector of $\Delta$.

Let $A = \mathsf{K}[x_1, \dots, x_n]$ be a polynomial ring over a field $\mathsf{K}$ and $\Delta$ be a simplicial complex over $n$ vertices $V = \{v_1, \dots, v_n\}$. The Stanley-Reisner ideal of $\Delta$ is the ideal $I(\Delta) \subset A$ generated by those square free monomials $x_{i_1} \cdots x_{i_m}$ where $\{v_{i_1}, \dots, v_{i_m}\} \notin \Delta$.

Let $\mathscr{H} = (V, E)$ be a hypergraph with $n$ vertices $V = \{v_1, \dots, v_n\}$. An independent set of $\mathscr{H}$ is a subset $W \subseteq V$ such that $\varepsilon \nsubseteq W$ for all $\varepsilon \in E$. The collection of $\Delta_{\mathscr{H}}$ of independent sets forms a simplicial complex, called the *independent complex*. Thus the Stanley-Resiner ideal of $\Delta_{\mathscr{H}}$ is the edge ideal of $\mathscr{H}$. More precisely, $I(\Delta_{\mathscr{H}}) = I(\mathscr{H}) \subset A$ is the ideal generated by the squarefree monomials $\prod_{x \in \varepsilon} x$ where $\varepsilon \in E$. Conversely, every squarefree monomial ideal $I \subset A$ can be associated with a hypergraph $\mathscr{H}_I = (V, E)$ where $V = \{v_1, \dots, v_n\}$ and $\varepsilon \in E$ if $\prod_{x_i \in \varepsilon} x_i$ is in the minimal generating set of $I$. So one has $I(\Delta_{\mathscr{H}_I}) = I$. We have the following easy and well known lemma.

LEMMA 3.1. *Let $(f_{-1}, f_0, \dots, f_{d-1})$ be the $f$-vector of the independent complex of a hypergraph $\mathscr{H}$. Then $P_{\mathscr{H}}(t, 0) = \sum_{i=0}^{d} f_{i-1} t^i$.*

Let $R = \oplus_{i \in \mathsf{N}} R_n$ be a finitely generated graded $\mathsf{K}$-algebra, where $R_0 = \mathsf{K}$ is a field. The Hilbert series of $R$ is the generating function defined by $H_R(t) = \sum_{i \in \mathsf{N}} \dim_{\mathsf{K}}(R_i) t^i$. If $I \subset A$ is a monomial ideal, the Hilbert series of the monomial ring $R = A/I$ is the rational function $H_R(t) = \frac{\mathscr{K}(R,t)}{(1-t)^n}$ where $\mathscr{K}(R, t) \in \mathsf{Z}[t]$. P. Renteln [13], and also D. Ferrarello and R. Fröberg [7] used the subgraph induced polynomial $S_G(x, y)$ of a graph $G$ to compute the Hilbert series of the Stanley-Reisner ring $R$ of the independent complex of $G$, namely:

$$H_R(t) = \frac{S_G(t, -1)}{(1-t)^n}.$$

Recently A. Goodarzi [9] generalized it for any squarefree monomial ideal by using the combinatorial Alexander duality and Hochster's formula. Below is a very short and direct proof of this result.

THEOREM 3.2. *Let $\mathscr{H}$ be a hypergraph on $n$ vertices, $I_{\mathscr{H}} \subset A = \mathsf{K}[x_1, \ldots, x_n]$ be its associated squarefree monomial ideal, and $R = A/I_{\mathscr{H}}$. Then*

$$H_R(t) = \frac{S_{\mathscr{H}}(t, -1)}{(1-t)^n}.$$

PROOF. We know by Lemma 3.1 that $P_{\mathscr{H}}(t, 0) = \sum_{i=0}^{d} f_{i-1} x^i$ is the polynomial of the $f$-vectors of the independent complex of $\mathscr{H}$. It follows that by [12, Proposition 51.3] that $H_R(t) = P_{\mathscr{H}}(\frac{t}{1-t}, 0)$ and by Theorem 2.1 we have

$$S_{\mathscr{H}}(t, -1) = (1-t)^n P_{\mathscr{H}}\left(\frac{t}{1-t}, 0\right) = H_R(t)(1-t)^n.$$

REMARK 3.3. Let $\mathscr{H}$ be a hypergraph and $R = A/I_{\mathscr{H}}$. It then follows by Lemma 3.1 and [12, Proposition 51.2] that $P_{\mathscr{H}}(t, 0)$ is the Hilbert polynomial of the algebra $R/(x_1^2, \ldots, x_n^2)$.

## 4. $P_{\mathscr{H}}(x, y)$ and $S_{\mathscr{H}}(x, y)$ in reconstruction conjecture

For a graph $G = (V, E)$ on a vertex set $V = \{v_1, \ldots, v_n\}$, the deck of $G$ is the collection $\mathscr{D}(G) = \{G_1, \ldots, G_n\}$ where $G_l = G - v_l$, $v_l \in V$ is the vertex deleted subgraph of $G$. An element of $\mathscr{D}(G)$ is called a card. The long standing graph reconstruction conjecture posed by Kelly and Ulam says that every simple graph on $n \geq 3$ vertices is uniquely determined, up to isomorphism, by its deck. Numerous unsuccessful attempts have been made to prove the conjecture, and a significant amount of work has been reported. The reader may see Bondy [2] for a survey on the subject. Reconstruction of hypergraphs is defined similarly to graphs. Kocay [10] and Kocay and Lui [11] have constructed a family of non-reconstructible 3-hypergraphs.

In recent years questions has been asked if a graph invariant is reconstructible, that is, if it can be obtained from its deck. Borzacchini in [3], [4] proved that both $S_G(x, y)$ and $P_G(x, y)$ are reconstructible. In fact, he proved that if $F_G(x, y)$ is any one of the subgraph polynomials and $F_{G_l}(x, y)$ is a subgraph polynomial of the card $G_l$, then

$$nF_G(x, y) = x\frac{\partial F_G(x, y)}{\partial x} + \sum_{l=1}^{n} F_{G_l}(x, y). \tag{5}$$

It is natural to extend this reconstructibility question to hypergraphs. Below we obtain a similar result.

PROPOSITION 4.1. *Let $\mathscr{H}$ be a hypergraph on $n \geq 3$ vertices. Then both $S_{\mathscr{H}}(x, y)$ and $P_{\mathscr{H}}(x, y)$ are reconstructible.*

PROOF. We prove the proposition for $S_{\mathscr{H}}(x, y)$ since the other will follow by Proposition 2.1. Let $S_{\mathscr{H}}(x, y) = \sum_{ij} \theta_{ij} x^i y^j$ and $S_{\mathscr{H}_l}(x, y) = \sum_{ij} \theta_{ij}^{(l)} x^i y^j$ for $l = 1, \ldots, n$. By direct calculation we have

$$nS_{\mathscr{H}}(x, y) - x\frac{\partial(S_{\mathscr{H}}(x, y))}{\partial x} = n + \sum_{l=1}^{n} \sum_{ij} (n - j)\theta_{ij} x^i y^j.$$

Now if $j < n$, then any edge-induced subhypergraph with $i$ vertices and $j$ edges is an edge-induced subhypergraph for $n - j$ cards. It follows that $\sum_{l=1}^{n} \theta_{ij}^{(l)} = (n - j)\theta_{ij}$. Putting this in the equation and recalling that $n = \sum_{l=1}^{n} \theta_{00}^{(l)}$ we obtain

$$nS_{\mathscr{H}}(x, y) = x\frac{\partial S_{\mathscr{H}}(x, y)}{\partial x} + \sum_{i=1}^{n} S_{\mathscr{H}_i}(x, y). \tag{6}$$

*4.1. Hilbert series and graded Betti numbers*

The authors in [6] studied reconstructibility of some algebraic invariants of the edge ideal of a graph $G$ such as the Krull dimension, the Hilbert series, and the graded Betti numbers $b_{i,j}$, where $j < n$. All their results can be extended to hypergraphs.

PROPOSITION 4.2. *Let $\mathscr{H}$ be a hypergraph on $n \geq 3$ vertices. The Hilbert function of $R = A/I_{\mathscr{H}}$ is reconstructible. In particular the Krull dimension, the dimension, and the multiplicity of $R$ are reconstructible, as is the $f$-vector of $\Delta_{\mathscr{H}}$.*

PROOF. We only prove that the Hilbert series is reconstructible since the other invariants are obtained from that. By Proposition 3.2 and (6) we have

$$\begin{aligned} nH_R(t) = \frac{nS_{\mathscr{H}}(t, -1)}{(1 - t)^n} &= \frac{t\frac{dS_{\mathscr{H}}(t,-1)}{dt}}{(1 - t)^n} + \sum_{i-1}^{n} \frac{S_{\mathscr{H}_i}(t, -1)}{(1 - t)^n} \\ &= \frac{t}{(1 - t)^n} \frac{dS_{\mathscr{H}}(t, -1)}{dt} + \sum_{i=1}^{n} \frac{H_{R_i}(t)}{1 - t}. \end{aligned}$$

Since $\frac{dH_R(t)}{dt} = \frac{d}{dt}\left(\frac{S_{\mathscr{H}}(t,-1)}{(1-t)^n}\right) = \frac{1}{t}\frac{t}{(1-t)^n}\frac{dS_{\mathscr{H}}(t,-1)}{dt} + \frac{n}{1-x}H_R(t)$, substituting this into the above, we obtain a first order ordinary linear differential equation

$$\frac{n}{1-t}H_R(t) = t\frac{dH_R(t)}{dt} - \frac{1}{1-t}\sum_{i=1}^{n} H_{R_i}(t).$$

For a monomial ideal $I \subset A$ the $\mathsf{Z}^n$-graded minimal free resolution of the $A$-module $R = A/I$ is :

$$\cdots \to \oplus_j A(-\mathbf{b})^{b_{i,\mathbf{b}}} \to \cdots \to \oplus_j A(-\mathbf{b})^{b_{2,\mathbf{b}}} \\ \to \oplus_j A(-\mathbf{b})^{b_{1,\mathbf{b}}} \to A \to A/I \to 0$$

where $\mathbf{b} = (b_1, \ldots, b_n) \in \mathsf{Z}^n$ and the modules $A(-\mathbf{b})$ are the graded shifts of $A$. The numbers $b_{i,\mathbf{b}}$ are *multi-graded Betti numbers* and $b_{ij} = \sum_{|\mathbf{b}|=j} b_{i,\mathbf{b}}$, where $|\mathbf{b}| = b_1 + \cdots + b_n$, are the *graded Betti numbers* of $R$. In particular, the $b_{in}$'s are the *super extremal graded Betti numbers* and they are useful in giving us the regularity and projective dimension of $I_{\mathscr{H}}$. It is well known that the graded Betti numbers are characterstic dependant, so we assume $\mathrm{char}(\mathsf{K}) = 0$.

By Hochester's formula, we can prove that the multi-graded Betti numbers $b_{i,\mathsf{b}}$ are reconstructible for $|\mathsf{b}| < j$, and so will the graded Betti numbers $b_{ij}$ for $j < n$. Reconstruction of the super extremal graded Betti numbers, however, seems a bit hard to determine. Since by Theorem 3.2, we have

$$S_{\mathscr{H}}(t,-1) = \sum_{i=0}^{n}\sum_{j}(-1)^i b_{ij} t^j. \tag{7}$$

It follows that the coefficient of $t^n$ in $S_{\mathscr{H}}(t,-1)$ is the alternating sum $\sum_i (-1)^i b_{in}$. So $b_{in}$ is reconstructible if there is only one $i$ such that $b_{in} \neq 0$. Cohen-macauley ideals or ideals with linear resolutions are examples of such ideals. There are also edge ideals with more than one non-zero super extremal graded Betti numbers, see [6, Example 5.3]. Summerizing, the following extends results in [6, §5] to a hypergraph. The proof is also similar, and hence ommited.

PROPOSITION 4.3. *Let $\mathscr{H}$ be a hypergraph on with a vertex set $V = \{v_1, \ldots, v_n\}$ and $n \geq 3$. Then the (multi-)graded Betti numbers $b_{ij}$ of the Stanley-Reisner ring $R = A/I_{\mathscr{H}}$ are reconstructible for all $j < n$. Moreover, if the super extremal graded Betti numbers $b_{in}$ of $I_{\mathscr{H}}$ are reconstructible, then the depth, projective dimension and regularity of $I_{\mathscr{H}}$ are reconstructible.*

We investigate if the Betti table of $I_{\mathscr{H}}$ is reconstructible. Let $\mathscr{B} = (b_{ij})$ be the Betti table of $I_{\mathscr{H}}$ and $\mathscr{B}_l = (b^{(l)}_{ij})$ be the Betti table of $I_{\mathscr{H}_l}$. Then combining

(6) and (7) and comparing the coefficients of $t^j$ we obtain

$$(n-j)\sum_i (-1)^i b_{ij} = \sum_i (-1)^i \sum_{l=1}^{n} b_{ij}^{(l)} \qquad \text{for} \quad j < n.$$

This equation shows it is difficult to determine each $b_{ij}$ only from the data $\{\mathscr{B}_l\}_{l=1}^n$ since anti-diagonals of $\mathscr{B}$ might contain more than one non-zero entry. We thus have the following which gives a partial answer to [6, Question 5.6].

PROPOSITION 4.4. *Let $\mathscr{H}$ be a hypergraph on $n \geq 3$ vertices. If each anti-diagonal of the Betti table of $I_{\mathscr{H}}$ contains at most one non-zero entry, then the Betti table of $I_{\mathscr{H}}$ is reconstructible.*

PROOF. Let $S_{\mathscr{H}}(t, -1) = \sum_{ij}(-1)^j \theta_{ij} t^i$. If $b_{ij}$ is the non-zero entry on the $j$'th anti-diagonal of the Betti table, using (7) we have $b_{ij} = \sum_k (-1)^{i+k}\theta_{jk}$. The result follows from Proposition 4.1.

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